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INFLUENCE OF CRITICAL CURRENT AND PARAMETERS OF JOSEPHSON JUNCTION ON FREQUENCY STABILITY OF OSCILLATOR

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Background. One of the important characteristics of generators based on Josephson junctions is the power spectral density of phase noises. A number of theoretical and experimental studies showed the spectral shape of generation line is Lorentzian and its width is tens or even hundreds of megahertz. Such form of generation line and its relatively large width complicate usage of JJ in high sensitivity receivers and digital radio engineering devices with frequency-pulse coding of information.

Objective. Problem of investigation the possibility to reduce generation linewidth and increase an output power is of current interest. In this context, the aim of this work is to study the dependence of Josephson junction generation linewidth from its geometrical dimensions and critical current.

Methods. This article was written by means of usage the operator calculation method and mathematical analysis to construct mathematical models and obtain graphical dependencies.

Results. Studies showed the generation linewidth decreases with an increase of Josephson junction width due to reduction in differential resistance of junctions. A magnification of critical current value leads to enlargement of generation linewidth since current of both normal electrons and Cooper pairs grows and this dependence is linear.

Conclusions. The analytical dependence of Josephson junction generation linewidth from its geometrical dimensions and currents of normal electrons and Cooper pairs were first obtained. Comparison of obtained dependencies with experimental results described in other works confirmed an authenticity of developed mathematical models.

Keywords: superconductors; Josephson junctions; generation linewidth; superconducting generators; frequency-pulse coded devices.

Introduction

In the development of digital radio engineering devices with frequency-pulse coding of information generators of auxiliary signals are widely used, frequency of which can exceed the frequency of information signals in several times [1]. Since the benefits of digital devices with frequency-pulse coding of information particularly reveal themselves in the range of ultra-high frequency (UHF), these generators must ensure generation of signals in this range. In addition, the ever-growing needs in expansion the band of operating frequencies and increasing the speed of wireless data transmission systems are increasingly more difficult to provide in traditional technologies with carrier frequencies in the range of 2...20 GHz (Wi-Fi, Wi-Max, 3G, LTE). It encourages researchers to develop wireless data transmission devices in the terahertz frequency range, one of the main components of which are generators [2, 3]. A promising method of constructing such generators is the usage of Josephson junctions (JJ) which can ensure the generation of electromagnetic waves in the range of wavelength from millimeter to infrared and are capable for rearrangement the frequency by change of voltage [3].

One of the important characteristics of generators based on Josephson junctions is the power spectral density of phase noises. A number of theoretical and experimental studies [3, 4, 5] showed the spectral shape of generation line is Lorentzian and its width is tens or even hundreds of megahertz. Such form of generation line and its relatively large width complicate usage of JJ in high sensitivity receivers and digital radio engineering devices with frequency-pulse coding of information. Therefore, problem of investigation the possibility to reduce generation linewidth and increase an output power is of current interest. In this context, the aim of this work is to study the dependence of Josephson junction generation linewidth from its geometrical dimensions and critical current.

Solving the Problem

In general, the generation linewidth is determined by spectral density of low-frequency voltage fluctuations [6]

$$\Delta f = \frac{1}{2}\left(\frac{2e}{\hbar}\right)^2 S_V(0) = \frac{1}{2}\left(\frac{2e}{\hbar}\right)^2 R_d^2 S_1^x(0), \qquad (1)$$

where R_d is a differential resistance at the operating point of the JJ current-voltage characteristic; $S_1(0)$ is an effective spectral density of low-frequency current fluctuations, which in the case of JJ resistive model can be written as

$$S_1^x(0) = S_1(0) + \left(\frac{I_{1_0}^2}{2I^2}\right) S_1(\omega), \tag{2}$$

where $S_1(0)$ and $S_1(\omega)$ are respectively low-frequency and high-frequency current fluctuations spectral densities.

Low-frequency constituent of current fluctuations spectral density can be calculated by the formula [3]

$$S_1(0) = \frac{1}{\pi U}\left(kTI_n + \frac{h\nu}{2}\operatorname{cth}\frac{eU}{kT} I_s\right), \tag{3}$$

where I_n and I_s are respectively normal and superconducting constituents of current; ν is a rate of the magnetic field change.

For junctions based on low-temperature superconductors (LTS), considering that $|eU| >> kT$, high-frequency constituent of the current fluctuations spectral density can be calculated by the formula [7]

$$S_1(\omega) = 2eI_n + 4eI_s. \tag{4}$$

In the case of high-temperature superconductors (HTS) $|eU| << kT$ and formula for high-frequency constituent of the current fluctuations spectral density takes on the form

$$S(\omega) = \frac{2I_n kT}{U} + \frac{4I_s kT}{U}. \tag{5}$$

In view of the above, for junctions based on HTS the dependence of the generation linewidth from current fluctuations spectral density can be determined by substituting (2), (3), (4) in (1):

$$\Delta f = \frac{1}{2}\left(\frac{2e}{\hbar}\right)^2 R_d^2 \left[\frac{1}{\pi U}\left(kTI_n + \frac{h\nu}{2}\operatorname{cth}\frac{eU}{kT} I_s\right) + \frac{I_0^2}{I^2} e\left(I_n + 2I_s\right)\right].$$

In the case of frequent contacts and high voltages, this formula can be rewritten as

$$\Delta f = \frac{1}{4\pi}\left(\frac{2e}{\hbar}\right)^2 R_d^2 \left[e\left(I_n + 2I_s\right) + \frac{I_0^2 e}{I_n + 2I_s}\right]. \tag{6}$$

If HTS is used for the formation of junctions, the generation linewidth can be calculated by substituting (2), (3), (5) in (1):

$$\Delta f = \frac{1}{4\pi}\left(\frac{2e}{\hbar}\right)^2 R_d^2 \left[e\left(I_n + 2I_s\right) + \frac{2I_0^2 kT}{U\left(I_n + 2I_s\right)}\right]. \tag{7}$$

To determine the dynamic resistance of JJ, we will use the formula for the current-voltage characteristics [7]

$$R_d = \frac{R_{sh} \cdot R_n}{R_{sh} + R_n} \frac{I}{\sqrt{I^2 - I_0^2}}, \tag{8}$$

where R_{sh} is a shunt resistance of resistive-shunted JJ.

To calculate the superconducting current component we will use the formula for the stationary Josephson effect

$$I_s = I_0 \sin\varphi.$$

To determine the phase difference between wave functions and current we should consider the equivalent circuit of resistive-shunted JJ shown in Fig. 1.

The difference between the current equivalent circuit from known ones is that it takes into account the inductance of Josephson junction. A value of this inductance depends on critical current on the phase difference of wave functions and can be calculated by the formula

$$L = \frac{n}{2eI_0\cos\varphi}.$$

The value of critical current I_0 depends on the type of superconductor and is determined by the formula [8]

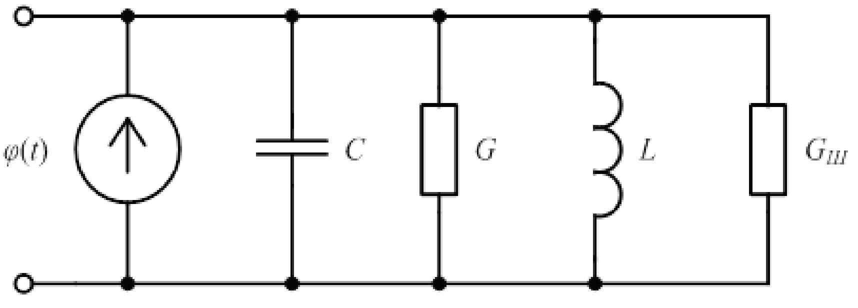

Fig. 1. Equivalent circuit of Josepshon junction

$$I_0 = \frac{2\Delta}{eR_n},$$

where R_n is a resistance of junction in normal non-conductive state; 2Δ is a width of superconductor energy gap.

In general, a phase difference of wave functions on Josephson junction depends on the voltage and it can be calculated by the formula

$$\varphi(t) = \varphi(0) + \frac{2eU}{\hbar}t = \varphi_0 + \omega t.$$

A value of Josephson junction characteristic voltage at which a generation takes place is associated with critical current and resistance in normal non-conductive state by the ratio

$$U_0 = I_0 R_U = \frac{2\Delta}{e}.$$

Value of characteristic voltage for Josephson junctions with LTS is 200-300 mV and for Josephson junctions from oxide superconductors (high-temperature) U_0 reaches 1 mV at nitrogen temperature and several mV's at lower temperatures. When voltage $U > U_0$ a generation of electromagnetic waves in the microwave range takes place the frequency of which is determined by the constant voltage and type of superconductors [8]

$$f_0 = \frac{2eU_0}{\hbar}.$$

Using the equivalent circuit of JJ a current that flows through the junction can be calculate by the formula

$$i(t) = I_0 \sin\varphi(t) + C\frac{dU(t)}{dt} + (G + G_{III})U(t) +$$

$$+L\int_0^{t_0} U(t)dt. \tag{9}$$

Using the known ratio for non-stationary Josephson effect, the value of alternating voltage on junction can be calculated by the formula

$$U(t) = \frac{\hbar}{2e}\cdot\frac{d\varphi(t)}{dt}. \tag{10}$$

In view of (10) the formula (9) can be rewritten as

$$\frac{I}{I_0} = \sin\varphi(t) + \frac{C}{I_0}\frac{\hbar}{2e}\frac{d^2\varphi(t)}{dt^2} + \frac{(G+G_{III})\hbar}{I_0 2e}\frac{d\varphi(t)}{dt} +$$

$$+\frac{\hbar^2}{4e^2 2I_0^2\cos\varphi(t)}\varphi(t). \tag{11}$$

Formula (11) will be used to determine the dependence on phase difference of wave functions from t and other elements of JJ equivalent circuit.

After a simple transformation (10) can be led to form

$$\frac{d^2\varphi(t)}{dt^2} + k\frac{d\varphi(t)}{dt} + \left[l\frac{\varphi(t)}{\cos\varphi(t)} + m\,sin\varphi(t) - n\right] = 0. \tag{12}$$

where $k = \frac{G + G_{III}}{C}$, $l = \frac{\hbar}{2e}\cdot\frac{1}{I_0 C}$, $m = \frac{I_0}{C}\frac{2e}{\hbar}$, $n = \frac{I}{C}\frac{2e}{\hbar}$.

Formula (12) can be written as a Lienard equation which is as follows:

$$\frac{d^2\varphi(t)}{dt^2} + f(\phi)\frac{d\varphi(t)}{dt} + g(\varphi) = 0.$$

For its solution replacement is used and then the differential equation of the 2nd order reduces to the differential equation of the 1st order, the general form of which is as follows:

$$\omega(\varphi)\frac{d\omega(\varphi)}{d\varphi} + f(\varphi)\omega(\varphi) + g(\varphi) = 0.$$

Aforecited formula is Abel equation of the second kind, which is solved analytically only in some cases. Therefore, for solution of equation (11) Runge-Kutta methods will be used.

For obtaining of analytical formula approximation by power polynomial of seventh order was used in our case. Initial data for calculation is the following: $C = 0{,}01\cdot 10^{-12}$ Ф, $G = 5/7$ См, $G_{III} = 7/5$ См, $I = 2\cdot 10^{-3}$ А, $I_0 = 1\cdot 10^{-3}$ А, $T = 2\cdot 10^{-12}$ с, $\varphi_0 = 90^0$ - the initial phase.

Polynomial formula for calculating the phase difference of the wave functions is:

$$\varphi(t) = p_0\cdot t^0 + p_1\cdot t^1 + p_2\cdot t^2 + p_3\cdot t^3 +$$

$$+p_4\cdot t^4 + p_5\cdot t^5 + p_6\cdot t^6 + p_7\cdot t^7.$$

Solution of equation is:

$$\varphi(t) = (-0{,}007911) + 0{,}008742\cdot t + 0{,}053569\cdot t^2 +$$

$$+(-0{,}013860\cdot t^3) + (-0{,}199074\cdot t^4) + (-0{,}606032\cdot t^5) +$$

$$+(-0{,}010983\cdot t^6) + 2{,}323014\cdot t^7.$$

Similar solutions were conducted for the cases when the variables were the parameters of the JJ equivalent circuit elements.

Using this representation for solution of differential equation dependence on phase difference of wave functions from parameters of the JJ equivalent circuit was investigated. The research results are presented in Fig. 2-4.

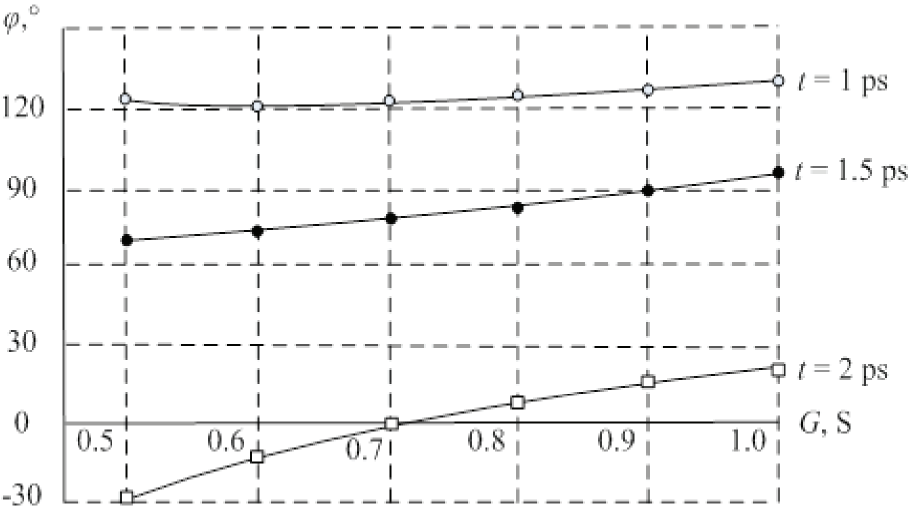

Fig. 2. Dependence on phase difference of wave functions from dynamic resistance of Josepshon junction

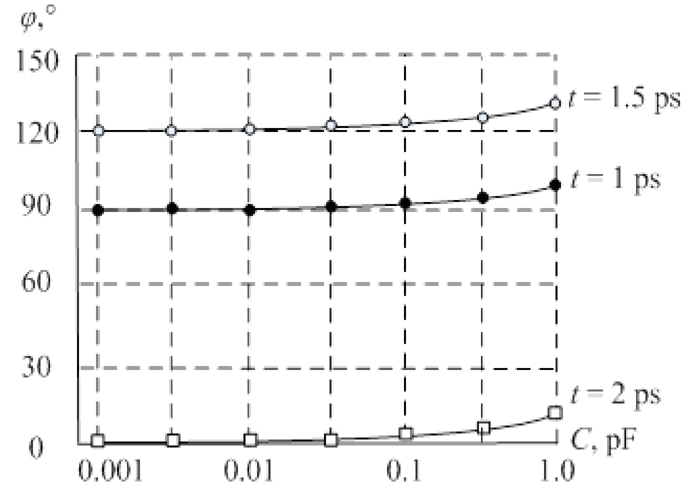

Fig. 3. Dependence on phase difference of wave functions from electrostatic capacitance of Josepshon junction

Analysis of the results shows that change in capacitance till 0.1 pF practically does not affect the phase difference of wave function and change of dynamic resistance leads to a significant change in phase difference of wave functions. Change of critical current also leads to a considerable change in phase difference of wave functions and this reaffirms the necessity to consider inductance of junction while calculating devices based on JJ.

Following data were used in the calculations: current density j_0 = 5÷10 kA/cm^2; width of energy gap 1÷3 meV; length of Josephson junction l = 40 µm; width of Josephson junction a = 5÷20 µm; surface resistance of superconductors in the normal state 0.02 Ohm/□; shunt resistance 0.1÷0.2 Ohm.

To calculate the dependence of generation linewidth from geometric dimensions of junctions, the dependence of normal electrons and Cooper pairs current value from current density and junction area must be considered.

Considering this, (6) and (7) can be rewritten as:

$$\Delta f=\frac{1}{4\pi}\left(\frac{2e}{\hbar}\right)^2\left(\frac{R_{sh}p_{\square}\frac{\ell}{a}}{R_{sh}+p_{\square}\frac{\ell}{a}}\right)^2\frac{\left(j_na\ell+2j_0a\ell\right)^2}{\left(j_na\ell\right)^2+\left(j_0a\ell\right)^2}\times$$

$$\times\left[e\left(j_na\ell+2j_0a\ell\right)+\frac{2ej_0^2a^2\ell^2}{j_na\ell+2j_0a\ell}\right]. \qquad (12)$$

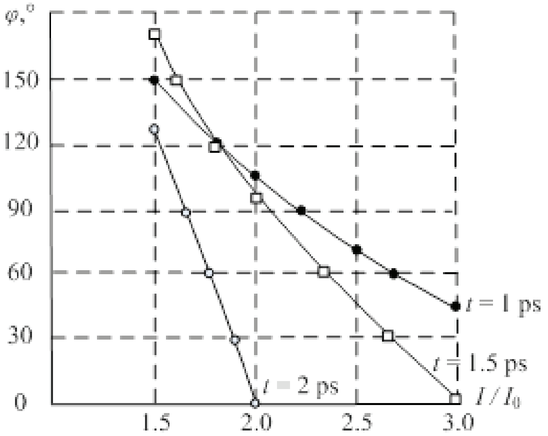

Fig. 4. Dependence on phase difference of wave functions from critical current of Josepshon junction

where $j_0 = (e\mu^2 / 8\pi U^2) \cdot (\Delta_0^2 / T_c)$ is a critical current density of JJ; μ is a chemical potential; Δ_0 is a module of superconductor energy gap parameter; T_c a critical temperature of superconductor; $p_{\square}$ is a surface resistance of superconductor; a and ℓ are width and length of JJ; j_n is a current density of normal electrons.

$$\Delta f = \frac{1}{4\pi}\left(\frac{2e}{\hbar}\right)^2 \left(\frac{R_{sh} p_{\square} \frac{\ell}{a}}{R_{sh} + p_{\square} \frac{\ell}{a}}\right)^2 \frac{\left(j_n a\ell + 2 j_0 a\ell\right)^2}{\left(j_n a\ell\right)^2 + \left(j_0 a\ell\right)^2} \times$$

$$\times\left[e\left(j_n a\ell + 2 j_0 a\ell\right) + \frac{2 e j_0^2 a^2 \ell^2 kT}{U\left(j_n a\ell + 2 j_0 a\ell\right)}\right]. \qquad (13)$$

Analysis (12), (13) shows the generation linewidth depends on dynamic resistance and current value of normal electrons and Cooper pairs. Increasing the width of JJ leads to an increase of current value and these dependences are proportional to junction area and respectively its width. Therefore, increasing the JJ width leads to an increase in generation linewidth. At the same time, a dynamic resistance with increasing the junction area, and so its width, leads to reducing the generation linewidth. Meanwhile, reduction of generation linewidth is inversely proportional to the square of increasing the JJ area, and so its width. Therefore, the generation linewidth must decrease with increase of JJ width.

Graphs on dependence of generation linewidth from critical current of JJ are given in Fig. 5, analysis of which shows an increase in critical current leads to an increase in generation linewidth.

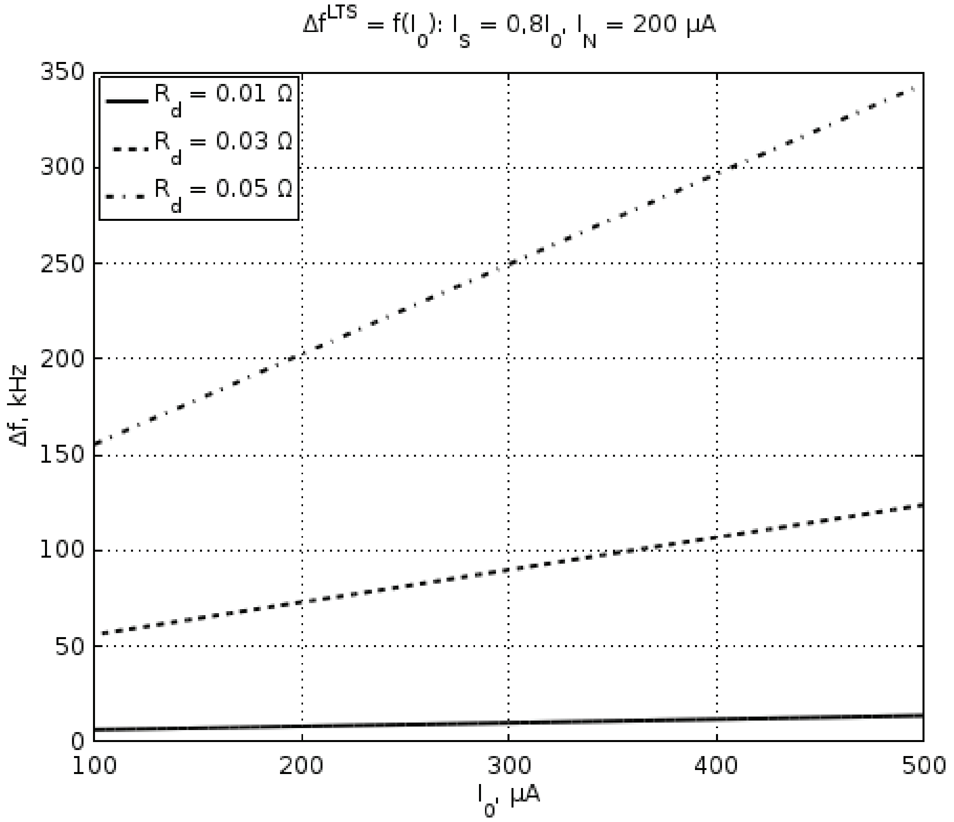

Fig. 5. Dependence of generation linewidth from critical current of JJ

Calculation results for dependence of generation linewidth from the width of JJ are presented in Fig. 6.

In Fig. 7 calculation results for a research of dependence on generation linewidth from constant

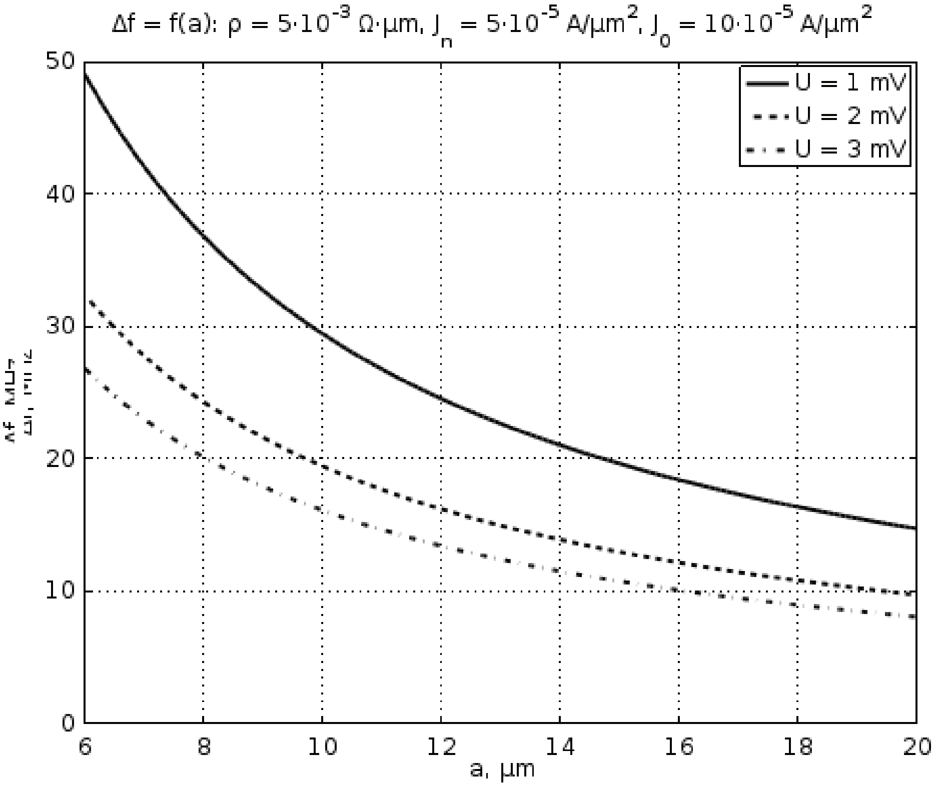

Fig. 6. Dependence of generation linewidth from the width of JJ

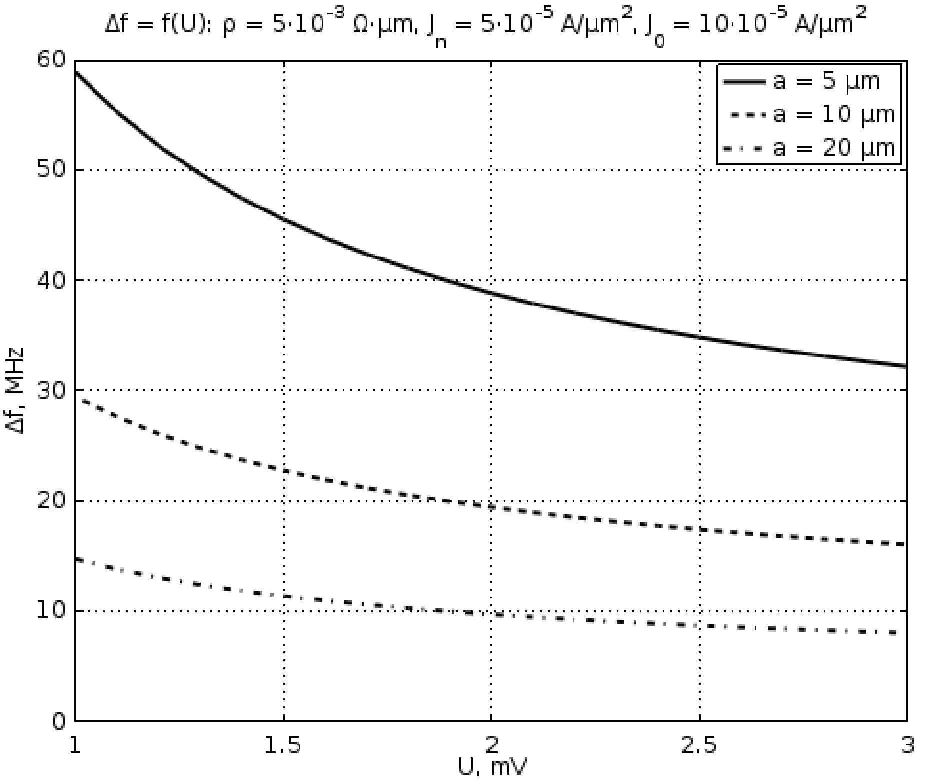

Fig. 7. Dependence of generation linewidth from constant voltage on JJ

voltage on JJ at different values of junction width that was conducted are shown. Analysis of them reveals that the increase in voltage leads to a reduction of generation linewidth and increase of JJ width also leads to a decrease in generation linewidth.

Analysis of the results reveals the generation linewidth decreases with an increase of JJ width, and this confirms the validity of obtained mathematical models. It should also be noted the results coincide with the experimental dependences given in the works [1, 3, 6].

Conclusions

1. The analytical dependence of Josephson junction generation linewidth from its geometrical dimensions and currents of normal electrons and Cooper pairs were first obtained.

2. Comparison of obtained dependencies with experimental results described in other works confirms an authenticity of developed mathematical models.

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Вплив критичного струму та параметрів переходу Джозефсона на стабільність частоти генератора

Проблематика. Однією із важливих характеристик генераторів на базі переходів Джозефсона є спектральна щільність потужності фазових шумів. В ряді теоретичних та експериментальних праць показано, що спектральна форма лінії генерації є лоренцевою, а її ширина становить десятки та навіть сотні мегагерц. Така форма лінії генерації і відносно велика її ширина ускладнюють застосування джозефсонівських переходів в приймальних пристроях високої чутливості та цифрових радіотехнічних пристроях з частотно-імпульсним кодуванням інформації.

Мета досліджень. Актуальною є задача дослідження можливості зменшення ширини лінії генерації та підвищення вихідної потужності. У зв'язку з цим, метою даної праці є дослідження залежності ширини лінії генерації від геометричних розмірів переходу Джозефсона та величини критичного струму.

Методика реалізації. Ця стаття була написана за допомогою використання методів операторного обчислення і математичного аналізу для побудови математичних моделей і отримання графічних залежностей.

Результати досліджень. Дослідження показали, що ширина лінії генерації зменшується зі збільшенням ширини переходу Джозефсона за рахунок зниження диференціального опору переходів. Збільшення величини критичного струму призводить до розширення ширини лінії генерації, оскільки струм нормальних електронів і куперівських пар зростає, і ця залежність носить лінійний характер.

Висновки. Вперше була отримана аналітична залежність ширини лінії генерації переходу Джозефсона від його геометричних розмірів і струму нормальних електронів та куперівських пар. Порівняння отриманих залежностей з експериментальними результатами, описаними в інших роботах, підтвердили достовірність розроблених математичних моделей.

Ключові слова: надпровідники; переходи Джозефсона; ширина лінії генерації; надпровідникові генератори; пристрої з частотно-імпульсним кодуванням.

Кичак В.М., Гузь М.Д., Громовый Д.С.

Влияние критического тока и параметров перехода Джозефсона на стабильность частоты генератора

Проблематика. Одной из важных характеристик генераторов на основе переходов Джозефсона является спектральная плотность мощности фазовых шумов. В ряде теоретических и экспериментальных работ показано, что спектральная форма линии генерации является лоренцевой, а ее ширина составляет десятки и даже сотни мегагерц. Такая форма линии генерации и относительно большая ее ширина затрудняют применение джозефсоновских переходов в приемных устройствах высокой чувствительности и цифровых радиотехнических устройствах с частотно-импульсным кодированием информации.

Цель исследований. Актуальной является задача исследования возможности уменьшения ширины линии генерации и повышение выходной мощности. В связи с этим, целью данной работы является исследование зависимости ширины линии генерации от геометрических размеров перехода Джозефсона и величины критического тока.

Методика реализации. Эта статья была написана с помощью использования методов операторного исчисления и математического анализа для построения математических моделей и получения графических зависимостей.

Результаты исследований. Исследования показали, что ширина линии генерации уменьшается с увеличением ширины перехода Джозефсона за счет снижения дифференциального сопротивления переходов. Увеличение величины критического тока приводит к расширению ширины линии генерации, поскольку ток нормальных электронов и куперовских пар растет, и эта зависимость носит линейный характер.

Выводы. Впервые была получена аналитическая зависимость ширины линии генерации перехода Джозефсона от его геометрических размеров и тока нормальных электронов и куперовских пар. Сравнение полученных зависимостей с экспериментальными результатами, описанными в других работах, подтвердили достоверность разработанных математических моделей.

Ключевые слова: сверхпроводники; переходы Джозефсона; ширина линии генерации; сверхпроводящие генераторы; устройства с частотно-импульсным кодированием.