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THE PRODUCTIVITY ANALYSIS OF DISCRETE INFORMATION TRANSFER SYSTEM NEAR THE SHANNON BORDER

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Background. The channel capacity and productivity assessments give a chance for system analysis principles development in applied information theory for telecommunication. The central task in the bounds of proposed research is the study of discrete communication channel (DCC) productivity.

Objective. The main task of current study is the analysis of discrete communication channel productivity through identification of the regularities in reliability behavior of modulation multi-position keying signals. Integral productivity assessment is the efficiency metric of communication channel resources usage.

Methods. Solving the main task of research generates bases on the finding the informational abilities of DCC. These informational abilities of DCC are expressed by channel capacity and productivity per one bit of information. Finally, the interdependence between channel capacity and productivity expressed via the information efficiency parameter η_C is the integral productivity estimation for DCC. The case of multiposition keying with its varieties (QPSK, QAM-16, and QAM-64) creates significantly bigger interest.

Results. The model of discrete information transfer system that includes source of discrete information, discrete communication channel with noises, defined method of signal forming and method of transmitted signal processing is proposed. The regularities of the channel capacity change and productivity of a discrete communication channel are established.

Conclusions. The hypothesis of the existence of extremes (upper limits) performance of DCC in discrete information transfer system without limitations on the received signals reliability is practically substantiated. Quantitative assessment to the discrete communication channel capacity and productivity extrema are presented. For achieving required reliability is noise-immunity coding as the recognized tool in the Shannon theory.

Key words: information theory; productivity; channel capacity; information efficiency.

Introduction

The traditional information theory indicators namely channel capacity and noise-immunity are studied peculiarly in current paper.

From one side, these indicators are functions of energy parameter h^2, which is applicable for transmitted signals reliability research. From other side, the main variable argument is the rate V_C.

Following proposed approach, the principal feature becomes apparent for applied information theory: the informational properties research of communication channels by way of telecommunication systems characteristics.

The main task of current study is analysis of discrete communication channel productivity through identification of the regularities in reliability behavior of modulation multi-position keying signals. Integral productivity assessment is the efficiency metric of communication channel resources usage.

Understanding and using the tool for right channel capacity and productivity assessment gives a chance for system analysis principles development in applied information theory for telecommunication.

Model description and task statement

The model of discrete information transfer system (DITS) is studied. The model includes:

- discrete information source that forms discrete digits with rate Vs;
- extended discrete channel for information transfer with interferences, which includes:
- coder on the transmission side, which forms coded sequence of digits with rate $V_C \geq V_S$ and source compressing (anti-interference) coding type 1 that is known at receiving side;

Scheme of discrete information transfer system is shown on Fig. 1.

- discrete communication channel with given spatially-power characteristics and known spectral density N_0 of additive noise interference;
- information recipient, which register discrete digits with rate V_S;
- modulator on the transmitting side, which forms channel digits with keeping channel rate V_C invariable. Channel digits have a given forming method type *m*, which is known in demodulator at receiving side. The method type *k* of signal processing is used to handle signal from communication channel at receiving side;

Elements of model have the following characteristics.

Discrete information source, that forms information sequence, is characterized by internal entropy *Hs* and amount of digits per time unit *Vs*. General information content that is generated by information source per time unit:

$$I_S = H_S \cdot V_S \ . \quad (1)$$

The received information sequence *B(t)* which went through extended DCIT with interferences, could be different from source information sequence *A(t)*.

Conditional probability P (B│A) includes measure of accordance for these information sequences. If *B(t) ≠ A(t)*, then the error in the sequence is considered. Middle probability value of such event is *Pe*.

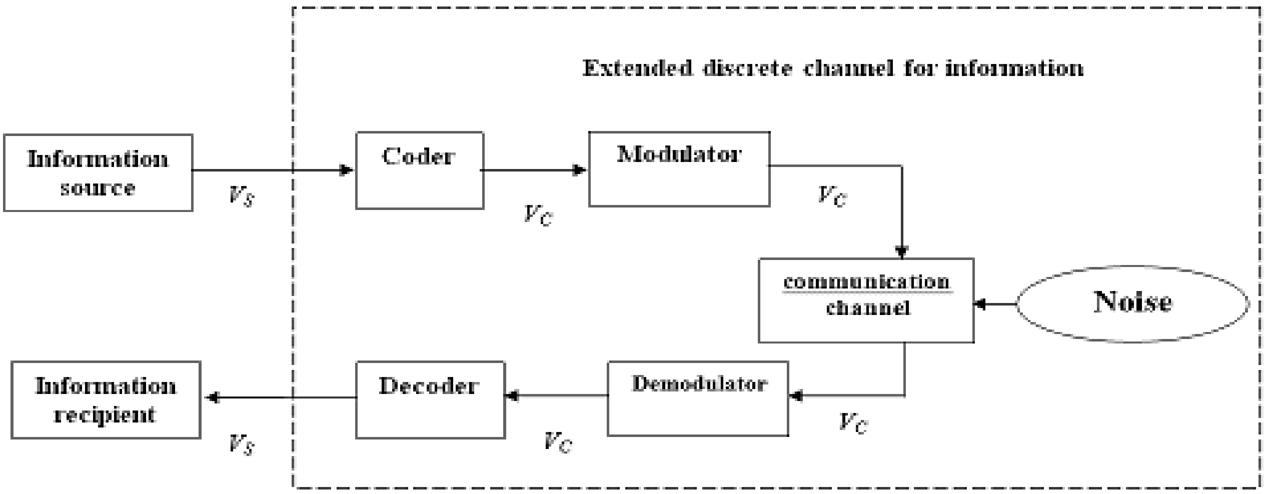

Fig.1. Discrete information transfer system

Modulator is characterized by forming method type *m* for channel digits with designated method of manipulation for sequence *X(t)*, and output speed of digits V_C. Demodulator is characterized additionally by coherent or incoherent method type *k* for processing signal from communication channel.

The sequence *Y(t)* from demodulator output goes to decoder input. The following statement is true is case of errors in channel digits: *Y (t) ≠ X(t)*. Then the middle value of error probability *p* for channel digits from binary symmetric channel without memory is estimated.

Discrete communication channel is characterized by given spatially-power characteristics and known additive noise interference spectral density N_0.

The measure of information content that transmitted through discrete communication channel (DCC) is the reciprocal information *I (X, Y)* for on channel digit.

Its volume depends on the probabilities p_j of errored or error-free receiving of channel digits.

Channel capacity C is the maximal average of transmitted information *I (X, Y)*, which can be transferred in one channel digit:

$$C = \max I(X,Y) = \max_{p_j} \sum p_j \log(p_j) \cdot \quad (2)$$

Maximal information content that can be transferred through DCC per time unit:

$$C_D = \max I_C = C \cdot V_C \cdot \quad (3)$$

The channel capacity C for binary symmetric channel without memory under mean probability of channel digit erroneous reception in accordance to (3) is equal:

$$C = 1 - H(p), \ H(p) = -\{p\log(p) + (1-p)\log(1-p)\}, (4)$$

where *H(p)* - entropy of random value, and has values *p* and *(1-p)*.

Random values are channel digits 1 or 0 that are registered by demodulator correctly or incorrectly, and have equal a priori appearance probabilities $P_0 = P_1 = 0{,}5$.

The statement (4) means channel informational abilities decreasing due to received results indefiniteness of channel digits.

In accordance with the Shannon theorem, arbitrarily small probability of error P_e in discrete channel can be achieved with using source compressing coding if information content from source is lower than channel capacity.

In the above notations is means that the following condition should meet always:

$$H_S \cdot R = H_C \leq C(p) \ , \quad (5)$$

where $H_S * R$ - information content from a source that is attributable to one channel digit.

Multiplying both parts of statement (5) to *Vc*, the following is true from (3) with taking into account (5):

$$I_{C_l} = H_S \cdot R_l \cdot V_C = I_{S_l} = H_S \cdot V_S \le \max I_C(p) = C_D(p) . \quad (6)$$

The system productivity is the average amount of transmitted information per time unit.

Therefore, the I_{S_l} in statement (6) is a measure of DITS productivity, and by fixed value *Hs* such measure could be *Vs*.

Thereby, if assume $H_S = const$, then it's followed from statement (11) that the condition for reaching maximal rate of undistorted source digits transfer V_S ($P_e \rightarrow 0$) in system with given characteristics coincides with condition of reaching maximal productivity I_{S_l}.

The Shannon limit should be supposed as the Shannon's channel capacity C, and maximal information amount that could be transferred through DCC per time unit C_D.

In this sense, the right part of statements (5) and (6) can be regarded as the Shannon limit for DCC.

The peculiarity of its behavior is that the Shannon limit can be moved in accordance with DCC parameters changes, resulting in a change of p values.

The classical Shannon model that connected with channel capacity analysis of DCC with interferences does not consider the parameters of the transmission system.

At the same, probability parameter p depends on both spatial-energy characteristics of DCC (line length, power radiation at al.) and forming / processing method for channel digits [3].

The expression (3) for maximal information amount C_D *t*hat could be transmitted through DCC per time unit, has multiplicands in right part that have opposite derivatives: the probability p grows with increasing *Vc* value, and, as result, channel capacity *C(p)* decreases. The final dependency behavior raises interest. The research of C_D dependency from rate *Vc* is the object of task that is solved in [4].

The following expression defines the channel capacity of DCC:

$$C_D = V_C \cdot \left(\log M + (1 - p_S) \cdot \log(1 - p_S) + p_S \cdot \log \frac{p_S}{M - 1} \right) \quad (7)$$

where *Vc* – digits rate in channel, p_s –error probability in communication channel.

The **central task** in the bounds of proposed research is study of DCC productivity.

Analytical presentation of DCC productivity in the general case is the following:

$$R_C = V_S \cdot \left(1 + (1 - p_b) \cdot \log(1 - p_b) + p_b \cdot \log p_b \right) \quad (8)$$

where *Vs* – rate of information source, p_b – bit error probability that transmitted through the DCC.

The information source rate *Vs* is equal to bits flow rate in channel *Vc* ($V_S = V_C$) when level of error-correcting coding is not involved.

Introduction of redundancy at the error-correcting coding stage causes $V_S < V_C$, but at the same time $p^*_b > p_b$, where p^*_b – bit error probability before decoding; p_b – bit error probability after decoding.

It's noteworthy that for BPSK in communication channel without coding, metrics p_s and p^*_b are equal: $p_s = p^*_b$.

For cases of multiposition keying (PSK-4 or QPSK, QAM-*N*, etc.) it's needed to distinguish p_s, p^*_b и p_b : $p_s > p^*_b > p_b$.

The expression (8) should be replaced by (9) in case of communication channel productivity analysis when antinoise coding is disabled:

$$R_D = \log M \cdot V_C \cdot \left(1 + (1 - p^*_b) \cdot \log(1 - p^*_b) + p^*_b \cdot \log p^*_b \right) \quad (9)$$

Solving the main task of research generates the derivative solution, which is connected with finding the informational abilities of DCC. These informational abilities of DCC are expressed by channel capacity and productivity per one bit of information (respectively expressions (10), (11)):

$$\overline{C} = \frac{\left(\log M + (1 - p_S) \cdot \log(1 - p_S) + p_S \cdot \log \frac{p_S}{M - 1} \right)}{\log M} \quad (10)$$

$$\overline{R} = \left(1 + (1 - p_b) \cdot \log(1 - p_b) + p_b \cdot \log p_b \right) \quad (11)$$

Finally, the interdependence between channel capacity and productivity expressed via the information efficiency parameter η_C is the integral productivity estimation for DCC:

$$\eta_C = \frac{R_C}{C_D} \quad (12)$$

or the same for the communication coding-free channel:

$$\eta_D = \frac{R_D}{C_D} \quad (13)$$

Obviously, for the case of binary manipulation in coding-free channel $\eta_D = 1$.

The characteristics of channel capacity with binary manipulation in coding-free channel are studied in detail in [4].

The case of multiposition keying with its varieties (QPSK, QAM-16, and QAM-64) creates significantly bigger interest.

Model implementation

The value of acceptable average error probability p_{acc} for channel digits traditionally is the criterion of maximal possible transfer rate in DCC:

$$\max V_D = V_C(p_{acc})\, max \tag{14}$$

Selection of p_{acc} is not associated usually with analysis of transmitted information volume. Therefore, achievement of (15) is just a consequence of approach expressed in (12):

$$\max I_C(p_{acc}) = C(p_{acc}) \cdot V_C(p_{acc}) \ . \tag{15}$$

If the basis of analysis is the information volume that transmitted through DITS, then it's possible to observe that expression (3) has contradictory trends of changing maximum information volume C_D transmitted through DCC when Vc changes. So, Vc growth causes decreasing of argument C{ $p = f(V_C)$}. In other words, the statement (4) can be expressed by functional:

$$C_D = C\{p = f(V_C)\} \cdot V_C = C_D(V_C) \, . \tag{16}$$

Thereby, existence of Vc value is possible so that indicator C_D reaches maximal value.

To use the expressions (7)…(11), let's address to formulas for assessment signals' reliability with multiposition keying. The following set of signals is studied in paper: QPSK, QAM-16, QAM-64.

Analytical expressions that allow us to define symbol p_S and bit p^*_b error probabilities for proposed set of multiposition keying types are presented below:

$$p_{SQPSK}(h^2) = \frac{3}{4} - \frac{1}{2}\Phi\left(\sqrt{h^2}\right) - \frac{1}{4}\Phi^2\left(\sqrt{h^2}\right) \tag{17}$$

$$p_{bQPSK}\left(h^2\right) = \frac{1}{2}\left[1 - \Phi\left(\sqrt{2h^2}\cos\frac{\pi}{4}\right)\right] \tag{18}$$

$$p_{SQAM}(h^2) = 1 - \left(1 - \frac{2(1 - \frac{1}{\sqrt{M}})}{\sqrt{2\pi}} \int_{\sqrt{\frac{3k}{M-1}h^2}}^{\infty} \exp\left(\frac{-u^2}{2}\right) du\right) \tag{19}$$

$$p_{bQAM}(h^2) = \frac{2^{k-1}}{2^k - 1} \cdot p_{SQAM} \tag{20}$$

General argument of errors function p_S and p_b is the energy parameter h^2 in presented expressions (17)…(20):

$$h^2 = \frac{E_C}{N_0} = \frac{P_C}{P_N} = \frac{\alpha}{N_0 \cdot V_C} \ , \tag{21}$$

where Ec is the average signal energy in receiving point

Pc – signal power in receiving point (W)

N_0 - One-sided noise spectral density (W/Hz)

P_N –noise power (W)

V_C –symbols rate in communication channel ($Mbps$)

$$\alpha = \frac{P_C}{N_0} \ – \quad \text{universal energy factor [4]}$$

At the same time, changing the dimension of parameter α unambiguously brings to changing rate dimension Vc without changing the h^2 value and respective probability values.

Other significant argument in expressions (19)…(20) is parameter M – multiplication factor. At that for studied multiposition keying types the M parameter takes values:

– for QPSK – M=4;
– for QAM-16 – M=16; (22)
– for QAM-64 –M=64.

The dependency of symbol error p_S and bit error p^*_b from digits rate in channel V_C is presented on Fig. 3 for multiposition keying types QPSK, QAM-16, and QAM-64.

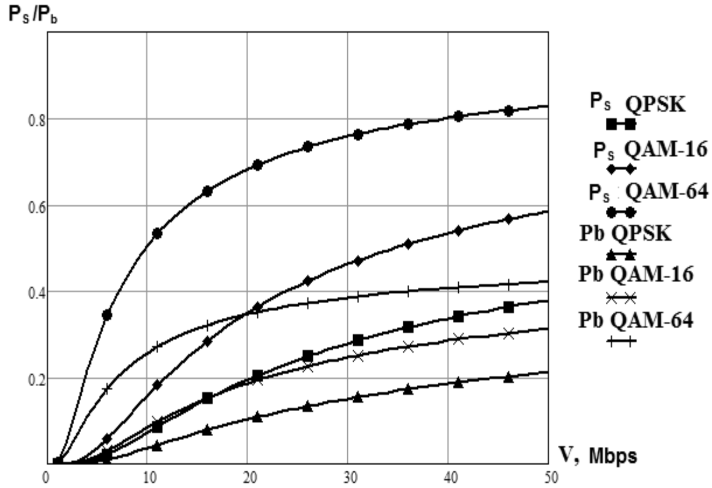

Fig. 3. Changing the symbol and bit errors probability for multiposition keying types QPSK, QAM-16, QAM-64

It's shown on the Fig. 3 with predefined value α=32 ms^{-1} that the difference between values p_S and p^*_b is growing when rates changing from 1 Mbps to 50 Mbps and when signal multiposition level increasing. The difference for QPSK is considerably smaller than for QAM-16/64, what causes prerequisites for different behavior of channel capacity C_D and productivity R_D as functions of respective arguments regarding (7) and (9).

The character of changing the channel capacity (Fig. 4) and channel productivity (Fig. 5) as functions of the growing information transfer rate in channel V_C under specified keying set is shown below.

These dependencies are plotted based on the expressions (7), (9), (17)…(19).

It's shown on the Fig. 4 that the channel capacity C_D as a function of parameter V_C has an extremum what means exceeding tempo of decreasing the nonlinear multiplier $C = 1 - H(p)$ in expression (4) comparably to linear growing of the V_C in expression (7).

The proportion in channel capacity value corresponds to initial multiposition keying factor according to (22) under the initial transfer rate value Vc = 1 Mbps.

The signal QAM-16 keeps maximum informational resource when digits transfer rate in channel Vc is changing (growing). The QAM-64 signal reaches an extremum of channel capacity earlier than signal QAM-16, but with lower values. The informational saturation of QPSK signal is slower, but QPSK has an advantage in noise immunity.

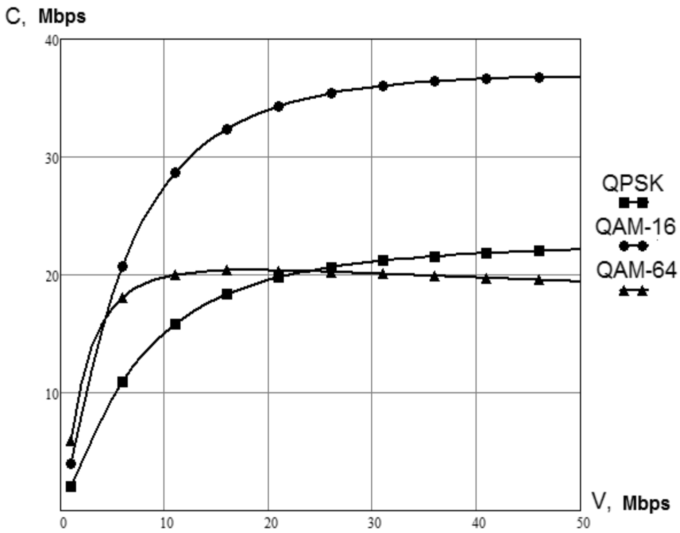

Fig. 4. The dynamics of channel capacity for QPSK, QAM-16, QAM-64

The productivity dynamic R_D on Fig. 5 includes an extremum when QAM-16 and QAM-64 are used, while R_D extremum is not expressed when QPSK is used.

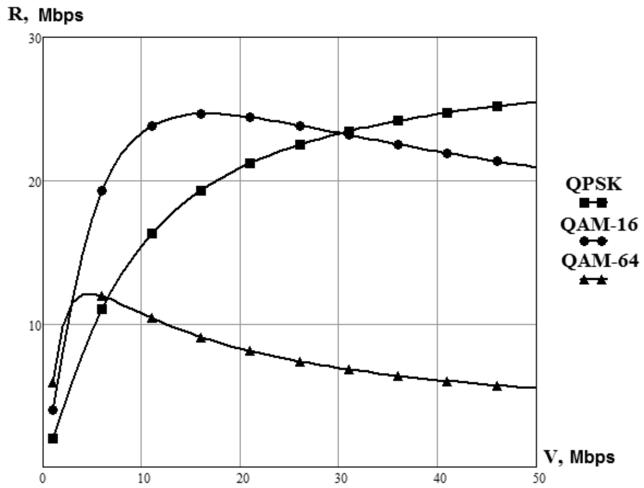

Fig. 5. The dynamics of channel productivity for QPSK, QAM-16, QAM-64

Development of central research task generates regularities that are connected with changing specific channel capacity and specific channel productivity, which are defined per 1 bit of information in DCC ((9)-(10)).

The dynamic of specific channel capacity is and specific channel productivity for QPSK, QAM-16, QAM-64 is shown on Fig. 6-7.

As shown on Fig. 6, informational resources of QPSK signal and QAM-16 signal are almost equal per one informational bit.

The dependencies on Fig. 7 and Fig. 8 correspond the tendency that is shown on the Fig. 3 – decreasing of noise immunity indicators hierarchy p_S and p^*_b with growing the multiposition keying factor M is represented by corresponding decreasing of channel capacity and channel productivity

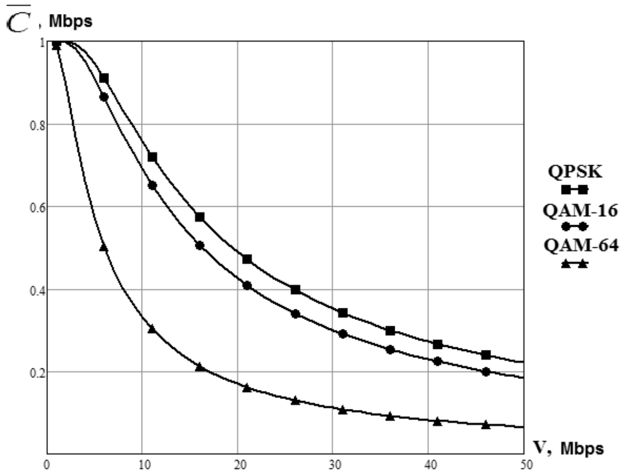

Fig. 6. The dynamics of specific channel capacity for QPSK, QAM-16, QAM-64

Table 1

α (ms^{-1})	Multiposition keying	V (Mbps)	C(Mbps)	V (Mbps)	R (Mbps)
50	QPSK	2,7 α	0,7 α	-	-
50	QAM-16	1,7 α	1,15 α	0,53 α	0,77 α
50	QAM-64	0,56 α	0,64 α	0,15 α	0,38 α
32	QPSK	2,7 α	0,7 α	-	-
32	QAM-16	1,7 α	1,15 α	0,53 α	0,77 α
32	QAM-64	0,56 α	0,64 α	0,15 α	0,38 α

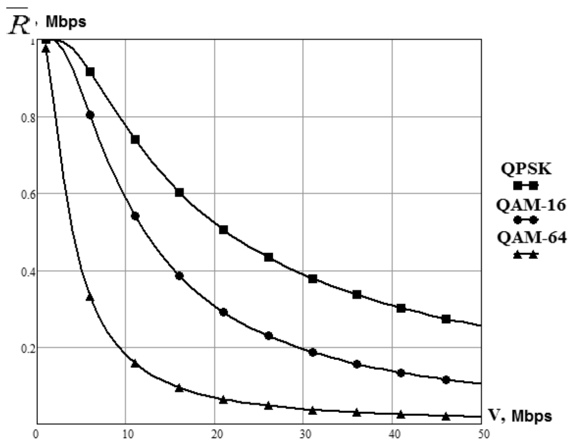

Fig. 7. The dynamics of specific channel productivity for QPSK, QAM-16, QAM-64

The results of channel capacity and channel productivity extrema calculation are combined in table 1.

The integrating factor of research is definition of channel usage efficiency. So, the dynamic of changing the signals informational efficiency in traditional and reduced forms is shown on Fig. 8-9 (for specific channel capacity and specific channel productivity).

The QPSK efficiency line is not matching the traditional idea of changing the channel usage informational efficiency because the η_D values are bigger than 1.

This is caused, from one side, by the fact that in channel without coding the limitations in Shannon theorem do not matter because not the task for reaching the required probability is solving. From other side the fact of exceeding by η_D the value 1 physically conditioned by insignificant difference of probabilities p_S and p^*_b (Fig. 3) when the analytical expressions (7) and (9) are different.

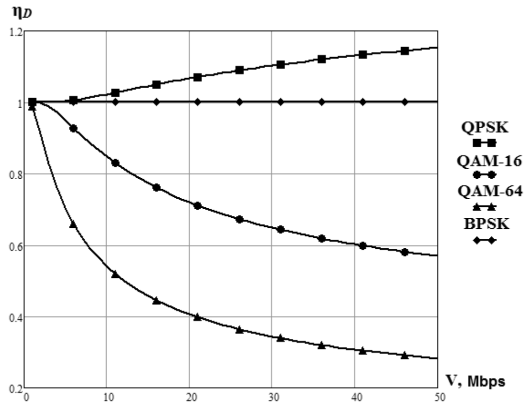

Fig. 8. The dynamics of channel efficiency usage for QPSK, QAM-16, QAM-64

The straight line on Fig. 8 and Fig. 9 is presented for BPSK signal for which the informational filling by channel capacity and channel productivity are equal ($C_D = R_D$).

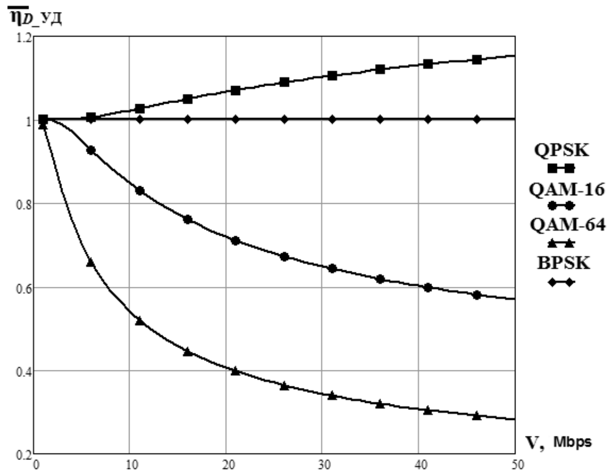

Fig. 9. The dynamics of channel efficiency for QPSK, QAM-16, QAM-64

Review of derived results

The model of discrete information transfer system is proposed that includes source of discrete information, discrete communication channel with noises, defined method of signal forming and method of transmitted signal processing.

The invariance of informational abilities values representation relatively to α values is proposed in tables 1-3.

As shown in table 2, the bit error probability in enough big and using the extremum of channel capacity is not constructive. At the same time, reasonable is usage of channel productivity extrema, because respective bit error probability potentially is liable to be corrected by proper noise-immunity code.

Table 2

Multiposition keying type	V (Mbps)	C(Mbps)	p^*_b	η_D
QPSK	2,7 α	0,7 α	0,271	1,207
QAM-16	1,7 α	1,15 α	0,321	0,556
QAM-64	0,56 α	0,64 α	0,334	0,424

Table 3

Multiposition keying type	V (Mbps)	R (Mbps)	p^*_b	η_D
QPSK	-	-	-	-
QAM-16	0,53 α	0,77 α	0,158	0,753
QAM-64	0,15 α	0,38 α	0,126	0,732

Conclusions

1. The research of channel capacity and productivity dependencies on Fig 5, 6 with different multiposition keying signals shown existence of extremum under respective energy circumstances in channel. Using the QAM-16 and QAM-64 signals states the existence of extremum, while the extremum of productivity is not expressed for QPSK. It should be noted that QAM-16 signals are more productive in the extremum point when compare with extremum for QAM-64 signal. It indicates the rationality of using the QAM-64 signals in small-cells wireless systems (e.g., LTE) unlike QAM-16 signal that is fit for use in big-cells wireless systems (e.g., UMTS).

2. The dynamic of dependencies on Fig. 7 and 8 corresponds the tendency that is shown on Fig. 3: decreasing the indicators of noise-immunity p_S and p^*_b with growing multiposition keying factor M are accompanied by respective decreasing of specific channel capacity and productivity.

3. The integrating circumstance of study is characteristic of channel efficiency usage. So, on the Fig. 8-9 the signals informational efficiency dynamic is shown in traditional and normalized forms (for specific channel capacity and specific channel productivity).

The QPSK efficiency line is not matching the traditional idea of changing the channel usage informational efficiency because the η_D values are bigger than 1.

4. The factor of received signals reliability is not considered when the conditions of channel productivity maximum approximation to the channel capacity extremums are explored. The recognized tool in the Shannon theory for achieving required reliability is noise-immunity coding.

Research of the productivity maximum is a separate task when the condition is defined to reach specified information reliability.

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А.В. Мошинська

Аналіз продуктивності системи передачі дискретної інформації поблизу границі Шеннона

Проблематика. Оцінка пропускної здатності каналу і його продуктивності розвиває принципи системного аналізу в області прикладної теорії інформації для телекомунікацій. Центральним завданням в рамках пропонованого дослідження є вивчення продуктивності дискретного каналу зв'язку (ДКЗ).

Мета досліджень. Головним завданням дослідження є аналіз продуктивності дискретного каналу зв'язку шляхом виявлення закономірностей в поведінці багатопозиційних сигналів. Пропонується оцінка інтегральної продуктивності та ефективності використання ресурсу каналу зв'язку.

Методика реалізації. Рішення центрального завдання основане на визначенні інформаційних можливостей дискретного каналу, які виражені через пропускну здатність та продуктивність, приведених на один біт переданої інформації. Інтегральною оцінкою продуктивності дискретного каналу зв'язку є коефіцієнт інформаційної ефективності η_C. Більший інтерес представляє випадок багаторазової маніпуляції з її різновидами: QPSK, QAM-16, QAM-64.

Результати досліджень. Запропоновано модель системи передачі дискретної інформації (СПДІ), що включає джерело дискретної інформації, дискретний канал зв'язку з перешкодами, заданий спосіб формування і спосіб подальшої обробки сигналу, переданого по лінії зв'язку. Встановлено закономірності зміни пропускної здатності і продуктивності дискретного каналу зв'язку.

Висновки. Практично обґрунтовано гіпотезу про існування екстремумів (максимумів) продуктивності ДКЗ в СПДІ без обмежень на достовірність повідомлень, що передаються. Надані кількісні оцінки екстремумів пропускної здатності и продуктивності ДКС. Визнаним в теорії Шеннона інструментом щодо забезпечення заданої достовірності є завадостійке кодування.

Ключові слова: теорія інформації; продуктивність; пропускна здатність; інформаційна ефективність.

А.В. Мошинская

Анализ производительности системы передачи дискретной информации вблизи границы Шеннона

Проблематика. Оценка пропускной способности канала и его производительности развивает принципы системного анализа в области прикладной теории информации для телекоммуникаций. Центральной задачей в рамках предлагаемого исследования является изучение производительности дискретного канала связи (ДКС).

Цель исследований. Главной задачей исследования является анализ производительности дискретного канала связи путем выявления закономерностей в поведении многопозиционных сигналов. Предлагается оценка интегральной производительности и эффективности использования ресурса канала связи.

Методика реализации. Решение центрального задания основано на определении информационных возможностей дискретного канала, которые выражены из-за пропускной способности и производительности, приведенных на один бит переданной информации. Интегральной оценкой производительности дискретного канала связи является коэффициент информационной эффективности η_C . Наибольший интерес представляет случай многократной манипуляции с ее разновидностями: QPSK, QAM-16, QAM-64.

Результаты исследований. Предложена модель системы передачи дискретной информации (СПДИ), включающая источник дискретной информации, дискретный канал связи с помехами, заданный способ формирования и способ последующей обработки сигнала, передаваемого по линии связи. Установлены закономерности изменения пропускной способности и производительности дискретного канала связи при изменении его энергетических параметров.

Выводы. Практически обоснована гипотеза о существовании экстремумов (максимумов) производительности ДКС в СПДИ без ограничений на достоверность передаваемых сообщений. Представлены количественные оценки экстремумов пропускной способности и производительности ДКС. Признанным в теории Шеннона инструментом для обеспечения заданной достоверности является помехоустойчивое кодирование.

Ключевые слова: теория информации; производительность; пропускная способность; информационная эффективность.