UDC 621.396

# THE CONVOLUTIONAL CODESANALYSIS TECHNIQUE ON THE OPTIMUM BLOCK CODES GROUNDS

Leonid O. Uryvsky, Kateryna A. Prokopenko, Anton M. Peshkin

National Technical University of Ukraine “Kyiv Polytechnic Institute”, Kyiv, Ukraine

*Abstract*— The convolutional codes analysis and synthesis technique is proposed. This technique is optimal according to near Shannon's bound criterion and guarantees a given bit error rate from message source. The proposed technique is based on the well-known optimum block codes synthesis method. As a result, equivalent parameters set of convolutional codes is received. This set corresponds the optimal block code. The algorithm for synthesis an optimal convolutional code is proposed. Examples of optimal convolutional codes analysis and synthesis in the channel with a given energy are presented.

*Keywords — convolutional codes, the Shannon bound, the Plotkin bound, the Varsharmov-Gilbert bound.*

## Introduction

An information theory, formulated by C. E. Shannon through postulates and fundamental theorems many years ago, opened the epoch for the bright achievements in industry of increase authenticity connection on the basis of error-correcting code [1].

The constructiveresult ofan appeal tothe Shannon-theory should be considered the concept of Shannon bound - *ShB* - curve, which is determined as the largest amount of information thatcan be transmittedover a communication channel with preset energy and frequency metrics. Like this theconcept of the Shannon bound –*ShB* – isidentified withthe concept ofchannel capacity*C*.

The formula forthe capacity ofa continuous channel$C_C$ is the mostcommon [1]:

$$C_C = F \cdot \log(1 + P_S / P_N), \quad (1)$$

where*F*– frequencybandof communication; $P_S$ / $P_N$ – signal $P_S$ to thenoise $P_N$ power ratio at the pointof his admission,which preventssignalat the same point.

Theformula forthe capacity ofdiscrete channel$C_D$ is lesscommon [2]:

$$C_D = V_C \cdot [\log M + P_{er} \cdot \log(\frac{P_{er}}{M-1}) + (1 - P_{er}) \cdot \log(1 - P_{er})], \quad (2)$$

where$V_C$– symbol rate ina discretecommunication channel; *M* - the number of levels per sample for discrete manipulation, $P_{er}$ – symbol error rateat the output ofthe demodulator, which is a function ofthe ratio$h^2$ = $P_S$ / $P_N$in a discretecommunication channel.

The expression (2) defines the maximum source symbol rate. At this source symbol rate, the required bit error rate in the channel with known $P_S$ / $P_N$ratio can be achieved due to error-correcting codes.

A huge variety ofcodesthat cancorrect errorsin channelswith varyingquality are synthesized on the basis of the Shannon theorem. Today the block codes are most investigational codes [3-5].

The basic parameters of the block codes are: length of code *n*, length of input message *k*, code rate $R_C = k / n$, minimum distance of the code *d* and, as a result, ability to correct $t \le (d - 1) / 2$ errors in a block with *n* symbols.

Knowing descriptions of channel through the symbol error rate$P_{er}$, it is possible to define given number of errors *m* in block with length *n* as follows:

$$P_m(m, n, h^2) = \frac{n!}{m!(n-m)!} P_{er}(h^2)^m (1 - P_{er}(h^2))^{n-m} \quad (3)$$

Then bit error rate $P_b$ ofsingle symbol on the decoder output, which corrects *t* errors in a block of *n* symbols, is evened:

$$P_b = P_t(m > t, n, h^2) = \frac{\sum_{j=0}^{t} C_n^j P_{er}(h^2)^j (1 - P_{er}(h^2))^{n-j} \cdot j}{n} \quad (4)$$

If it is necessary to provide required bit error rate $P_{b\ req}$, the value of *t* in expression (4) should be found.

Should be noted that the expression for the largest amount of information *R*, which is contained in a single binary symbol ($M$ = 2), transmitted over a channel with interference and over-coded is:

$$R=[\log M+P_b\cdot\log(P_b)+(1-P_b)\cdot\log(1-P_b)]=R_{c\,\max}, \quad (5)$$

this valuequantify coincideswith the value ofthe largestcoding rate$R_C$= $k$ / $n$,which allows to correctany errorsinthe transmittedcodedblockregardless ofthe values $n$and$k$.Realcodes have $R_C$<$R_{C\,max}$.

Another class of codes is convolutional codes. Their basic parameters are: constraint length $v$, code rate $R_C$, minimum free distance $d_f$.

Substantial advantage of convolutional codes is simplicity of realization. In block codes bit error rate can be estimated due to known block length $n$ and minimum distance of code $d$, but in convolutional codes a task of bit error rate estimation is not obvious due to absence of analogical parameters and direct analytical relations between the structural parameters of convolutional codes and its error-correcting parameters.

The purpose of this work is forming of convolutional codes estimation technique on the basis of the known optimal block codes synthesis technique on the criterion of the maximal Shannon bound approaching.

## Raising of task

The basic task of error-correcting codes is an increase of information transfer authenticity. For block codes there are methods which provide determination of optimal block codes [4]. In the described technique an optimal block code is a block code, which is maximally approached the Shannon bound (code has the higher code rate) at the formulated requirements to authenticity and at initial limitations which are imposed on a channel and code - code length $n$ and power ratio $h^2$.

For error-correcting codes the Plotkin bound - *PB* - and Varsharmov - Gilbert bound - *VGB* exist [1], that determine the possible values interval for code parameters with the set properties:

$$k\le n-2d+2+\log_2 d\ , \quad (6)$$

$$n-k\le\log_2\sum_{i=0}^{d-2}C_{n-1}^{i}\ , \quad (7)$$

when - *PB* in accordance with (6) determines bound of theoretical existence of codes length $n$ and minimum code distance of $d$ with code rate $R_{PB}$=$k$/$n$ = $R_{max}$; VGB in accordance with (7) determines bound of the assured existence of code with length $n$ and minimum code distance of $d$ and code rate $R_{VGB}$<$R_{PB}$.

From a theory, it is known, the parameters of the real error-correcting codes will be between *PB* and *VGB* bounds.

In [4] the technique, that allows choosing an error-correcting code which provides necessary bit error probability on the output of decoder $P_b$at preset power ratio $h^2$, is described.

The basic elements of this method are:

1)Construction of Shannon bound – *ShB* – in the ($h^2$, $R$) coordinates (Fig. 1a) according to expression (5).

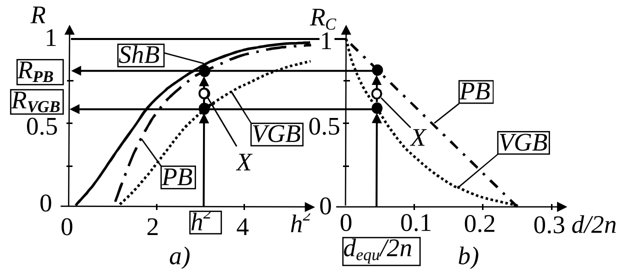


Fig.1.Transformation of *PB* and *VGB* from ($R_C$ , $d/2n$) coordinates into the ($R$,$h^2$) coordinates.

2) Determination of errors number of $t_{req}$, which must be corrected by a surplus block code with length $n$, for achievement of required bit error rate of $P_{b\,req}$, going out the known values of index $h^2$ and type of modulation with subsequent conversionthe parametervalue$t_{req}$ inparameter$d_{req}$ / 2$n$,where$d$ = 2$t$ +1 (depending$t_{req}$ ($h^2$)and$d_{req}$/ 2$n$ ($h^2$)are not shownin Fig.1);

3) Determinations of possible values of code rate $R_c$ $\to${$R_{PB}$, $R_{VGB}$<$R_{PB}$} by the using of *PB* and *VGB* bounds, which can be attained in a block code (Fig.1,b). Eachvalue of $h^2$corresponds tothe value of$d_{req}$ / 2$n$,whichcorresponds to the values of $R_{cmax}$ ($d_{req}$/2$n$)and$R_{VGB}$ ($d_{req}$/2$n$).Movementalongdepending$d_{req}$/2$n$ ($h^2$)generatesan appropriatereflection of boundary*PB*and*VGB*in the {$R$, $h^2$} coordination. According to basedinflictedvalue$h^2$electiontakes placeamong famouscodethat bestmeets the criterion$R_c \le \max R_c(n, t)$ -point X(Fig. 1, b).

4) Indexes of block code *(n, k)* are represented on one plane with Shannon bound – *ShB* – in ($h^2$, $R$) coordinates –point X(Fig. 1b).

So, {$n$, $k$, $d$}-code, that provides the required bit error rate$P_{breq}$ at set conditionscaused becomes known.

Displayedin Fig. 1projection of PBandVGBon the planein ($R$, $h^2$)coordinatesmakes it possible toassess theextent ofreaching theselectedcodeandShannon bound.

For convolutional codes there is no similar technique of optimal convolutional codes parameters determination. And applying technique which are used for block codes is impermissible, because block codes are determined through the parameters {$n$, $k$, $d$} and convolutional codes - {$v$, R,$d_f$,}. Thus, the parameters of these different classes of codes are formally unconnected.

Taking into account these facts, the basic task of this paper is development of parameters choosing technique for optimal convolutional codes. As technique of determination of block codes are known, there is a task of convolutional codes equivalent parameters searching, which would respond the parameters of optimal block codes.

The criterion of convolutional codes equivalence parameters ($n_{equ}$, $d_{equ}$) to the corresponding optimal block codes is a condition, that the searched sought after convolutional codes {$v$, $R$,$d_f$} and but corresponding to him block codes {$n_{equ}$, $R$, $d_{equ}$} had identical code rate $R$, and provided necessary probability of bit error of $P_{b\ req}$ on the decoder output at the identical values of $h^2$ and type of modulation.

## Technique of convolutional codes error possibilities analysis

It is necessary to define the parameters of convolutional codes equivalent to the block code that will be able to provide the set bit error rate. These equivalent parameters are code distance of $d_{equ}$ and code block length$n_{equ}$.

For convolutional codes expression for the estimation of bit error rate $P_b$ acquires a next kind [3]:

$$P_b \le \sum_{l=d_f}^{\infty} W_l \cdot P_l \,, \qquad (8)$$

where $W_l$ is the tabbed values of coefficients of code spectrum (depend on $v$ and $d_f$), which are equal to the amount of errors on the decoder output, which arose up, when accepted code combination defended from passed on distance of $d_f = l$; $P_l$ is the value of probability of choice of erroneous way of weight of $l$ that take into account thevalues of$P_{er}$ -error probabilityat the demodulator output [3].

Using dependency$P_b = f\ (h^2)$for convolutional codeswith different constraint length$v$for differentcode rate$R_C$(eg, ν = 2, 5, 7 for$R_C$= 1/2, 1/3, 2/3, 3/4) which are obtained from analytical dependencesforbitprobabilities(8), the parametersof convolutionalcodesthat providea fixedbit error rate (eg,$P_b$=$10^{-6}$) are defined.

The defined set of convolutional codes parameters of {$v_i$, $h^2_i$, $R_{ci}$} is the result of the first step, where $v_i$ - length restrictions for the $i$-th code, $R_{ci}$ - code rate of the $i$-th code that provides bit error probability $P_{breq}$ at a certain power ratio $h^2_i$.

At the second step, the found values of convolutional codes are presented in coordinates ($h^2$, $R$ =$R_{ci}$). Considering that these coordinates are general for convolutional and block codes (fig.1b), convolutional codes will represent together bounds *PB* and *VGB* (for the fixed value of length $n$)of block codes and bound of *ShB* (Fig.2).

The points, which have corresponding value of $h^2$ and $R$ at bit probabilities of error equal to $10^{-6}$ is presented on the Fig.2. The plots of *PB* and *VGB* bounds are executed for a block code with block $n$ = 511 for providing of bit error probability $10^{-6}$ (in Fig. 2codewith$v$ = 5, $h^2_i$= 4and$R_{si}$= 1/2 is marked with dash-dottedline).

On Fig.2 it is shown, that for every code the rate $R_c$ convolutional codes are located below and more right than Varsharmov – Gilbert bound on the criterion of error probability. That means theircorrectiveability isworsethan the selectedprototype of blockcode.

On the Fig.2 projected linepasses through thevalueof the power ratio$h^2$ =4 andthe projected pointof convolutionalcodeswith parameters$v$ =5, $R_c$ = 0.5 (dotted line 2) and$v$ =2,$R_c$ = 0.33 (dotted line 1).

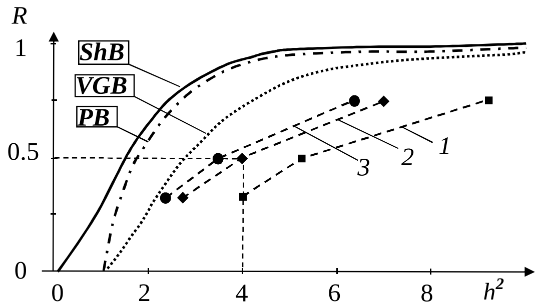


Fig.2.Illustrations of convolutional codes projection together with the lines of *PB* and *VGB* for block code of $n$ = 511 : 1 – $v$=2, 2 – $v$=5, 3 – $v$=7 for $R_c$=0.33 (points in the bottom), 0.5 (points in middle), 0.75 (point at the top).

At the third stepaprojection of selectedin the first stepof convolutional codes{$v_i$, $h^2_i$, $R_{ci}$} fromregion ($h^2_i$, $R$) in theregion ($h^2_i$, $d$ / $2n_i$) is built. Executionof this projection is an action that corresponds tothe inversealgorithmfor blockcodes.

The projection of code parameters {$v_i$, $h^2_i$, $R_{ci}$} in the region$d$ /2$n$ ($h^2$) is constructed by means of thesequentialmovement ofselected pointswith coordinates ($h^2_i$, $R$ = $R_{ci}$) in the coordinates ($R_{ci}$, $d/2n_i$) and then, using *PB*and*VGB*boundinregion withcoordinates ($h^2_i$,$d$ /2$n_i$).

Simultaneously,a line$d_{req}$/ $2n$ ($h^2$)fora block code with$n$ = const (for example, $n$ = 511) is displayed in the coordinates ($h^2$, $d$ /2$n$), according to the step 2 of knownmethod mentioned above.

Result of projection is illustrated in Fig. 3.

The belonging of convolutional code projected point to the line of block code $d$ /2$n$ ($h^2$) for $n$ = const is the equality condition of correctional ability of block and convolutional codes (points 1-5 on Fig.3). Equality of code rate follows at the terms of construction of dependences for convolutional and block codes with identical $R_c$.

Obviously, that in order that the block code line $d/2n$ ($h^2$) crossed the projections of convolutional codes, it is necessary to change the parameters of block code, for example, diminishing block length. Thus the line will move on the right till the crossing with corresponding projections.

The result of this step is a value of block code length that equals convolutional code with the same bit error rate.

At the last step, knowing $n_{\text{equ}}$ and value $d/2n$ it is possible to find the equivalent value of minimum code distance of $d_{\text{equ}}$.

As a result of application of this technique the sets of parameters, equivalent to block codes $\{n_{\text{equ}}, d_{\text{equ}}\}$ with equal code rate $R_c$ and $h^2$ are obtained.

Using the found parameters, it is possible to compare efficiency of convolutional and block codes and propose the technique optimal error-correcting code search on the criterion of the maximal Shannon bound approaching.

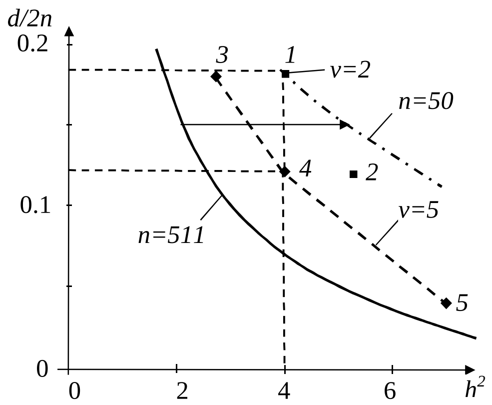


Fig.3. Illustration of projected points for convolutional codes (points 1-5) in the area of error-correcting line for block codes ($h^2$, $d/2n$) with the block length of code block $n$ = 511 (solid line) and $n$=50 (dot-and-dash line). 1 – $v$=2, $R_c$=0.33; 2 – $v$=2, $R_c$=0.5, 3 – $v$=5, $R_c$=0.33; 4 – $v$=5, $R_c$=0.5, 5 – $v$=5, $R_c$=0.75.

## Comparative Analysis of optimal convolutional codes on the criterion of Shannon bound approaching

The aims of proposed technique is to determine the optimal parameters of convolutional code$\{v, R_c\}$ on criterion forprovidingcriterion of the highercoding ratefor givenparameters($h^2_{req}$, $P_{er}$, $P_{breq}$).

The technique of analysis consists of three steps.

The first step is to use the noise immunity line$P_b = f(h^2, P_{er})$ and the definition of convolutional codes $\{v_i, R_{ci}\}$, which guarantees to provide the required reliability $P_{breq}$ in $h^2_{req}$ (such as in Fig. 4a).

The second step is a projection of the obtained values ($h^2_{req}$,$P_{breq}$) in the area of error-correcting code bounds and Shannon bound ($h^2$, $R$) (such as in Fig. 4b).

The third stage is determination of optimal error-correcting code with parameters $\{\nu, R, d_f\}$, on the criterion of the maximal Shannon bound approaching for set power ratio $h^2$.

The realization of the offered technique of analysis is consider on the example of communication channel with the power parameter $h^2$ = 4 for the case of non-coherent reception of phase manipulation signals in order to achieve bit error rate not worse than $10^{-6}$ (Fig.4).

It is known Fig.2, that for $h^2$ = 4 there are convolutional codes with options $\{\nu_i, R_{ci}\}$: {5;1/2} and {2;1/3}.

It should be noted that the use of this code for $h^2$> 4 is possible (Fig.4b it is a code {2;1/2}, dotted-line 1, dot-and-dash projection). The increase of energy in a channel leads to decreasing of probability of error on the decoder output, and thus an authenticity of transmission is improved.

At the same time, when the energy increases, it is possible to choose a convolutional code which would provide necessary authenticity and would have higher code rate, and thus it was optimal for the improvement of power terms (it is a code {7;3/4} on Fig.4b – dotted-line 3).

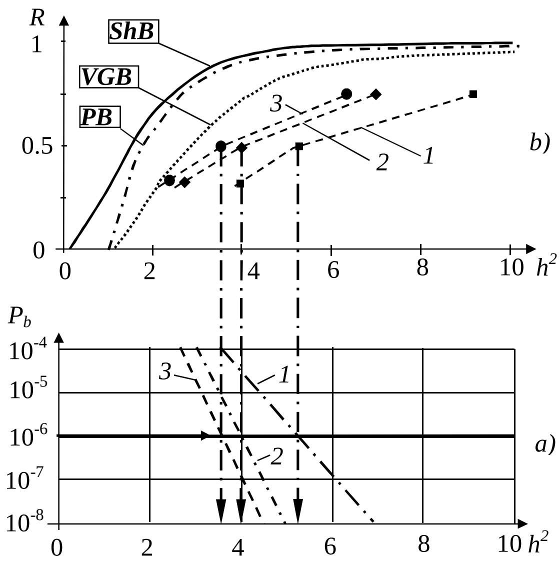


Fig.4 Illustration of optimal error-correcting code searching algorithm for the value of power rate $h^2$= 4: 1 – $v$=2, 2 – $v$=5, 3 – $v$=7 ($R_c$ = 1/2).

Decreasing of energy ($h^2$< 4) makes it impossible to provide the necessary bit error rate. And in this case, when energy is not enough, it is necessary to choose another error-correcting code (with less code rate *R*, or greater value of constraint length *ν*- it is code {7;1/2} on Fig.4b – dotted-line 3, dot-and-dash projection) to elect.

## Analysis of efficiency bounds for convolutional and block codes

The results of equivalent parameters search of convolutional codes are listed in the tables 1 and 2. The results show that equivalent parameters, found by means of bounds of *PB* and *VGB* can be determined for convolutional codes.

TABLE I. EQUIVALENT PARAMETERS OF CONVOLUTIONAL CODES, OBTAINED WITH THE USE VARSHARMOV - GILBERT BOUND

| *R, ν* | $h^2$ | $d_{екв}$ | $n_{екв}$ |
|---|---|---|---|
| 1/2, 2 | 5.25 | 7.27 (7) | 40 |
| 1/2, 3 | 5.00 | 7.27 (7) | 40 |
| 1/2, 4 | 4.35 | 8.95 (9) | 55 |
| 1/2, 5 | 4.00 | 12.9 (13) | 90 |
| 1/2, 6 | 3.80 | 15.1 (15) | 110 |
| 1/2, 7 | 3.50 | 21.2 (21) | 165 |

TABLE II. EQUIVALENT PARAMETERS OF CONVOLUTIONAL CODES, OBTAINED WITH THE USE PLOTKIN BOUND

| *R, ν* | $h^2$ | $d_{екв}$ | $n_{екв}$ |
|---|---|---|---|
| 1/2, 3 | 5.00 | 6,89 (7) | 18 |
| 1/2, 4 | 4.35 | 6,89 (7) | 18 |
| 1/2, 6 | 3.80 | 8,82 (9) | 25 |

For example, a convolutional code rate $R_c$ = 1/2 and constraint length ν = 3:

($n_{equ\_PB}$, $d_{equ\_PB}$) = (18, 7);

($n_{equ\_VGB}$, $d_{equ\_VGB}$) = (40, 7).

It means that corresponding block codes can have parameters within the limits of $n \in$ [18;40] at $d$ =7.

For the estimation of convolutional codes the parameters of bound of VGB is used, as it corresponds the case of the assured existence of codes.

The maximum equivalent values of convolutional codes, which probability of error on the decoder output of at the level of $10^{-6}$ is provided, are represented in a separate table for every code rate – Table 3.

TABLE III. MAXIMUM EQUIVALENT PARAMETERS OF CONVOLUTIONAL CODES

| *R* | *1/3* | | *1/2* | | *2/3* | | *3/4* | |
|---|---|---|---|---|---|---|---|---|
| | min | max | min | max | min | max | min | max |
| ν | 2 | 7 | 2 | 8 | 2 | 8 | 6 | 8 |
| $d_{equ}$ | 11 | 59 | 7 | 28 | 3 | 11 | 5 | 7 |
| $n_{equ}$ | 45 | 320 | 40 | 225 | 11 | 145 | 55 | 110 |
| $h^2$ | 4 | 2.35 | 5.25 | 3.2 | 8 | 4,85 | 6.85 | 5.75 |
| $P_{er}$*$10^{-3}$ | 22.7 | 65.6 | 10.9 | 36.8 | 2.33 | 13.8 | 4.43 | 8.24 |

The equivalent values of convolutional codes for two maximum points for every code rate are represented in a Table 3. Point minimum "min" responds to a convolutional code with the least code constraint length *ν* and channel with the best energy. The peak-point of "max", on the contrary, responds to a code with higher code constraint length *ν* and channel with worst energy.

The found values are maximum, because at worsening of energy in the channel, the use of the indicated convolutional codes will not allow to correct the necessary amount of errors for providing of the set probability of error. At the same time, at the improvement of energy in a communication channel marked convolutional codes will not allow to increase code rate, increasing useful information content, which is passed. Let's compare the possibilities of convolutional codes for code rate 1/3 (table 4) and 3/4 (table 5) and possibilities of BCH-block codes.

TABLE IV. COMPARISON OF CONVOLUTIONAL CODE WITH R = 1/3 WITH THE BCH BLOCK CODES

| Code | $R_c$ | *n* | *t* | $h^2$ | $P_b$ |
|---|---|---|---|---|---|
| Convolutional, ν = 8 | 1/3 | (equv.) 320 | 29 | 2.35 | $10^{-6}$ |
| BCH | 0.34 | 1023 | 87 | 2.35 | $2*10^{-10}$ |
| BCH | 0.34 | 1023 | 87 | 2.20 | $10^{-6}$ |
| BCH | 0.43 | 1023 | 73 | 2.35 | $10^{-6}$ |

TABLE V. COMPARISON OF CONVOLUTIONAL CODE WITH R = 3/4 WITH THE BCH BLOCK CODES

| Code | *R* | *n* | *t* | $h^2$ | $P_{бит}$ |
|---|---|---|---|---|---|
| Convolutional, ν = 8 | 3/4 | (equv.) 110 | 3 | 5.75 | $10^{-6}$ |
| BCH | 0,75 | 1023 | 26 | 5.35 | 0 |
| BCH | 0,75 | 1023 | 26 | 3.20 | $10^{-6}$ |
| BCH | 0.99 | 1023 | 1 | 5,75 | $10^{-6}$ |

The results, which are summarized in the tables 4 and 5, show that the use of codes of BCH gives advantage in code rate, or in energy, or in authenticity in comparing to the best maximum convolutional codes at the fixed requirements to authenticity of reception. It appears thus, that code length of block code is anymore than equivalent block of convolutional code length.

The dependence for a maximum code with speed ¾ will be built.

Fig. 5illustrates that convolutional code{8, 3/4} has the samecorrectiveopportunitiesasblockcodeBCH(*n* =1023)at code rate$R_c$=0.99 ($h^2$=3,25), and at equal code rate$R_c$=0.75 provides the necessaryreliabilitywhen$h^2$=5,75.

The found power bound $h^2$ for convolutional codes application allows to formulate structural conclusions about expedience of the use of different types of error-correcting code. For example, in a channel with the non-coherent reception of signals of phase manipulation, when on the decoder output it is necessary to achieve probability of error $10^{-6}$, application of convolutional codes expedient at terms overstatement by energy of critical value $h^2_{crit} > 2.35$.

In this case application of convolutional codes at worst power ratio $h^2$ is ineffective according to the criterion of the maximal Shannon bound approaching.

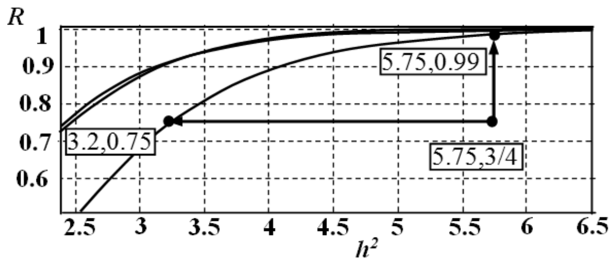


Fig.5 Comparison of error-correcting possibilities of maximum convolutional code with parameters (ν, Rc, h2) = (8, 3/4, 5.75) and block codes of BCH (n, R, h2) = (1023, 0.75, 3.2) with t = 26 and BCH (1023, 0.99, 5.75) with t =1.

If the power ratio is higher than $h^2_{crit}$ for the fixed requirements to quality of connection, kind of modulation and signal processing, it is profitable to use convolutional codes, as they are better, than block codes due to procedures realization simplicity of forming and treatment of signals.

## Conclusions

Technique of convolutional codes analysis on the criterion of the maximal Shannon bound approaching is proposed. This technique is based on the synthesis of optimal on the same criterion block codes. The set of convolutional codes equivalent parameters that correspond to the initial parameters of optimal block code is obtained as a result. Each convolutional code {ν, $R$,$d_f$} it is possible to characterize with the block codes equivalent parameters {$n_{equ}$, $R$, $d_{equ}$}, what can be found due to Plotkin and Warshamov-Gilbert bounds.

Formulated technique of convolutional codes synthesis in a channel with preset parameter and energy allows defining an optimal convolutional code which would have the least surplus, most code rate and assuredly provided necessary bit error rate.

It is shown on the example of convolutional codes with the most value of code constraint length ν = 8 for code rate $R$ = 1/3; 3/4, that it is possible to find block codes which can provide the best error-correcting, or greater code rate for set $h^2$. In this case, the length of block code *n* should be more than equivalent block code length of convolutional code $n_{equ}$ at $R$ = *const*. In the case when block and convolutional code are equivalent in error-correcting possibilities for power ratio, better than critical ($h^2 > h^2_{crit}$), the convolutional code should be used, as it is simpler in realization.

## References

1. C. E. Shannon "A Mathematical Theory of Communication" // Bell System Technical Journal, 1948, Vol. 27, pp. 379–423, 623–656.
2. Gallager R.G. “Information Theory and Reliable Communication”/ USA. – 1968, 588 pp.
3. G. C. Clark, J. B. Cain. "Error-Correction Coding for Digital Communications". Plenum Press: New York, NY, 1981.
4. Uryvsky L.O., Prokopenko K.A.,Peshkin A.M. " Error - Correcting Codes acording to Shannon’s limit" // Telecommunication sciences.–Kyiv: NTUU 'KPI' – 2011, vol. I, pp. 41-47.
5. W.E. Ryan and Shu Lin. "Channel Codes Classical and Modern". New York, NY: Cambridge University Press, 2009.

Received in final form May 13, 2014