

UDC 629.052.3

DOI: 10.20535/2411-2976.22025.63-67

## CORRECTION OF FLIGHT SPEED MEASUREMENTS USING RADIO NAVIGATION ANGLE-RANGE MEASURING SYSTEM DATA

Volodymyr M. Vasyliev

Faculty of Radio Engineering  
Igor Sikorsky Kyiv Polytechnic Institute, Kyiv, Ukraine

**Background.** Integration of various navigation devices and systems by data combining is widely used for improving the reliability and accuracy. The classical combining method is that the greatest effect is achieved when combined systems measure the same parameters, and the frequency characteristics of the measurement errors differ significantly. This makes it possible to use the classical compensation and filtering scheme using a Kalman filter.

**Objective.** This article presents the scheme, algorithm and program for combining flight speed data received from the on-board air data system and data received from the ground-based radio navigation angle-range measuring system. The result of such combining is used to correct the airspeed measurements.

**Methods.** The peculiarity of the proposed integration procedure is that it consists of several stages. First, the ground speed is recovered by optimal estimation based on the angle-range measurement data. At the second stage, the estimated ground speed is integrated with the airspeed measurements using the compensation and filtering scheme, simultaneously correcting the airspeed measurements.

**Results.** To verify the proposed integration scheme and algorithm, computer statistical modelling has been performed using a Kalman filter. The obtained results demonstrate the possibility of implementing the proposed scheme and algorithm for correcting the airspeed measurement system with increased accuracy.

**Conclusions.** The advantages of the proposed design are that the process of ground speed recovery and the process of data combining are performed separately, which makes it possible to control the correction process.

**Keywords:** *navigation; integration; flight speed; air data measuring system; angle-range measuring system; optimal estimation.*

### Introduction

An effective method that improves the performance of navigation systems is their integration with optimal signal processing.

Modern aircraft are equipped with a large number of sensors and devices that can operate on different physical principles to determine the same navigation parameters, as well as parameters that may be functionally related.

The methods of integrating various navigation systems are quite fully described in many works [1-4].

Unlike known methods and algorithms for integrating various navigation devices and systems, which describe the combined mathematical model for navigation parameters estimation, this article proposes the scheme and algorithm for phased integration of the on-board air data system (ADS), which provides information about airspeed, with measurements from a ground-based angle-range measuring system (ARMS). As a result of such integration, the airspeed is corrected and refined.

### Mathematical models of measurement errors

To solve the problem under consideration, it is necessary to have mathematical models of the

integrated system errors and the functional relationship between measurements and the parameters being estimated.

In general, an expression describing the aircraft's motion can be written as

$$\mathbf{r} = \mathbf{V}, \quad (1)$$

where  $\mathbf{r}$  is the radius vector from a reference angle-range system to the aircraft;  $\mathbf{V}$  is the aircraft velocity relative to the ground.

The velocity in (1) can be written, taking into account the errors of velocity determination, in the form

$$\mathbf{V} = \mathbf{V}_0 + \Delta \mathbf{V}, \quad (2)$$

where  $\mathbf{V}_0$  is the aircraft velocity determined by the air data system;  $\Delta \mathbf{V}$  is the air data errors. These errors may be caused by wind and heading system errors.

Considering the process of aircraft motion in a rectangular coordinate system  $\mathbf{x}$ - $\mathbf{y}$ , we can write

$$\begin{aligned} \dot{x} &= V_{0x} + \Delta V_x; \\ \dot{y} &= V_{0y} + \Delta V_y, \end{aligned} \quad (3)$$

where  $V_{0x}, V_{0y}, \Delta V_x, \Delta V_y$  are the corresponding components of rectangular coordinates according to expression (2).

The errors  $\Delta V_x, \Delta V_y$  are independent random processes and each of them is described by an exponential correlation function.

Shaping filters that generate the processes  $\Delta V_x, \Delta V_y$  (3) are given by equations

$$\begin{aligned}\Delta \dot{V}_x &= -\alpha_x \Delta V_x + \sigma_x \sqrt{2\alpha_x} n_x; \\ \Delta \dot{V}_y &= -\alpha_y \Delta V_y + \sigma_y \sqrt{2\alpha_y} n_y,\end{aligned}\quad (4)$$

where  $n_x, n_y$  are uncorrelated white noise processes with the parameters of  $N(0,1)$ ;  $\sigma_x, \sigma_y$  and  $\alpha_x, \alpha_y$  are RMS values and the correlation time, respectively.

A radio navigation angle-range measuring system measures the distance  $\rho^*$  from the ground station to the object and the azimuth  $\theta^*$ .

It is assumed that distance measurement errors  $\Delta \rho = v_\rho$  and azimuth measurement errors  $\Delta \theta = v_\theta$  are random, independent, normally distributed variables with root-mean-square values  $\sigma_\rho, \sigma_\theta$  respectively.

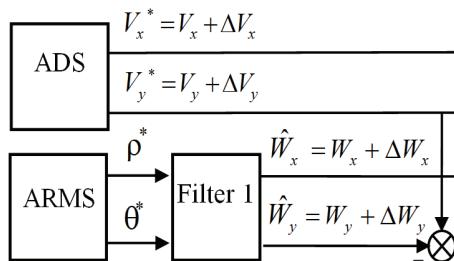
### Flight speed measurements correction scheme based on the angle-range measurements

Fig. 1 shows the proposed scheme for correcting airspeed measurements with two-stage processing of information coming from the air data system and the angle-ranging measuring system.

At the first stage, the components  $\hat{W}_x, \hat{W}_y$  of the ground speed are recovered by an optimal estimation from the measurements  $\rho^*, \theta^*$  of the angle-range measuring system.

The result of the recovery performed using the optimal filter can be represented as

$$\hat{W}_x = W_x + \Delta W_x; \quad \hat{W}_y = W_y + \Delta W_y, \quad (5)$$



where  $W_x, W_y$  are the real values of ground speed components;  $\Delta W_x, \Delta W_y$  are the corresponding errors.

Next, the velocity estimations  $\hat{W}_x, \hat{W}_y$  (5) are integrated with the data from the on-board air data  $V_x^*, V_y^*$  using the compensation and filtering method.

Now, the velocity can be written as the sum of the actual velocity value and measurement errors, i.e.

$$V_x^* = V_x + \Delta V_x; \quad V_y^* = V_y + \Delta V_y. \quad (6)$$

As a result of compensation, the differences between air data measurement error and the recovery error are formed as

$$\Delta V_x - \Delta W_x + s_x; \quad \Delta V_y - \Delta W_y + s_y, \quad (7)$$

where  $s_x = V_x - W_x$ ;  $s_y = V_y - W_y$  are the differences between the deterministic components of airspeed and ground speed, which can be treated as forcing functions for the next filter.

Subtraction results (7) are fed to a second filter, which extracts the air data measurement errors as the filter estimations  $\Delta \hat{V}_x, \Delta \hat{V}_y$ .

At the final stage, the airspeed values are corrected with the result as

$$\hat{V}_x = V_x + (\Delta V_x - \Delta \hat{V}_x); \quad \hat{V}_y = V_y + (\Delta V_y - \Delta \hat{V}_y). \quad (8)$$

The corrected airspeed values (8) get close to the true value because the measurement errors are reduced according to the relations

$$(\Delta V_x - \Delta \hat{V}_x) < \Delta V_x, \quad (\Delta V_y - \Delta \hat{V}_y) < \Delta V_y.$$

### Preparation for the computer simulation

Discretisation of the modelled processes (4), describing the errors of the airspeed, yields the following equations:

$$\Delta V_{x(i)} = e^{-\alpha_x \Delta t} \Delta V_{x(i-1)} + \sigma_x \sqrt{2\alpha_x} (1 - e^{-\alpha_x \Delta t}) / \Delta t n_{x(i-1)}; \quad (9)$$

$$\Delta V_{y(i)} = e^{-\alpha_y \Delta t} \Delta V_{y(i-1)} + \sigma_y \sqrt{2\alpha_y} (1 - e^{-\alpha_y \Delta t}) / \Delta t n_{y(i-1)}.$$

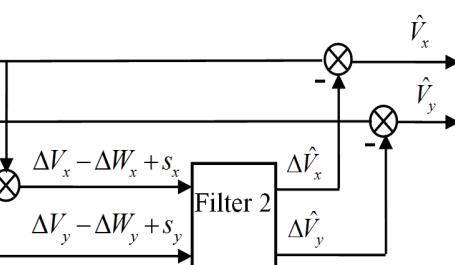


Fig. 1. The scheme of airspeed correction

where  $\Delta t = t_i - t_{i-1}$  is the discretisation step.

To recover the ground speed values based on the angle-range measuring data, it is necessary to determine the relation between the range and azimuth measurements  $\rho^*, \theta^*$  with the elements of the state vector  $\mathbf{X} = [x, V_x, y, V_y]^T$  in a rectangular coordinate system.

This relation is nonlinear

$$\mathbf{Z}_i = h(\mathbf{X}_i) + \mathbf{v}_i, \quad (10)$$

$$\text{where } \mathbf{Z} = \begin{bmatrix} \rho^* \\ \theta^* \end{bmatrix}; \quad h(\mathbf{X}) = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} \sqrt{x^2 + y^2} \\ \arctg \frac{x}{y} \end{bmatrix}; \quad \mathbf{v} = \begin{bmatrix} v_\rho \\ v_\theta \end{bmatrix}$$

is the vector of random, uncorrelated distance and azimuth measurement errors with covariance matrix

$$\mathbf{R} = M[\mathbf{v}_i \mathbf{v}_i^T] = \begin{bmatrix} \sigma_\rho^2 & 0 \\ 0 & \sigma_\theta^2 \end{bmatrix}, \quad (11)$$

where  $\sigma_\rho$ ,  $\sigma_\theta$  are the known root mean square errors of distance and azimuth measurements, respectively.

When applying the Kalman filter, it is necessary to provide a linear formulation of the problem. For example, linearization can be performed in two ways [5].

The first one is the linearisation of the equation (10) by expanding it into a Taylor series with respect to the estimated state vector and obtaining the model in linearised form

$$\Delta \mathbf{Z} = \mathbf{H} \Delta \mathbf{X} + \mathbf{v}, \quad (12)$$

where  $\Delta \mathbf{Z} = \mathbf{Z} - h(\hat{\mathbf{X}})$ ;  $\Delta \mathbf{X} = \mathbf{X} - \hat{\mathbf{X}}$ ;  $\hat{\mathbf{X}}$  is the estimated state vector;

$$\mathbf{H} = \frac{\partial h}{\partial \mathbf{X}} = \begin{bmatrix} \frac{x}{\sqrt{x^2 + y^2}} & 0 & \frac{y}{\sqrt{x^2 + y^2}} & 0 \\ \frac{y}{x^2 + y^2} & 0 & -\frac{x}{x^2 + y^2} & 0 \end{bmatrix}.$$

Another way is as follows. The range and azimuth measurements are converted into a rectangular coordinate system

$$x^* = \rho^* \sin \theta^*; \quad y^* = \rho^* \cos \theta^*. \quad (13)$$

This allows applying a simple linear filter algorithm with

$$\mathbf{X} = [x, V_x, y, V_y]^T; \quad \mathbf{Z} = \begin{bmatrix} x^* \\ y^* \end{bmatrix}; \quad \mathbf{v} = \begin{bmatrix} v_x \\ v_y \end{bmatrix}; \quad (14)$$

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

In this case, the covariance matrix of converted measurement errors according to (13) will be written as

$$\mathbf{R} = M[\mathbf{v}_i \mathbf{v}_i^T] = \begin{bmatrix} \sigma_x^2 & \sigma_{xy}^2 \\ \sigma_{yx}^2 & \sigma_y^2 \end{bmatrix}, \quad (15)$$

where

$$\begin{aligned} \sigma_x^2 &= \sigma_\rho^2 \sin^2 \theta + \rho^2 \sigma_\theta^2 \cos^2 \theta; \\ \sigma_y^2 &= \sigma_\rho^2 \cos^2 \theta + \rho^2 \sigma_\theta^2 \sin^2 \theta; \\ \sigma_{xy}^2 &= \sigma_{yx}^2 = \sin \theta \cos \theta (\sigma_\rho^2 - \rho^2 \sigma_\theta^2). \end{aligned}$$

### Computer simulation results

Verification of the proposed airspeed values correction scheme using angle-range measuring system data was carried out by computer simulation using the Kalman filter.

When simulating ground speed recovery based on measurements of the angle-range measuring system, the following initial data for (10) - (15) were used: object flight speed  $V = 100$  m/s;  $\sigma_\theta = 0.25$  degree;  $\sigma_\rho = 50$  m; time discretisation step  $\Delta t = 1$  s.

For modelling the processes (9), it was assumed that  $\sigma_x = \sigma_y = 20$  m/s;  $1/\alpha_x = 1/\alpha_y = 360$  s.

Fig. 2 shows the result of estimating the error of air data measurement (that is the output of the final filter) (curve 1) in comparison with the error modelled according to equation (9) (curve 2).

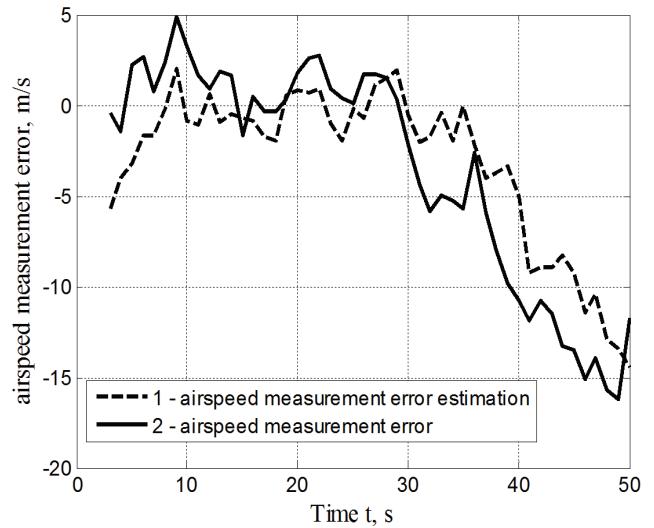


Fig 2. Estimation of the airspeed measurement error

Fig. 3 shows the root mean square error of air data system measurement error estimation (curve 1) as a statistical evaluation result, and curve 2 shows the result obtained by solving the Kalman filter equation for the covariance matrix of the estimation errors.

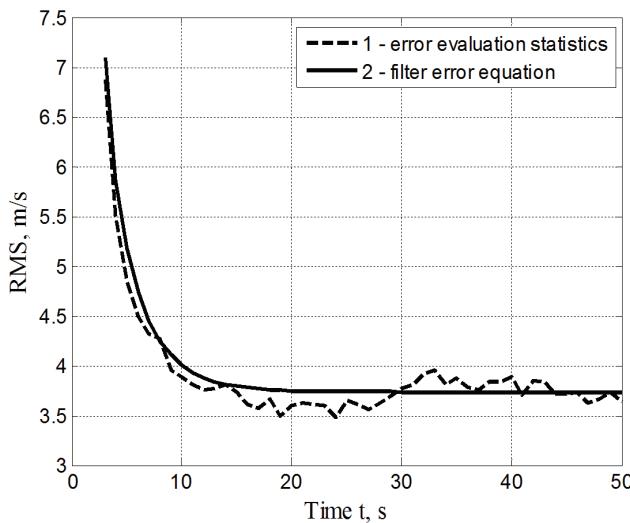


Fig. 3. RMS error of the airspeed measurement errors estimation

## References

1. Mohinder S. Grewal, Angus P. Andrews, Chris G. Bartone. Global Navigation Satellite Systems, Inertial Navigation and Integration. – New York, NY: John Wiley & Sons, 2001. – 392 p.
2. Rogers R. M. Applied Mathematics in Integrated Navigation Systems (3rd ed.). – Reston, VA: American Institute of Aeronautics and Astronautics, 2007. – 408 p.
3. Zakharin F.M., Syniehlazov V.M., Filiashkin M.K. Alhorytmichne zabezpechennia inertsialno-sputnykovykh system navihatsii [Algorithmic software of inertial-satellite navigation systems]. – Kyiv: Vyd-vo Nats. Avia. Un-tu “NAU-druk”, 2011. – 320 p. [in Ukrainian].

## Conclusions

The proposed scheme and procedure of airspeed measurement correction based on measurement data incoming from the radio navigation angle-range measuring system consists of several stages.

At the first stage, the object's ground speed is recovered using the data of the angle-range measuring system.

At the second stage, the recovered and estimated velocity is combined with the data from the on-board air data measurement system using a compensation and filtering scheme, while simultaneously correcting the air data measurement values.

Computer simulation results demonstrate the feasibility of implementing the proposed scheme and algorithm for improving the accuracy of the airspeed measurement system.

The advantage of the proposed design is that ground speed recovery and data combining are performed separately, allowing for control over the correction process.

4. Vasyliev V.M., Dolintse B.I. Integration of Inertial and Satellite Navigation Systems with using Corrective Circuits and Filtering //IEEE 4rd International Conference on Methods and Systems of Navigation and Motion Control. Proceedings. – 2016. – pp. 275-278.
5. Vasyliev V. M., Naumenko K. V. Tochnist alhorytmiv traiektornoi otsinky za danymi dalekomirnykh system sposterezhennia pry riznykh sposobakh linearyzatsii [Accuracy of trajectory estimation algorithms based on distance-measuring surveillance data using different linearization methods] //Visnyk Natsionalnoho tekhnichnogo universytetu Ukrayny “KPI”, seriia Radiotekhnika. Radioaparatobuduvannia. – 2012. – No 51. – pp. 77-87 [in Ukrainian].

**Васильєв В.М.**

**Корекція вимірювань швидкості польоту за даними радіонавігаційної кутомірно-далекомірної системи**  
Радіотехнічний факультет КПІ ім. Ігоря Сікорського, м. Київ, Україна

**Проблематика.** Інтеграція різних навігаційних пристрій та систем з об'єднаною обробкою даних широко використовується для підвищення надійності та точності. Класичний метод об'єднання полягає в тому, що найбільший ефект досягається при об'єднанні систем, які вимірюють однакові параметри, за умови, що частотні характеристики їх похибок вимірювання суттєво відрізняються. Це дозволяє використовувати класичну схему компенсації та фільтрації з використанням фільтра Калмана.

**Мета дослідження.** У цій статті представлено схему, алгоритм та програму для об'єднання даних про швидкість польоту, отриманих від бортової системи повітряних сигналів, та даних, отриманих від наземної радіонавігаційної

кутомірно-далекомірної системи. Результат об'єднання використовується для корекції даних вимірювання швидкості польоту.

**Методика реалізації.** Особливістю запропонованої процедури інтегрування є те, що вона складається з кількох етапів. Спочатку відновлюється шляхова швидкість польоту з її оптимальною оцінкою за даними вимірювань кутомірно-далекомірної системи. На другому етапі оцінена шляхова швидкість комплексується з вимірюваннями повітряної швидкості за схемою компенсації та фільтрації, одночасно коригуючи вимірювання повітряної швидкості.

**Результати дослідження.** Для перевірки запропонованої схеми та алгоритму інтегрування було проведено комп'ютерне статистичне моделювання з використанням фільтра Калмана. Отримані результати демонструють можливість реалізації запропонованої схеми та алгоритму для корекції системи вимірювання швидкості польоту з підвищеннем точності.

**Висновки.** Перевагою запропонованої проекта є те, що процес відновлення та оцінки шляхової швидкості і процес комплексної обробки даних виконуються окремо, що дозволяє контролювати процес корекції.

**Ключові слова:** *навігація; інтеграція; швидкість польоту; система повітряних сигналів; кутомірно-далекомірна система; оптимальна оцінка.*

Received by the Editorial Office  
October 2, 2025

Accepted for publication  
November 25, 2025