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COUPLING OSCILLATIONS OF LATTICES OF DIFFERENT DIELECTRIC RESONATORS

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Background. The development of many elements of modern communication systems is increasingly based on the use of various types of dielectric resonators (DR). The theory of coupled oscillations of resonators is the basis for further calculations and optimisation of the scattering matrices of electromagnetic waves on various devices. When calculating devices built on a large number of resonators, direct numerical methods are often not effective. They usually require the use of powerful computers, therefore, the calculation of elements on a large number of DR is impossible without building analytical models of complex structures based on electrodynamic modelling.

Objective. The study aims to find analytical expressions for the frequencies and distributions of electromagnetic fields of natural oscillations of lattices, consisting of a large number of various types of dielectric resonators for use in various devices of optical communication systems. To solve this problem, a linear system of equations, which relates the complex amplitudes and frequencies of the resonators, obtained earlier from the perturbation theory, was used.

Methods. To find analytical expressions, methods of matrix theory are used. In this case, both known methods of calculating the determinants of tri-diagonal and circulant matrices are used, as well as their modifications related to the calculations of more complex matrices, which, after transformations, are reduced to much simpler formulas. The final result is the receipt of new general analytical formulas for describing coupled oscillations of lattices consisting of a large number of dielectric resonators of various types.

Results. Coupled oscillations of one-dimensional linear lattices of two types of dielectric resonators are considered. New analytical expressions for complex frequencies and amplitudes of resonators, as well as Q-factor expressions without restrictions on their number, are obtained. A new model of natural oscillations of two-dimensional lattices, consisting of dielectric resonators of two different types, is constructed. General analytical solutions are found for the frequencies and amplitudes of coupled oscillations for two types of two-dimensional lattices with different arrangements of resonators. Analytical solutions are found for the amplitudes and frequencies of coupled oscillations of two axially symmetric ring lattices with different types of resonators, which are characterised by different placement symmetry in free space. The obtained general analytical expressions for the frequencies of coupled oscillations are compared with the results of calculations obtained by numerically, by solving linear systems of equations. A very good agreement between the solutions obtained by the two methods is demonstrated.

Conclusions. The developed theory is the basis for the design of many devices of the optical wavelength range, which are built on the basis of the use of a large number of dielectric resonators of various types. The obtained new analytical expressions for calculating coupled oscillations of dielectric resonators allow building new more efficient models of scattering for optimisation of various optical communication devices.

Keywords: *dielectric resonator; eigen oscillations; lattice; tridiagonal matrix; circulant.*

I. INTRODUCTION

Today lattices of dielectric resonators (DR) [1 – 30] find application in different devices of the microwave, theraherce, infrared and optical wavelength ranges. Most widely it is used in filters [1, 3, 6, 7, 11, 17, 18 – 21, 29]; multiplexers [2, 5, 8, 9, 12, 15]; antennas [13, 14, 16, 24]; modulators [22, 26, 27]; as well as lasers [23, 28]; switches [4]; sensors [10]; meta lenz [25], to name a few. Such lattices, as a rule, contain a very large number of resonators, which significantly complicates the calculation and optimisation of their parameters.

The calculation and optimisation of devices built on lattices of dielectric resonators is usually based on the use of direct numerical methods for solving the system of Maxwell's equations, as well as on the use of various analytical solutions of these equations for scattering problems, obtained using various approximations. In work [34], an analytical theory of scattering on systems of dielectric resonators is developed, based on the use of perturbation theory. The indicated theory is based on expansions of solutions of scattering problems in terms of basis functions of coupled oscillations of resonators. Obtaining such basis functions is a rather complex independent task, especially in cases where the number of resonators is large. Dielectric resonator lattices typically contain hundreds of elements, so constructing basis functions for their natural oscillations is a complex task. Numerical calculation of such basis functions is also often very difficult. However, in some cases, such basis functions can be calculated in analytical form.

The aim of this study is to obtaining analytical solutions of basis functions and analyses problems of coupled oscillations of 1-2 dimensional lattices of different dielectric resonators.

II. Coupled oscillations of Dielectric Resonators

In the [34] it's looked for a solution to the problem of coupled oscillations of a system of N DRs obtained in the form of an expansion their field $(\mathbf{e},\mathbf{h})$ in terms of the natural oscillations of the same, but isolated resonators $(\mathbf{e}_s,\mathbf{h}_s)$:

$$\begin{pmatrix}\mathbf{e}\\ \mathbf{h}\end{pmatrix}=\sum_{s=1}^{N} b_s \begin{pmatrix}\mathbf{e}_s\\ \mathbf{h}_s\end{pmatrix}; \qquad (1)$$

In general, was obtained an equation system for amplitudes $\|b_s\|$ (1):

$$\sum_{s=1}^{N}\kappa_{st}\ b_s-\lambda b_t=0\,; \qquad \left(s,t=1,\ 2,...,\ N\right), \qquad (2)$$

where

$$\lambda\ =2(\tilde{\omega}-\omega_0)/\omega_0=2(\delta\omega/\omega_0+i\omega''/\omega_0)\,; \qquad (3)$$

$\tilde{\omega}$ - complex frequency of coupled oscillations; ω_0 - real part of the frequency of isolated DRs; $\delta\omega=\mathrm{Re}(\tilde{\omega}-\omega_0)$; $\omega''=\mathrm{Im}(\tilde{\omega})$.

The distribution of the amplitudes of coupled oscillations of a system of resonators $\|b_s\|$ was formulated as an eigenvalues problem for a finite-dimensional coupling operator $K=\|\kappa_{st}\|$:

$$K=\begin{pmatrix} i\tilde{k}_1 & \kappa_{21} & \kappa_{31} & \cdots & \kappa_{N1}\\ \kappa_{12} & i\tilde{k}_2 & \kappa_{32} & \cdots & \kappa_{N2}\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ \kappa_{1,N-1} & \kappa_{2,N-1} & \kappa_{3,N-1} & \cdots & \kappa_{N,N-1}\\ \kappa_{1,N} & \kappa_{2,N} & \kappa_{3,N} & \cdots & i\tilde{k}_N \end{pmatrix}. \qquad (4)$$

where $\kappa_{st}\neq\kappa_{ts}$ - are coupling coefficients of a s–th and t–th for different DR. Diagonal elements of the coupling operator matrix K determined only by the magnitude of the radiation of s - th partial resonators, represented by coupling coefficients $\tilde{k}_s$.

Equating to zero, the determinant of system (2),

$$\det\left\|\kappa_{st}(1-\delta_{st})+(i\tilde{k}_s-\lambda)\delta_{st}\right\|=0\,, \qquad (5)$$

the characteristic equation was obtained, the solution of which determines the frequency splitting that arises due to the electromagnetic influence of the resonators. In this case, each non-degenerate value of the frequency $\tilde{\omega}^s=\omega^s+i\omega^{s''}$ $(s=1,2,...,N)$ of s-th natural oscillations of the system corresponds to its own column vector:

$$\left\|b_t^s\right\|=\begin{pmatrix} b_1^s\\ b_2^s\\ \vdots\\ b_N^s\end{pmatrix}, \qquad (s,t=1,2,...,N) \quad (6)$$

of the coupling operator K (4), determining the distribution of amplitudes of partial resonators. Thus, in the absence of degeneracy of the DR natural oscillations, for a system consisting of N resonators is characterised by a $N\times N$ matrix of amplitudes of coupled oscillations:

$$B=\begin{pmatrix} b_1^1 & b_1^2 & \cdots & b_1^N\\ b_2^1 & b_2^2 & \cdots & b_2^N\\ \vdots & \vdots & \vdots & \vdots\\ b_N^1 & b_N^2 & \cdots & b_N^N\end{pmatrix}. \qquad (7)$$

In general cases, the solution of the equation system (2) is carried out numerically, but, in some cases can be found in analytical form, which significantly simplified and increases the speed of calculations, especially for large systems ($N \gg 1$). We have considered particular solutions of (2) for lattices of different DRs.

III. Coupling oscillations of one-dimensional lattices of Dielectric Resonators

To find the eigenvalues (3) and eigenvectors of K matrix (4) for one-dimensional DR lattice, a relatively simple case was considered, when each resonator is coupled only to its neighbours. We call it the first approximation (Fig. 1, a). The second approximation determined the solution obtained under the condition of taking into account the coupling of resonators with neighbours and neighbouring neighbours (Fig. 1, b). Continuing this process, if desired, we can calculate more accurately the parameters of the lattice.

In the simple case of a one-dimensional lattice of N identical DRs, the system of equations (2) takes the form:

$$\kappa_{t-1,t} b_{t-1} + (i\tilde{k}_t - \lambda) b_t + \kappa_{t+1,t} b_{t+1} = 0, \ (t = 1, 2, ..., N). \quad (8)$$

For identical DRs, the coupling coefficients of the resonators with open space are equal to each other: $\tilde{k}_t = \tilde{k}_0$. And if the resonators are located at equal distances from each other: $\kappa_{t-1,t} = \kappa_{t+1,t} = \kappa_{12}$ then the system is simplified

$$\kappa_{12} b_{t-1} + (i\tilde{k}_0 - \lambda) b_t + \kappa_{12} b_{t+1} = 0 \quad (9)$$

The solution of (9) is well known [32]; we represented it as a set of normalized eigenvectors with an amplitude distribution:

$$b_t = b_0 \sin(\theta t), \quad (t = 1, 2, ..., N) \quad (10)$$

Where θ is a constant that determines the phase of the natural oscillations of the t-th resonator in the lattice.

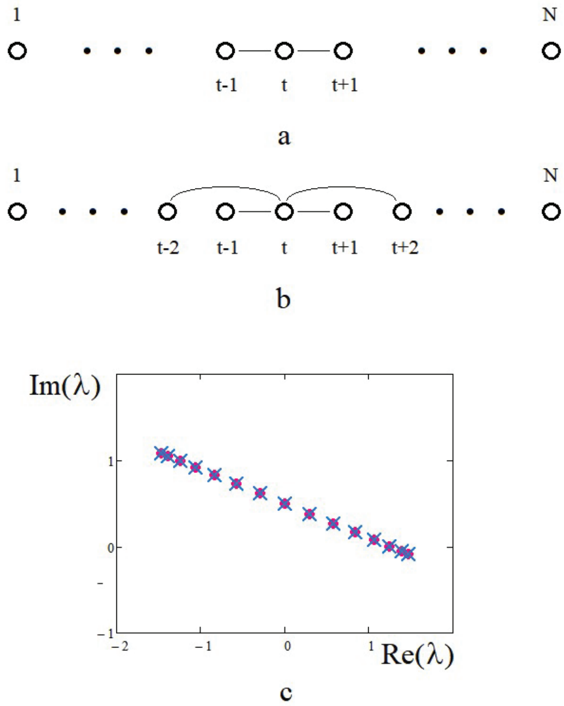

Fig. 1. One-dimensional lattices of N (a, b) identical DRs. Results of the numerical calculation of the eigenvalues are dots; the analytical ones are crosses (a, c), obtained for the first approximation (a): $N = 15$; $\tilde{k}_0 = 0,5$; $\kappa_{12} = 0,75 - 0,3i$; (c). (Here and below, the numerical values of the coupling coefficients are taken arbitrarily).

Substituting (10) into equations (9), we obtained:

$$2\kappa_{12} \cos\theta + (i\tilde{k}_0 - \lambda) = 0 \quad (11)$$

We supplemented equations (10), (11) with the condition of symmetry of the coupling oscillations amplitude distribution for all resonators of the lattice $|b_v| = |b_{N-v+1}|$ ($v = 1, 2, ..., N$); from which we found:

$$|\sin(\theta v)| = |\sin[\theta(N - v + 1)]|. \quad (12)$$

The solution to equation (12) has also known form:

$$\theta = \theta^s = \frac{s\pi}{(N+1)}, \ (s = 1, 2, ..., N) \quad (13)$$

The characteristic equation (11) together with (13) determines the N eigenvalues found in the first approximation:

$$\lambda^s = i\tilde{k}_0 + 2\kappa_{12}\cos\left[\frac{s\pi}{(N+1)}\right], \qquad (14)$$

where each value λ^s corresponds s-th normalized vector (6) of natural oscillations of the resonator system:

$$b_t^s = b_0^s \sin(\theta^s t),\ (t = 1,2,...,N) \qquad (15)$$

An interesting feature of the found solution is the reality of the phase distribution functions of the amplitudes of coupled oscillations, obtained in the approximation of interaction of only neighbouring resonators. As follows from (13), these functions do not depend on the electromagnetic parameters but are determined only by the number of resonators in the lattice.

Fig. 1, c shows the result of comparison of the eigenvalues of a linear lattice consisting of 15 DRs, calculated using formula (14) (crosses) and the eigenvalues obtained numerically for the truncated coupling matrix (4) (dots) for the first approximation.

From (3), (14) we easily find an analytical expression for the quality factor of coupled oscillations of a linear one-dimensional lattice:

$$Q^s = \frac{1 + \mathrm{Re}(\kappa_{12})\cos\theta^s}{\tilde{k}_0 + 2\,\mathrm{Im}(\kappa_{12})\cos\theta^s},$$

It is interesting that even if isolated resonators do not radiate into the structure under consideration: $\tilde{k}_0 = 0$, the appearance of other resonators coupled to each other can lead to the appearance of visible resonances, with a quality factor

$$Q^s = [1 + \mathrm{Re}(\kappa_{12})\cos\theta^s]/[2\,\mathrm{Im}(\kappa_{12})\cos\theta^s].$$

The absence of coupling between the resonators determines the quality factor of the lattice oscillations, equal to the quality factor of one isolated resonator $1/\tilde{k}_0$.

From (13) it also follows that for $N \to \infty$, the eigenvalues occupy an "interval" of frequencies determined by the deviation $\theta^s \in (0,\pi)$, therefore from (14), (3) we find:

$$\mathrm{Re}[\delta\omega^s/\omega_0] = 1/2\,\mathrm{Re}[\lambda^s] \in (-\mathrm{Re}[\kappa_{12}], \mathrm{Re}[\kappa_{12}]),$$

and

$$\omega''/\omega_0 = 1/2\,\mathrm{Im}[\lambda^s] \in (\tilde{k}_0/2 - \mathrm{Im}[\kappa_{12}], \tilde{k}_0/2 + \mathrm{Im}[\kappa_{12}])$$

for each type of oscillation of partial resonators.

The formula for the second approximation may be found taking into account the coupling with the two closest resonators on each side (Fig. 1, b).

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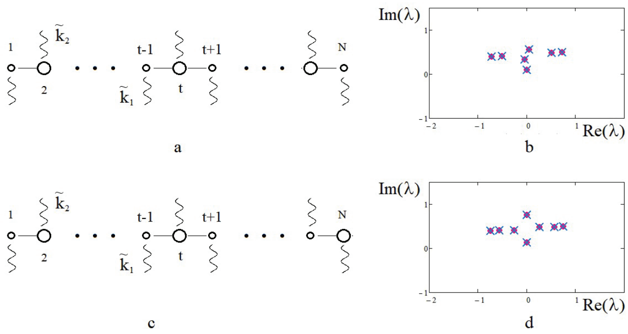


Fig. 2. One-dimensional lattices of different DR; odd number of resonators (a); even number of resonators (c). (b) $N = 7$; (d) $N = 8$; $\tilde{k}_1 = 0{,}1$; $\tilde{k}_2 = 0{,}8$; $\kappa_{12} = 0{,}8 - 0{,}3i$; $\kappa_{21} = 0{,}2 + 0{,}1i$.

Equations describing coupled oscillations of resonators of different types, are described formally by the same system of equations (2), but it is necessary to take into account that the coefficients of mutual coupling become asymmetrical: $\kappa_{12} \neq \kappa_{21}$. In this case, obtaining simple solutions in analytical form can be performed only for lattices of alternating DR (Fig. 2).

We have examined lattices of this type (Fig. 2) in more detail. For this purpose, we represented the system (2), also considering the coupling only between adjacent resonators:

$$\begin{cases} \cdots \\ \kappa_{21}b_{t-1} + (i\tilde{k}_1 - \lambda)b_t + \kappa_{21}b_{t+1} = 0 \\ \kappa_{12}b_t + (i\tilde{k}_2 - \lambda)b_{t+1} + \kappa_{12}b_{t+2} = 0 \\ \cdots \end{cases}; \qquad (16)$$

where $(t = 1,2,...,N)$ and $\kappa_{12} \neq \kappa_{21}$.

A non-trivial solution to the system of equations (16) is determined by the condition:

$$\det\begin{pmatrix} w_1 & \kappa_{21} & 0 & \dots & 0 & 0 & 0 \\ \kappa_{12} & w_2 & \kappa_{12} & \dots & 0 & 0 & 0 \\ 0 & \kappa_{21} & w_1 & \dots & 0 & 0 & 0 \\ . & . & . & \dots & . & . & . \\ 0 & 0 & 0 & \dots & w_1 & \kappa_{21} & 0 \\ 0 & 0 & 0 & \dots & \kappa_{12} & w_2 & \kappa_{12} \\ 0 & 0 & 0 & \dots & 0 & \kappa_{21} & w_1 \end{pmatrix} = 0 . \quad (17)$$

where $w_s = i\tilde{k}_s - \lambda$; $(s = 1,2)$

By dividing the rows (17) by κ_{21} and κ_{12}, and then successively multiplying and dividing the rows and columns by

$$\left[\frac{(i\tilde{k}_2 - \lambda)}{(i\tilde{k}_1 - \lambda)}\frac{\kappa_{21}}{4\kappa_{12}}\right]^{1/2},$$

determinant (17) [33] is reduced to a symmetric form. So up to a constant factor, (17) can be conveniently represented as:

$$\det\begin{pmatrix} 2x & 1 & 0 & \dots & 0 & 0 & 0 \\ 1 & 2x & 1 & \dots & 0 & 0 & 0 \\ 0 & 1 & 2x & \dots & 0 & 0 & 0 \\ . & . & . & \dots & . & . & . \\ 0 & 0 & 0 & \dots & 2x & 1 & 0 \\ 0 & 0 & 0 & \dots & 1 & 2x & 1 \\ 0 & 0 & 0 & \dots & 0 & 1 & 2x \end{pmatrix} = \quad (18)$$

$$= C_N^1(\cos\theta) = \frac{\sin(N+1)\theta}{\sin\theta},$$

here $C_N^1(x)$ - Gegenbauer polynomial [33].

$$x = \cos\theta = \left[\frac{(i\tilde{k}_1 - \lambda)(i\tilde{k}_2 - \lambda)}{4\kappa_{12}\kappa_{21}}\right]^{1/2} \quad (19)$$

Conditions (17) and (18) again define the possible values of the phase constants:

$$\theta^s = s\pi/(N+1); \quad (s = 1,2,...,N) \quad (20)$$

Equations (20), (19) allow us to calculate the eigenvalues of the coupling operator (4):

$$\lambda^{s\pm} = \frac{i}{2}(\tilde{k}_1 + \tilde{k}_2) \pm d^s; \quad (21)$$

where

$$d^s = \frac{1}{2}[(4\cos\theta^s)^2\kappa_{12}\kappa_{21} - (\tilde{k}_1 - \tilde{k}_2)^2]^{1/2} . \quad (22)$$

At first glance, it seems that expressions (20), (21), (22) contain $2N$ independent solutions. In fact, each of these solutions, corresponding to the chosen sign of the (21), determines a double set of identical values λ. As a result, the total number of distinct eigenvalues remains equal to N.

The amplitudes of the system of different resonators, found on the basis of using the (17), (18), were represented in the form:

$$b_n^s = b_{0n}^s(-1)^{N+n}\sin[n\theta^s] . \quad (23)$$

where

$$b_{0n}^s = \left|\sin(n\frac{\pi}{2})\right| - \sqrt{\frac{\kappa_{21}(i\tilde{k}_2 - \lambda^s)}{\kappa_{12}(i\tilde{k}_1 - \lambda^s)}}\left|\cos(n\frac{\pi}{2})\right|$$

As follows from the obtained expressions, if the resonators become identical, the relations (21) – (23) become (14), (15).

In the general case, the amplitudes of the coupled oscillations of different resonators b_{0n}^s are not equal to each other.

Fig. 2, b, d show the results of comparison of eigenvalues calculated numerically (dots) with the values obtained from (21), (22) (crosses) for two lattices with an odd and even number of resonators.

IV. COUPLING OSCILLATIONS OF TWO-DIMENSIONAL LATTICES OF DIFFERENT DIELECTRIC RESONATORS

Obtained results for one-dimensional lattices, can be generalized to the case of two-dimensional rectangular lattices of $N \times M$ various resonators (Fig. 3, 4, a). To do this, we defined the “coordinates” of each resonator by two numbers (s,t), where s denotes the number of the horizontal position, and t the number of the vertical position of the DR in a rectangular lattice. Equation (2) for the s,t th DR, also taking into account the coupling only with adjacent resonators of the lattice, was written in the form:

$$\kappa_{s-1,t-1|s,t} b_{s-1,t-1} + \kappa_{s-1,t|s,t} b_{s-1,t} +$$
$$+\kappa_{s-1,t+1|s,t} b_{s-1,t+1} + \kappa_{s,t-1|s,t} b_{s,t-1} + (i\tilde{k}_1 - \lambda) b_{s.t} +$$
$$+\kappa_{s,t+1|s,t} b_{s,t+1} + \kappa_{s+1,t-1|s,t} b_{s+1,t-1} +$$
$$+\kappa_{s+1,t|s,t} b_{s+1,t} + \kappa_{s+1,t+1|s,t} b_{s+1,t+1} = 0 . \qquad (24)$$

($s = 1,2,...,N$; $t = 1,2,...,M$).

Here we have designated $\kappa_{u,v|s,t}$ the coefficients of mutual coupling between the resonator u,v and the resonator s,t.

Next we considered the case of periodic placement of two different types of resonators, shown in Fig. 4, 5, a.

In the general system (2), two equations were identified for describing the oscillation amplitudes of resonators of the first and second types.

For resonators of the first type (Fig. 3, a), shown by small circles, we designated coupling coefficients by

$$\kappa_{s,t-1|s+1,t} = \kappa_{s,t+1|s+1,t} = \kappa_{s+2,t-1|s+1,t} = \kappa_{s+2,t+1|s+1,t} = \kappa_{xy}^{11} ;$$
$$\kappa_{s+1,t-1|s+1,t} = \kappa_{s+1,t+1|s+1,t} = \kappa_{y}^{21} ;$$
$$\kappa_{s,t|s+1,t} = \kappa_{s+2,t|s+1,t} = \kappa_{x}^{21} .$$

For resonators of the second type, shown in Fig. 3, a by large circles, the mutual coupling coefficients were designated by

$$\kappa_{s-1,t-1|s,t} = \kappa_{s-1,t+1|s,t} = \kappa_{s+1,t-1|s,t} = \kappa_{s+1,t+1|s,t} = \kappa_{xy}^{22} ;$$
$$\kappa_{s,t-1|s,t} = \kappa_{s,t+1|s,t} = \kappa_{y}^{12} ; \; \kappa_{s-1,t|s,t} = \kappa_{s+1,t|s,t} = \kappa_{x}^{12} .$$

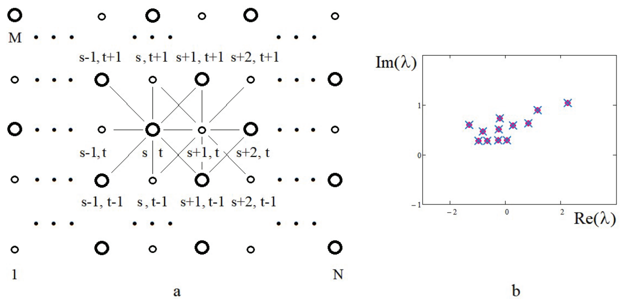


Fig. 3. Two-dimensional "chessboard" lattice of different resonators (a). The results of the numerical calculation of the eigenvalues are dots; the analytical ones are crosses. (b): $N = 4$; $M = 3$; $\tilde{k}_1 = 0,35$; $\tilde{k}_2 = 0,75$; $\kappa_{x}^{12} = 0,7 - 0,3i$; $\kappa_{x}^{21} = 0,3 + 0,2i$; $\kappa_{y}^{12} = 0,5 + 0,2i$; $\kappa_{y}^{21} = 0,25 + 0,15i$; $\kappa_{xy}^{12} = 0,35 + 0,3i$; $\kappa_{xy}^{21} = 0,15 + 0,2i$; $\kappa_{xy}^{11} = 0,5 - 0,2i$; $\kappa_{xy}^{22} = 0,5 + 0,3i$.

Then equations (24) could be rewritten as:

$$\kappa_{x}^{21} (b_{s,t}^{2} + b_{s+2,t}^{2}) + \kappa_{y}^{21} (b_{s+1,t-1}^{2} + b_{s+1,t+1}^{2}) +$$
$$+(i\tilde{k}_1 - \lambda) b_{s+1.t}^{1} + \kappa_{xy}^{11} (b_{s,t-1}^{1} + b_{s,t+1}^{1} + b_{s+2,t-1}^{1} + b_{s+2,t+1}^{1}) = 0 \qquad (25)$$
$$\kappa_{x}^{12} (b_{s-1,t}^{1} + b_{s+1,t}^{1}) + \kappa_{y}^{12} (b_{s,t-1}^{1} + b_{s,t+1}^{1}) +$$
$$+(i\tilde{k}_2 - \lambda) b_{s.t}^{2} + \kappa_{xy}^{22} (b_{s-1,t-1}^{2} + b_{s-1,t+1}^{2} + b_{s+1,t-1}^{2} + b_{s+1,t+1}^{2}) = 0$$

We were looking for a solution in the form:

$$b_{s,t}^{1,2} = b_{0}^{1,2} \sin(\theta_x s) \sin(\theta_y t) , \qquad (26)$$

where the amplitudes $b_0^{1,2}$ and phase constants θ_x, θ_y do not depend on the number of resonators.

Substituting (26) into (25), after simple transformations, we obtained a system of two equations for the amplitudes of resonators of different types:

$$\left\{\begin{array}{l} [(i\tilde{k}_1-\lambda)+4\kappa_{xy}^{11}\cos\theta_x\cos\theta_y]\cdot b_0^1+ \\ +2[\kappa_x^{21}\cos\theta_x+\kappa_y^{21}\cos\theta_y]\cdot b_0^2=0; \\ 2[\kappa_x^{12}\cos\theta_x+\kappa_y^{12}\cos\theta_y]\cdot b_0^1+ \\ +[(i\tilde{k}_2-\lambda)+4\kappa_{xy}^{22}\cos\theta_x\cos\theta_y]\cdot b_0^2=0 \end{array}\right. . \quad (27)$$

By analogy with the representation for a one-dimensional lattice, we put:

$$\theta_x=\theta_x^n=\frac{n\pi}{(N+1)};\quad \theta_y=\theta_y^m=\frac{m\pi}{(M+1)};$$
$$(n=1,2,...,N;\ m=1,2,...,M) \qquad (28)$$

Having equated the determinant of system (27) to zero, we found:

$$\lambda^{\pm}=\frac{i}{2}(\tilde{k}_1+\tilde{k}_2)+2(\kappa_{xy}^{11}+\kappa_{xy}^{22})\cos\theta_x\cos\theta_y\pm d; \quad (29)$$

where

$$d=\frac{1}{2}\Big\{16[\kappa_x^{12}\cos\theta_x+\kappa_y^{12}\cos\theta_y][\kappa_x^{21}\cos\theta_x+\kappa_y^{21}\cos\theta_y]+$$
$$+[i(\tilde{k}_1-\tilde{k}_2)+4(\kappa_{xy}^{11}-\kappa_{xy}^{22})\cos\theta_x\cos\theta_y]^2\Big\}^{1/2}.$$

Relations (28) together with (29) determine the $N\times M$ eigenvalues of a rectangular two-dimensional lattice of different DRs, shown in Fig. 3, a. The result of comparison of analytical and numerical values is shown in Fig. 3, b.

In the case of a “linear” arrangement of different resonators, shown in Fig. 4, a, the solution was carried out in a completely similar manner. In this case, in the equation for resonators of the first type, we designated the coupling coefficients as:

$$\kappa_{s-1,t-1|s,t}=\kappa_{s-1,t+1|s,t}=\kappa_{s+1,t-1|s,t}=\kappa_{s+1,t+1|s,t}=\kappa_{xy}^{21};$$
$$\kappa_{s,t-1|s,t}=\kappa_{s,t+1|s,t}=\kappa_y^{11};\ \kappa_{s-1,t|s,t}=\kappa_{s+1,t|s,t}=\kappa_x^{21},$$

and in the equations for resonators of the second type, the coupling coefficients were designated as:

$$\kappa_{s,t-1|s+1,t}=\kappa_{s,t+1|s+1,t}=\kappa_{s+2,t-1|s+1,t}=\kappa_{s+2,t+1|s+1,t}=\kappa_{xy}^{12};$$
$$\kappa_{s,t|s+1,t}=\kappa_{s+2,t|s+1,t}=\kappa_x^{12};$$
$$\kappa_{s+1,t-1|s+1,t}=\kappa_{s+1,t+1|s+1,t}=\kappa_y^{22}.$$

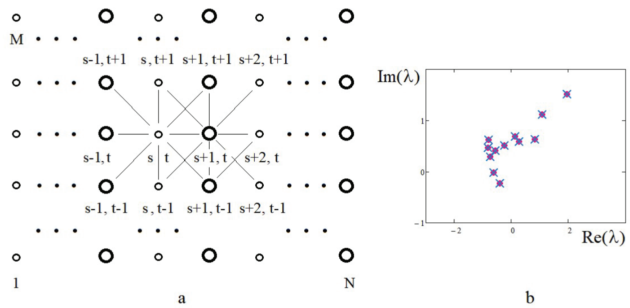


Fig. 4. Two-dimensional "linear" lattice of various resonators (a). The results of the numerical calculation of the eigenvalues are dots; the analytical ones are crosses. (b): $N=4$; $M=3$; $\tilde{k}_1=0,35$; $\tilde{k}_2=0,75$; $\kappa_x^{12}=0,7-0,3i$; $\kappa_x^{21}=0,3+0,2i$; $\kappa_y^{11}=0,5+0,25i$; $\kappa_y^{22}=0,35+0,25i$; $\kappa_{xy}^{12}=0,35+0,3i$; $\kappa_{xy}^{21}=0,15+0,2i$.

Then the system of equations obtained from (24) for a resonator of different types takes the form:

$$\kappa_x^{21}(b_{s-1,t}^2+b_{s+1,t}^2)+\kappa_y^{11}(b_{s,t-1}^1+b_{s,t+1}^1)+$$
$$+(i\tilde{k}_1-\lambda)b_{s,t}^1+\kappa_{xy}^{21}(b_{s-1,t-1}^2+b_{s-1,t+1}^2+b_{s+1,t-1}^2+b_{s+1,t+1}^2)=0$$
$$(30)$$
$$\kappa_x^{12}(b_{s,t}^1+b_{s+2,t}^1)+\kappa_y^{22}(b_{s+1,t-1}^2+b_{s+1,t+1}^2)+$$
$$+(i\tilde{k}_2-\lambda)b_{s+1,t}^2+\kappa_{xy}^{12}(b_{s,t-1}^1+b_{s,t+1}^1+b_{s+2,t-1}^1+b_{s+2,t+1}^1)=0$$

Again we sought a solution in the form of (26). Substituting (26) into (30), after small transformations, we obtained a system of equations that also does not depend on the number of resonators:

$$\begin{cases} [(i\tilde{k}_1-\lambda)+2\kappa_y^{11}\cos\theta_y]\cdot b_0^1 + \\ +2[\kappa_x^{21}+2\kappa_{xy}^{21}\cos\theta_y]\cos\theta_x\cdot b_0^2=0; \\ 2[\kappa_x^{12}+2\kappa_{xy}^{12}\cos\theta_y]\cos\theta_x\cdot b_0^1 + \\ +[(i\tilde{k}_2-\lambda)+2\kappa_y^{22}\cos\theta_y)]\cdot b_0^2=0 \end{cases} \qquad (31)$$

where also:

$$\theta_x=\theta_x^n=\frac{n\pi}{(N+1)}; \quad \theta_y=\theta_y^m=\frac{m\pi}{(M+1)}; \qquad (32)$$

$$(n=1,2,...,N;\; m=1,2,...,M)$$

Expressions (32) together with (31) defined the $N\times M$ eigenvalues of a rectangular two-dimensional "linear" lattice of different DRs:

$$\lambda^{\pm}=\frac{i}{2}(\tilde{k}_1+\tilde{k}_2)+(\kappa_y^{11}+\kappa_y^{22})\cos\theta_y\pm d; \quad (33)$$

where

$$d=\frac{1}{2}\Big\{16[\kappa_x^{12}+2\kappa_{xy}^{12}\cos\theta_y][\kappa_x^{21}+2\kappa_{xy}^{21}\cos\theta_y](\cos\theta_x)^2+$$

$$+[i(\tilde{k}_1-\tilde{k}_2)+2(\kappa_y^{11}-\kappa_y^{22})\cos\theta_y]^2\Big\}^{1/2}.$$

In the case of identical resonators: if $\tilde{k}_1=\tilde{k}_2=\tilde{k}_0$; $\kappa_x^{11}=\kappa_x^{22}=\kappa_x$; $\kappa_y^{11}=\kappa_y^{22}=\kappa_y$; $\kappa_{xy}^{12}=\kappa_{xy}^{21}=\kappa_{xy}$, the found expressions (29), (33) become

$$\lambda=i\tilde{k}_0+2\kappa_x\cos(\theta_x)+2\kappa_y\cos(\theta_y)+$$

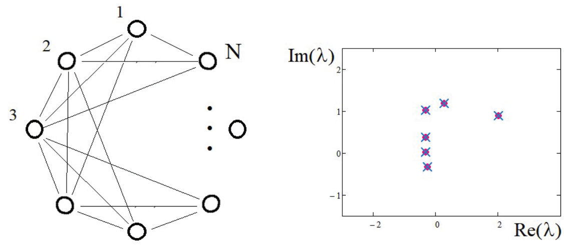
$$+4\kappa_{xy}\cos(\theta_x)\cos(\theta_y).$$

A comparison of the calculation results obtained from expressions (33) with the numerical calculations of the eigenvalues is given in Fig. 4, b.

V. Coupling Oscillations of Ring Lattices of Dielectric Resonators

At the beginning we have considered particular solutions of (2) for a lattice of identical DRs ($\tilde{k}_s=\tilde{k}_0$) ($s=1,2,...,N$), indicated by circles, located at the vertices of a regular polygon in a ring structure (Fig. 5, a).

It's assumed that the coefficients of mutual coupling of the identical resonators satisfy two conditions: the symmetry: $\kappa_{st}=\kappa_{ts}$ and translation: $\kappa_{st}=\kappa_{t+w,s+w}$ ($w=1,...,N-1$). The last condition can also be rewritten as: $\kappa_{st}=\kappa_{|s-t|}=\kappa_v=\kappa_{-v}$, where $v=|s-t|$. In this case, matrix (4) becomes a circulant matrix [31, 32]; the elements of each row are obtained by cyclically permuting the elements of the previous one.



a b

Fig. 5. Ring lattices of N identical DRs (a). (b): $N=11$; $\tilde{k}_0=0,5$; $\kappa_1=0,3+0,3i$; $\kappa_2=0,25+0,1i$; $\kappa_3=0,2-0,1i$; $\kappa_4=0,15-0,2i$; $\kappa_5=0,1+0,1i$

The eigenvalues of such a circulant matrix are well known [32], for the coupling operator (4):

$$\lambda^s=i\tilde{k}_0+\sum_{v=1}^{N-1}\kappa_v\cdot(\eta_s)^v \qquad (34)$$

where

$$\eta_s=\exp\left(\frac{2s\pi i}{N}\right); \qquad (s=0,1,...,N-1)$$

is the s-th complex N-th root of unity.

The matrix of normalized eigenvectors of a circulant has the form [32]:

$$B=\frac{1}{\sqrt{N}}\cdot\begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & \eta_1 & \ldots & \eta_{N-1} \\ 1 & \eta_1^2 & \ldots & \eta_{N-1}^2 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \eta_1^{N-1} & \cdots & \eta_{N-1}^{N-1} \end{pmatrix}. \qquad (35)$$

As follows from (34), (3), the frequencies of natural oscillations of regular ring structures of identical DRs are linear functions of the coupling coefficients, and the eigenvectors (6) do not depend on the coupling coefficients at all, but are determined only by the number of resonators in the system.

Fig. 5, b shows the result of comparison of the eigenvalues of the ring structure 11 DR, found numerically (points) and calculated using formula (34) (crosses).

VI. Coupling oscillations of Ring Lattices of Different Dielectric Resonators

Next we considered two ring lattices of different DR. The parameters of the first type of resonators were designated by the number **1**; the parameters of the second type of resonators by the number **2**. The equations describing the coupled oscillations of ring lattices of resonators of different types are also described by the same system of equations (2), also taking into account the asymmetry of the coupling coefficients between resonators of different types: $\kappa_{12}^{12} \neq \kappa_{12}^{21}$.

The amplitudes of natural oscillations of resonators **1** and **2** of the lattice were designated as:

$$\begin{pmatrix} b_t^1 \\ b_t^2 \end{pmatrix} = \begin{pmatrix} a^1 \\ a^2 \end{pmatrix} b_t \ , \qquad (36)$$

where b_t are the amplitudes (35).

For the lattice shown in Fig. 6, a, the system of equations for the amplitudes, taking into account the definition of the Toeplitz matrices of each individual lattice: $\kappa_{t-s,t}^{1,2} = \kappa_s^{1,2} = \kappa_{-s}^{1,2}$, takes the form:

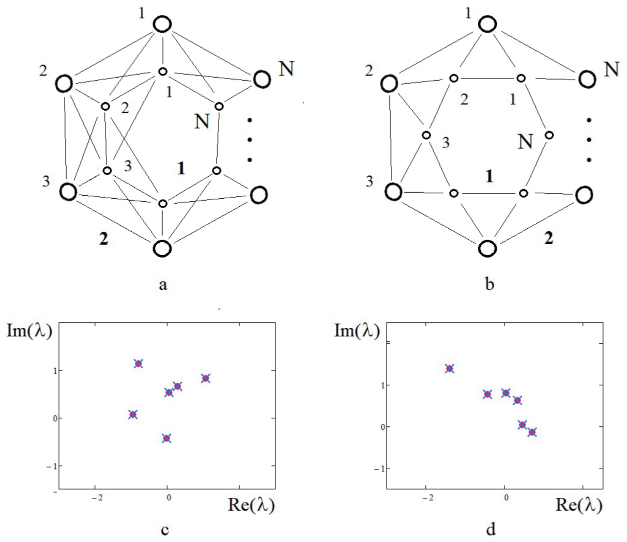


Fig. 6. Ring lattices of 2N (a, b) different DR. (e): $N=4$; $\tilde{k}_1 = 0,25$; $\tilde{k}_2 = 0,75$; $\kappa_1^1 = 0,4-0,5i$; $\kappa_2^1 = -0,2-0,1i$; $\kappa_0^{12} = 0,5+0,25i$; $\kappa_0^{21} = 0,35+0,15i$; $\kappa_1^{12} = 0,15+0,25i$; $\kappa_1^{21} = 0,25-0,15i$; $\kappa_2^{12} = 0,2+0,2i$; $\kappa_2^{21} = 0,1+0,1i$; $\kappa_1^2 = 0,3+0,3i$; $\kappa_2^2 = -0,15-0,1i$;

(d): $N=4$; $\tilde{k}_1 = 0,5$; $\tilde{k}_2 = 0,75$; $\kappa_1^1 = 0,6-0,5i$; $\kappa_2^1 = -0,2-0,1i$; $\kappa_0^{12} = 0,45+0,25i$; $\kappa_0^{21} = 0,15+0,1i$; $\kappa_2^{12} = 0,2+0,2i$; $\kappa_2^{21} = 0,1+0,15i$; $\kappa_1^2 = -0,3+0,3i$; $\kappa_2^2 = -0,15-0,1i$.

The results of the numerical calculation of the eigenvalues are dots; the analytical ones are crosses (c - d).

$$\begin{cases} [2\sum\limits_{s\in\Xi} \Delta_s \cdot \kappa_s^1 \cos(2s\pi j/N) + (i\tilde{k}_1 - \lambda)] \cdot a^1 + \\ \quad +[2\sum\limits_{s\in\Xi'} \Delta_s \cdot \kappa_s^{21} \cos(2s\pi j/N)] \cdot a^2 = 0; \\ \quad [2\sum\limits_{s\in\Xi'} \Delta_s \cdot \kappa_s^{12} \cos(2s\pi j/N)] \cdot a^1 + \\ +[2\sum\limits_{s\in\Xi} \Delta_s \cdot \kappa_s^2 \cos(2s\pi j/N) + (i\tilde{k}_2 - \lambda)] \cdot a^2 = 0 \end{cases} ; \qquad (37)$$

where $\Delta_s = 1/(1+\delta_{s,0}+\delta_{s,\frac{N}{2}})$; $\Xi = 1,2,...,(N-1)/2$; $\Xi' = 0,1,2,...,(N-1)/2$ for odd number of resonators; $\Xi = 1,2,...,(N/2)$; $\Xi' = 0,1,2,...,(N/2)$ for an even number of resonators.

The condition for the existence of a non-trivial solution of system (37) determines the eigenvalues:

$$\lambda^{\pm} = \frac{i}{2}(\tilde{k}_1 + \tilde{k}_2) + \sum_{s\in\Xi} \Delta_s[\kappa_s^1 + \kappa_s^2]\cos(2s\pi j / N) \pm d \, , \quad (38)$$

where

$$d = \frac{1}{2}\left\{16\left[\sum_{s\in\Xi}\Delta_s \cdot \kappa_s^{12}\cos(\frac{2s\pi j}{N})\right]\left[\sum_{s\in\Xi}\Delta_s \cdot \kappa_s^{21}\cos(\frac{2s\pi j}{N})\right] + \right.$$

$$\left. + \left[i(\tilde{k}_1 - \tilde{k}_2) + 2\sum_{s\in\Xi}\Delta_s \cdot (\kappa_s^1 - \kappa_s^2)\cos(\frac{2s\pi j}{N})\right]^2\right\}^{1/2} ;$$

and amplitudes:

$$a^1 / a^2 = -[2\sum_{s\in\Xi}\Delta_s \cdot \kappa_s^1 \cos(2s\pi j / N) + (i\tilde{k}_1 - \lambda)] /$$

$$/[2\sum_{s\in\Xi'}\Delta_s \cdot \kappa_s^{21}\cos(2s\pi j / N)] .$$

For the structure shown in Fig. 5, b, by grouping symmetric resonators in different ring sublattices, we obtained:

$$\begin{cases} [2\sum\limits_{s=1}^{(N-1)/2} \kappa_s^1 \cos(2\pi js / N) + (i\tilde{k}_1 - \lambda)] \cdot a^1 + \\ \quad + \sum\limits_{s=1}^{(N+1)/2} \nu_{s,(N+1)/2}\kappa_s^{21} f_s^j \cdot a^2 = 0; \\ \quad \sum\limits_{s=1}^{(N+1)/2} \nu_{s,(N+1)/2}\kappa_s^{12} f_s^j \cdot a^1 + \\ + [2\sum\limits_{s=1}^{(N-1)/2} \kappa_s^2 \cos(2\pi js / N) + (i\tilde{k}_2 - \lambda)] \cdot a^2 = 0 \end{cases}$$

for an odd number of resonators;

$$\begin{cases} [2\sum\limits_{s=1}^{N/2-1} \kappa_s^1 \cos(2\pi js / N) + (i\tilde{k}_1 - \lambda)] \cdot a^1 + \\ \quad + \sum\limits_{s=1}^{N/2} \kappa_s^{21} f_s^j \cdot a^2 = 0; \\ \quad \sum\limits_{s=1}^{N/2} \kappa_s^{12} f_s^j \cdot a^1 + \\ + [2\sum\limits_{s=1}^{N/2-1} \kappa_s^2 \cos(2\pi js / N) + (i\tilde{k}_2 - \lambda)] \cdot a^2 = 0 \end{cases} \quad (39)$$

for an even number of resonators.

Here $\nu_{s,t} = 1/(1+\delta_{s,t})$;

$$f_s^j = e^{-j\pi i/N}[e^{2j(s)\pi i/N} + e^{2j(N-s+1)\pi i/N}] . \quad (40)$$

The appearance of the common phase factor (40) in (39) is due to the condition of correspondence between the solutions for two lattices consisting of N DR and the solution for one ring lattice of 2N resonators, when the resonators are placed at the same distance from the common axis of the structure.

From the condition of existence of non-trivial solutions of the system of equations (39), we also determined the eigenvalues:

$$\lambda^{\pm} = \frac{i}{2}(\tilde{k}_1 + \tilde{k}_2) + \sum_{s\in\Xi} \Delta_s[\kappa_s^1 + \kappa_s^2]\cos(2s\pi j / N) \pm d \, , \quad (41)$$

where

$$d = \frac{1}{2}\left\{4\left[\sum_{s\in\Xi}\Delta_s \cdot \kappa_s^{12} f_s^j\right]\left[\sum_{s\in\Xi}\Delta_s \cdot \kappa_s^{21} f_s^j\right] + \right.$$

$$\left. + \left[i(\tilde{k}_1 - \tilde{k}_2) + 2\sum_{s\in\Xi}\Delta_s \cdot (\kappa_s^1 - \kappa_s^2)\cos(\frac{2s\pi j}{N})\right]^2\right\}^{1/2} ;$$

Fig. 6, c, d show the results of comparison of the eigenvalues of the coupling matrices calculated numerically (dots) with the values obtained from (41) (crosses).

VII. CONCLUSION

General analytical solutions of equations describing coupled oscillations of one-dimensional and two-dimensional rectangular and ring lattices of dielectric resonators of various types are obtained.

The solutions found significantly simplify the calculation of the parameters of coupled oscillations of complex lattices of different DR and are the basis for constructing a more effective scattering theory.

The developed theory is the basis for constructing a wide class of optical communication devices built using dielectric resonators of various types.

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Зв'язані коливання ґраток різних видів діелектричних резонаторів

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Проблематика. Розробка багатьох елементів сучасних систем зв’язку все частіше базується на використанні різних видів діелектричних резонаторів. Теорія зв’язаних коливань резонаторів є основою для проведення подальших розрахунків матриць розсіювання електромагнітних хвиль на різних пристроях, побудованих на їх основі. При розрахунках цих пристроїв, побудованих на великої кількості резонаторів, прямі чисельні методи часто не є ефективними. Вони як правило потребують використання потужних комп’ютерів, тому розрахунок і оптимізація елементів, побудованих на великої кількості діелектричних резонаторах, неможливо без побудови аналітичних моделей складних структур, які базуються на електродинамічному моделюванні.

Мета досліджень. Метою даного дослідження є пошук аналітичних виразів для частот та розподілів електромагнітних полів власних коливань ґраток, які складаються з великої кількості діелектричних резонаторів різних видів для використання їх в різноманітних пристроях систем оптичного зв’язку. Для вирішення цієї задачі використовувалась лінійна система рівнянь, яка пов’язує між собою комплексні амплітуди та частоти резонаторів, отримана раніше із теорії збурень для рівнянь Максвелла.

Методика реалізації. Для пошуку аналітичних виразів використовуються методи теорії матриць. При цьому, використовуються як відомі методи розрахунків три діагональних та циркулянтних матриць, так і запропоновані їх модифікації, пов’язані з розрахунками більш складних матриць, які після перетворень, зводяться до значно більш простих співвідношень. Кінцевим результатом є отримання нових загальних аналітичних формул для опису зв’язаних коливань решіток, які складаються із великого числа діелектричних резонаторів різних видів.

Результати досліджень. Розглянуто зв’язані коливання лінійних одно-вимірних ґраток діелектричних резонаторів двох видів. Отримані нові аналітичні вирази для комплексних частот та амплітуд резонаторів, а також вирази добротності без обмеження на їх кількість. Побудована нова модель власних коливань двовимірних ґраток, які складаються із діелектричних резонаторів двох різних типів. Знайдено загальні аналітичні рішення для частот та амплітуд зв'язаних коливань для двох видів двовимірних ґраток з різним розташуванням резонаторів. Знайдено аналітичні рішення для амплітуд та частот зв'язаних коливань двох аксіально-симетричних кільцевих ґраток з різними типами резонаторів, які характеризуються різною симетрією розміщення у вільному просторі. Отримані загальні аналітичні вирази для частот зв’язаних коливань порівнюються з результатами, розрахунків, отриманими шляхом чисельного вирішення лінійних систем рівнянь. Демонструється дуже гарне збіг рішень, отриманих двома способами.

Висновки. Розвита теорія є основою для конструювання багатьох пристроїв оптичного діапазону довжин хвиль, які будуються на основі використання великого числа діелектричних резонаторів різних видів. Отримані нові аналітичні вирази для розрахунку зв’язаних коливань діелектричних резонаторів дозволяють будувати нові більш ефективні моделі різноманітних пристроїв оптичного зв’язку.

Ключові слова: *діелектричний резонатор; власні коливання; ґратка; тридіагональна матриця; циркулянтний.*

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