10.20535/2411-2976.22024.39-52

UDC 621.391

OPTIMIZATION OF THE SCATTERING MATRIX OF FREQUENCY - DETUNED ADD/DROP FILTERS FOR MULTIPLEXERS BUILT ON SYSTEMS OF OPTICAL DIELECTRIC RESONATORS WITH WHISPERING GALLERY MODES

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Background. A significant increase in the speed of information transmission in fiber-optic communication networks is determined by the strict requirements imposed on the elemental base of receiving and transmitting devices. One of the important components of such devices is diplexers built on different notch and bandpass filters, which are often performed on dielectric resonators (DR) with whispering gallery mode (WGM) oscillations. Calculation and optimization of the parameters of multilink filters and diplexers built on DR is impossible without further development of the theory of their design. The development of the theory of diplexers today is often based on electrodynamic modelling, which is built on preliminary calculations of filter scattering parameters in various transmission lines with a complex topology of connections.

Objective. The aim of this study is to construct electrodynamics models of wave scattering on complex multi-connected DR structures with degenerate types of WGM natural oscillations, which contain several frequency-detuned bandpass or notch filters located in one or several transmission lines. To solve the scattering problem, we proposed a system of equations derived from perturbation theory for Maxwell's equations [24], modified to describe the DR fields with whispering gallery oscillations in transmission lines. The construction of such solutions is complicated by the fact that each of the partial optical resonators of the filters, in the case of excitation of azimuthally inhomogeneous WGM in it, has, as a rule, two degenerate types of natural oscillations. Moreover, each such type of oscillation is characterized by different complex values of the coupling coefficients with the line, open space, and also with other resonators. The latter circumstance leads to the fact that the systems of equations for the amplitudes of natural and forced oscillations of resonators are doubled. The signs of the coefficients of mutual coupling are usually different, this leads to the fact that the behaviour of DRs in the system becomes more difficult to predict, therefore the second objective of this work is to study the patterns of the scattering characteristics of line waves on systems of frequency-detuned DRs with degenerate types of oscillations with the possibility of constructing multiplexers for modern optical communication systems.

Methods. The methods of technical electrodynamics are used for calculating and analysing scattering matrices. The end result is obtaining new analytical equations and formulas for new complex structures of coupled dielectric resonators with whispering gallery oscillations in the different transmission lines.

Results. Frequency dependences of scattering matrices on complex structures of frequency-detuned filters built on coupled optical DR with whispering gallery oscillations located in one or more transmission lines are considered. Electromagnetic models of bandstop and add/drop filters are built, consisting of various optical resonators with degenerate types of natural oscillations. General analytical expressions of vector coefficients and matrices for building systems of equations describing coupled oscillations, as well as forced oscillations of resonators in cases of their use in optical filters with serial and parallel arrangement, are given. General solutions for the scattering field on frequency-detuned resonators located in different optical transmission lines have been found. Examples of calculating frequency dependences of the scattering matrix for the most interesting structures consisting of two different frequency-detuned filters are given. The frequency scattering characteristics of several types of devices are calculated, which consist of two notch filters with different blocking bands, made on detuned DRs in one transmission line. The possibilities of the earlier proposed method are demonstrated in the example of calculating the scattering characteristics of known types of diplexers built on the basis of the use of two add/drop filters with different frequency bandwidths. The frequency dependences of the scattering matrices of the two most common types of devices, located in parallel between two or four regular transmission lines of add/drop filters with different numbers of resonators; laterally coupled add/drop filters; parallel-coupled add/drop filters; twisted double-channel side-coupled integrated space sequence of resonators (SCISSORs), were studied. New scattering models of diplexers consisting of optical resonators of different connection topologies were built: serial, parallel with the use of laterally coupled add/drop filters; parallel-coupled add/drop filters; twisted double-channel SCISSORs. The frequency dependences scattering matrix of the diplexers were also calculated. The characteristics of the designed diplexers obtained from the examined filters of different types were compared.

Conclusions. The theory of diplexer construction, which takes place on the simultaneous optimization of the scattering matrix of several filters built on complex systems of dielectric resonators with degenerate types of whispering gallery oscillations is expanded. A calculation method was developed and new analytical relations were found for the scattering matrix coefficients of optical diplexers of various types.

Keywords: *scattering; dielectric resonator; scattering matrix; notch filter; laterally coupled add/drop filter; parallel-coupled add/drop filter; twisted double-channel SCISSOR; diplexer.*

I. INTRODUCTION

Multiplexers of various types, built on the basis of filters on dielectric resonators (DRs) with whispering gallery mode (WGM) [1 - 24], are important elements of modern optical communication systems. The design and tuning of optical multiplexers, which typically contain a large number of different multi-section filters, is a complex technical task. An important stage in its solution is the optimization of scattering characteristics on complex systems of coupled DRs, which are usually located in multi-connected structures of optical transmission lines and also have degenerate types of natural oscillations.

The aim of this study is to construct electrodynamics' models of wave scattering on multi-connected complex DR structures with degenerate types of natural oscillations, which contain several frequency-detuned bandpass or notch filters located in one or several transmission lines. To solve the scattering problem, we use a system of equations derived from perturbation theory for Maxwell's equations [24], modified to describe the DR fields with whispering gallery oscillations in a transmission line. The construction of such solutions is complicated by the fact that each of the partial optical resonators of the filters, in the case of excitation of azimuthally inhomogeneous WGM in it, has at least two degenerate types of natural oscillations. Moreover, each such type of oscillation is characterized by different complex values of the coupling coefficients with the line, open space, and also with other resonators. The latter circumstance leads to the fact that the systems of equations for the amplitudes of natural and forced oscillations of resonators become more complex. The signs of the coefficients of mutual coupling are usually different, this leads to the fact that the behaviour of DRs in the system becomes more difficult to predict, therefore the second objective of this work is to study the patterns of the scattering characteristics of line waves on systems of frequency-detuned DRs with degenerate types of WGM oscillations with the possibility of constructing multiplexers for modern optical communication systems.

II. SCATTERING THEORY ON DETUNED DRS WITH DEGENERATE OSCILLATIONS IN TRANSMISSION LINES

Let us first consider the simplest case of wave scattering on two notch filters made on the basis of different frequency DR with WGM oscillations (Fig. 1, a). Let us assume that we know the field of degenerate natural oscillations in frequency for all resonators of both filters. We will characterize each type of oscillations by a given parity of the field distribution relative to a given plane of symmetry of the resonator in the transmission line: even $(\mathbf{e}_s^e, \mathbf{h}_s^e)$, or odd $(\mathbf{e}_s^o, \mathbf{h}_s^o)$. We will designate the frequencies of these oscillations, respectively, as: $\tilde{\omega}_s^e = \omega_s + i\omega_s^{e\prime\prime}$, or $\tilde{\omega}_s^o = \omega_s + i\omega_s^{o\prime\prime}$. The coupling coefficients of the DR with the transmission line $\tilde{k}_s^e$, $\tilde{k}_s^o$ can be calculated using analytical expressions for the field of the line and the field of each of the N resonators $(\mathbf{e}_s^e, \mathbf{h}_s^e)$, $(\mathbf{e}_s^o, \mathbf{h}_s^o)$, $(s = 1, 2, ..., N)$ [27]. We can also calculate the mutual coupling coefficients k_{12}^e; k_{12}^o for non-propagating and propagating waves $\tilde{k}_{12}^e$; $\tilde{k}_{12}^o$, using the expressions for the field of the line and the field of the resonator [27]. We will assume that the coupling coefficients with the open space $\tilde{k} = \tilde{k}^{e,o}$ of each of the resonators with rotational symmetry are equal for identical oscillations of different parity. It is also obvious that such degenerate oscillations are orthogonal to each other; the coupling coefficients between them in each resonator are equal to zero.

In the case of scattering on two different filters, the first of which consists of N_1, and the second of N_2 DRs to construct the theory, we use scattering field expansions in terms of v-th coupled oscillations of the $(N_1 + N_2)$ DR system $\left(\mathbf{e}^v, \mathbf{h}^v\right)$ [27] ($v = 1, 2, ..., 2(N_1 + N_2)$).

$$\mathbf{e}(\mathbf{r}) = \sum_{s=1}^{2N_1} b_s^{1g} \mathbf{e}_s^{1g}(\mathbf{r}) + \sum_{s=2N_1+1}^{2(N_1+N_2)} b_s^{2g} \mathbf{e}_s^{2g}(\mathbf{r}) = \sum_{s=1}^{2(N_1+N_2)} b_s \mathbf{e}_s(\mathbf{r});$$

$$\mathbf{h}(\mathbf{r}) = \sum_{s=1}^{2N_1} b_s^{1g}\mathbf{h}_s^{1g}(\mathbf{r}) + \sum_{s=2N_1+1}^{2(N_1+N_2)} b_s^{2g}\mathbf{h}_s^{2g}(\mathbf{r}) = \sum_{s=1}^{2(N_1+N_2)} b_s\mathbf{h}_s(\mathbf{r})\,. \quad (1)$$

Here $g = e,o$, depending on the type of parity of the partial resonator oscillations. To reduce the length of the formulas, in some sums (as, for example, (1)) we will omit the indices denoting the filter number or the parity index. Next, we will agree that the first indices will denote $2N_1$ the amplitudes of the natural oscillations of the resonators or other parameters of the 1st filter and that the indices changing from $2N_1+1$ to $2(N_1+N_2)$ will denote the amplitudes of the resonators and other parameters of the 2nd filter. Let us also denote by $M = 2(N_1+N_2)$ the total number of resonances of both filters taken into account.

We determine complex amplitudes b_s and complex frequencies $\tilde{\omega} = \omega' + i\omega''$ of coupled oscillations of detuned DRs system, solving the problem of eigenvalues of the coupling operator K [27]:

$$\sum_{s\neq t}^{M} \kappa_{st} b_s + (i\tilde{k}_t - \lambda_t) b_t = 0\,, \quad (2)$$

$(t = 1,2,...,M)$;

$\lambda_t = 2\cdot(\dfrac{\tilde{\omega}-\omega'_t}{\omega'_t})$, which we also transform into the form:

$$\sum_{s\neq t}^{M} \hat{\kappa}_{st} b_s + (\hat{\kappa}_t - \lambda) b_t = 0\,, \quad (3)$$

$(t = 1,2,...,M)$;

$$\lambda = 2\cdot(\frac{\tilde{\omega}-\omega_0}{\omega_0})\,.$$

Where for each eigenvalue λ (determined by the frequency $\tilde{\omega}$): the field of coupled oscillations takes the form (1). Here

$$\hat{\kappa}_{st} = \frac{\omega'_t}{\omega_0}\kappa_{st}\,;$$

$$\hat{\kappa}_t = i\frac{\omega'_t}{\omega_0}(\tilde{k}+\tilde{k}_t) - 2\frac{\omega_0-\omega'_t}{\omega_0}\,;$$

$\kappa_{st} = \kappa_{st}^{e,o} = k_{st}^{e,o} + i\tilde{k}_{st}^{e,o}$ mutual coupling coefficient of the s-th and t-th DR for even or odd mode; $\tilde{k}$ coupling coefficients of resonators with open space; $\tilde{k}_t = \tilde{k}_t^{e,o}$ coupling coefficients of resonators with transmission line; $\omega'_t = \omega_t'^{e,o} = \omega_t$ - is the real part of the frequency of t-th DR` even or odd mode; $b_t = b_t^{e,o}$ - complex amplitudes; ω_0 - arbitrary frequency [27], defined to give a canonical form to the solution of the problem of natural oscillations (2).

In the second stage, we construct a solution to the problem of the $(\mathbf{E}_l^+,\ \mathbf{H}_l^+)$ wave scattering on a system of coupled detuned DRs with WGM, also using the perturbation theory of Maxwell's equations and expansions (1). Here l - is the multi-index characterizing transmission line mode.

We will assume that each of the resonators of two filters may be made of a loss dielectric: $\tilde{\varepsilon}_s = \varepsilon'_s - i\varepsilon''_s$ ($s = 1,2,...,2(N_1+N_2)$), wherein $\varepsilon''_s << \varepsilon'_s$. We present the solution to the scattering problem in the form [24], also decomposing it into coupled oscillations of the resonators of both filters:

$$\mathbf{E}(\omega) \approx \mathbf{E}_l^+ + \sum_{s=1}^{M} a^s(\omega)\mathbf{e}^s;$$
$$\mathbf{H}(\omega) \approx \mathbf{H}_l^+ + \sum_{s=1}^{M} a^s(\omega)\mathbf{h}^s, \quad (4)$$

where $(\mathbf{e}^s,\ \mathbf{h}^s)$ - is the field of coupled oscillations of a resonator system (3), corresponding to the eigenvalues $\lambda = \lambda^s$ ($s = 1,2,...,M$).

Using expressions of perturbation theory, written for the scattered field, transmission line field, and also the fields of partial resonators, we arrive at the equations for amplitudes $a^s(\omega)$ [27]:

$$\sum_{s=1}^{M} a^s(\omega) b_t^{gs} Q_{st}(\omega) = -Q_t^D \left(c_t^{g+}\right)^* / \omega_t w_t\,, \quad (5)$$

where

$$Q_{st}(\omega) = \omega/\omega_t + 2iQ_t^D(\omega/\omega_t - 1 - \lambda^s/2)\,; \quad (6)$$

$Q_t^D = \omega_t w_t / P_t^D$ - loss quality factor in the dielectric of the t - th DR for even or odd degenerate oscillations; here $P_t^D = \dfrac{\omega_t}{2}\varepsilon''_t \int_{V_t} \left|\mathbf{e}_t^g\right|^2 dv$) determines the dielectric power loss of the t - th resonator; $c_t^{g\pm}$ - field expansion coefficient of t – th DR by transmission line waves in the region $z > z_t$ (sign +), or $z < z_t$ (sign -) [27]; here z_t - longitudinal coordinate t -th DR along the z -axis of the transmission line.

The transmission T and the reflection coefficient R of the complex detuned DR system in the transmission line we obtained in the form taking into account simultaneously the natural oscillations of the resonators of both filters:

$$T(\omega) = T_0 + \sum_{u=1}^{M}\left(\sum_{s=1}^{M} b_s^u c_s^{g+}\right)a^u(\omega) = $$
$$= T_0 - \frac{1}{\det B(\omega)}\sum_{s=1}^{M}\det B_s^{+}(\omega);$$
$$R(\omega) = R_0 + \sum_{u=1}^{M}\left(\sum_{s=1}^{M} b_s^u c_s^{g-}\right)a^u(\omega) = \quad (7)$$
$$= R_0 - \frac{1}{\det B(\omega)}\sum_{s=1}^{M}\det B_s^{-}(\omega).$$

Where T_0, (R_0) is the transmission (reflection) coefficients of the transmission line without DRs. From (5) we find:

$$B_s^{\pm}(\omega) = $$
$$= \begin{bmatrix} b_1^1 Q_{11}(\omega) & \dots & Q_1^D\sum_{u=1}^{M} b_u^s\tilde{k}_{u1}^{\pm+} & \dots & b_1^M Q_{M1}(\omega) \\ b_2^1 Q_{12}(\omega) & \dots & Q_1^D\sum_{u=1}^{M} b_u^s\tilde{k}_{u2}^{\pm+} & \dots & b_2^M Q_{M2}(\omega) \\ . & . & . & . & . \\ b_{M-1}^1 Q_{1(M-1)}(\omega) & \dots & Q_2^D\sum_{u=1}^{M} b_u^s\tilde{k}_{u(M-1)}^{\pm+} & \dots & b_{M-1}^M Q_{M(M-1)}(\omega) \\ b_M^1 Q_{1M}(\omega) & \dots & Q_2^D\sum_{u=1}^{M} b_u^s\tilde{k}_{uM}^{\pm+} & \dots & b_M^M Q_{MM}(\omega) \end{bmatrix} \quad (8)$$

$$B(\omega) = \begin{bmatrix} b_1^1 Q_{11}(\omega) & b_1^2 Q_{21}(\omega) & \dots & b_1^M Q_{M1}(\omega) \\ b_2^1 Q_{12}(\omega) & b_2^2 Q_{22}(\omega) & \dots & b_2^M Q_{M2}(\omega) \\ . & . & \dots & . \\ b_M^1 Q_{1M}(\omega) & b_M^2 Q_{2M}(\omega) & \dots & b_M^M Q_{MM}(\omega) \end{bmatrix};$$

Here $\tilde{k}_{sn}^{++} = \left(c_s^{v+}c_n^{g+*}\right)/\left(\omega_n w_n\right) = (\tilde{k}_{sn})_0 e^{-i\Gamma(z_s - z_n)}$; $\tilde{k}_{sn}^{-+} = \left(c_s^{v-}c_n^{g+*}\right)/\left(\omega_n w_n\right) = (\tilde{k}_{sn})_0 e^{-i\Gamma(z_s + z_n)}$; $c_s^{v\pm}$, $c_s^{g\pm}$ - is the expansion coefficients of the s -th DR field: $(\mathbf{e}_s^e, \mathbf{h}_s^e)$, or $(\mathbf{e}_s^o, \mathbf{h}_s^o)$ ($v, g = even\_or\_odd\_mode$; $s = 1,2,...,M$) on the propagation wave field of the line $(\mathbf{E}_1^{\pm}, \mathbf{H}_1^{\pm})$; Γ - longitudinal wave number of a line; z_s - coordinate of the s -th DR; Q_t^D - dielectric loss Q-factor of t -th resonator.

Let us define below, for brevity, the determinant of a matrix $B_s^{\pm}(\omega)$ as the determinant of a matrix $B(\omega)$ in which the s -th column is replaced by a column vector:

$$\mathbf{V}_s = \Big(Q_1^D\sum_{u=1}^{M} b_u^s\tilde{k}_{u1}^{\pm+} \quad Q_1^D\sum_{u=1}^{M} b_u^s\tilde{k}_{u2}^{\pm+} \quad \dots$$
$$\dots \quad Q_2^D\sum_{u=1}^{M} b_u^s\tilde{k}_{u(M-1)}^{\pm+} \quad Q_2^D\sum_{u=1}^{M} b_u^s\tilde{k}_{uM}^{\pm+}\Big)^T \quad (9)$$

where T transposition sign. The structure of the vector $\mathbf{V}_s$ is determined, on the one hand, by the interaction of the partial resonators of the filters with the field of the incident wave $\left(\mathbf{E}_1^{+}, \mathbf{H}_1^{+}\right)$, and on the other hand, by the topology and quantity of it radiation into a given transmission lines.

III. SCATTERING ON DIFFERENT NOTCH FILTERS OF DETUNED DRS

Using the results of theory (2) – (8), we investigate the frequency dependences of the scattering matrix of two notch filters with detuned frequencies, located in series in the transmission line. The study of such filters is of interest for the design of a new type of bandpass filters for optical communication systems, proposed earlier in [26].

Let us assume that all resonators of each filter are identical and located at the same distance from each other. Here and below, circles of different sizes indicate detuned resonators.

To minimizing the mutual coupling coefficients, we proposed that adjacent filter resonators are located on different sides of the line (Fig. 1, a).

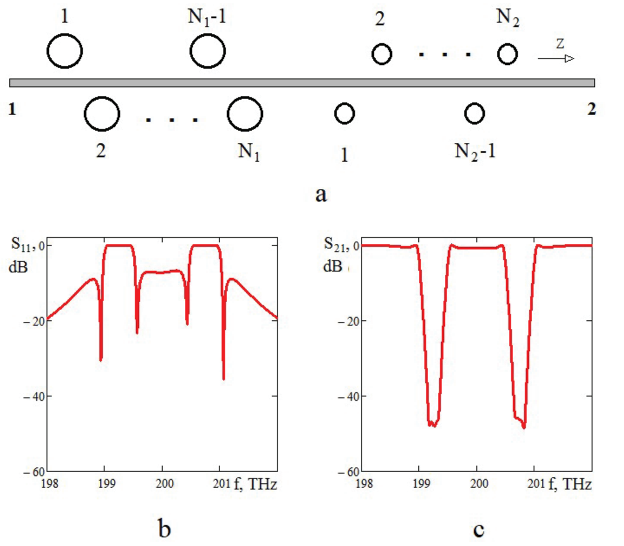

Fig. 1. Scattering characteristics (b, c) of the system of two different optical notch filters (a): $f_0 = 200$ THz; detuned in frequencies $df = \pm 0,75$ THz relative to f_0; $N_1 = N_2 = 4$ DRs. The distance between adjacent DRs $\Gamma\Delta z_{s,s+1} = 31\pi/2$; the distance between nearest filter DRs: $\Gamma\Delta z_{1,2} = 64\pi/2$. DR coupling coefficients with a transmission line: for even mode: $\tilde{k}_s^e = 0,002$; for odd mode: $\tilde{k}_s^o = 0,0002$; coupling coefficients of the resonators with open space: $\tilde{k} = 10^{-7}$; coupling coefficients between neighbouring DRs for even mode: $k_{12}^e = 10^{-5}$; for odd mode: $k_{12}^o = -10^{-6}$.

Considering that the coefficients of mutual coupling are rapidly decreasing functions of the coordinates, we will also neglect the interaction of non-adjacent DRs along non-propagating waves of the line: $k_{st}^{e,o} = k_{12}^{e,o}(1-\delta_{st})\delta_{|s-t|1}$, here δ_{st} Kronecker symbol.

The coupling matrix K (2) of two bandstop filters we represented in the form:

$$K = \left\| i(\tilde{k} + \sum_{v=1}^{2N_1} \tilde{k}_v^{1g}\delta_{sv} + \sum_{v=2N_1+1}^{M} \tilde{k}_v^{2g}\delta_{sv})\delta_{st} + \kappa_{st}(1-\delta_{st}) \right\|.$$

Fig. 1, b, c show the frequency dependences of the S-matrix elements obtained by solving equations (2), (5), taking into account the approximations made. Where $S_{21} = 20\lg|T|$; $S_{11} = 20\lg|R|$.

Let us pay attention to the minimization of parasitic oscillations of the obtained frequency dependences, which arises as a consequence of the minimization of mutual coupling between partial resonators.

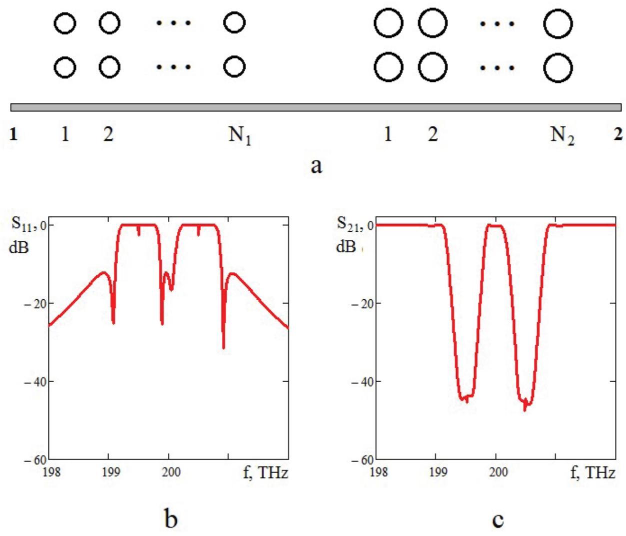

Fig. 2. Two band stop complex design SCISSOR (side-coupled integrated spaced sequence of optical resonators) filters (a). Scattering characteristics (b, c) of the system of filters (a): $f_0 = 200$ THz; $df = \pm 0,5$ THz; $N_1 = N_2 = 4$ resonators; $\Gamma\Delta z_{s,s+1} = 31\pi/2$; $\Gamma\Delta z_{1,2} = 64\pi/2$. DR coupling coefficients with a transmission line: for even mode: $\tilde{k}_s^e = 0,002$; for odd mode: $\tilde{k}_s^o = 0,001$; coupling coefficients of the resonators with open space: $\tilde{k} = 10^{-8}$; coupling coefficients between neighbouring resonators for even mode: $k_{12}^e = 10^{-7}$; for odd mode: $k_{12}^o = -10^{-7}$; coupling coefficients between vertically adjacent resonators for even mode: $kv_{12}^e = 2\cdot 10^{-4}$; for odd mode: $kv_{12}^o = -10^{-5}$.

An example of calculating the frequency dependences of the scattering matrix of two frequency-detuned notch filters of a more complex design is shown in Fig. 2. The main difference of this structure is the possibility of more flexible control frequency characteristics of the scattering matrix by varying the coupling between adjacent filter resonators.

The coupling matrix of these filters K (2) is represented in the form:

$$K = \left\| i(\tilde{k} + \sum_{v=1}^{2N_1} \tilde{k}_v^{1g}\delta_{sv} + \sum_{v=4N_1+1}^{4N_1+2N_2} \tilde{k}_v^{2g}\delta_{sv})\delta_{st} + \kappa_{st}(1-\delta_{st}) \right\|.$$

In contrast to the sequential arrangement of the DR (Fig. 1, a), half of the filter resonators (Fig. 2, a) are not coupled to the transmission line; this leads to the fact that the system of equations (2), (5) has a more complex structure; s-th column of $\mathbf{V}_s$ rows are sparse due to: $c_t^{g\pm} = 0$, if $2N_1 \le t \le 4N_1$, or $4N_1 + 2N_2 \le t \le 4N_1 + 4N_2$. In this case, vectors (9) take the form:

$$\mathbf{V}_s^{\pm} = \left(Q_1^D \sum_{u\in L} b_u^s \tilde{k}_{u1}^{\pm+} \ \ldots \ Q_1^D \sum_{u\in L} b_u^s \tilde{k}_{u(2N_1)}^{\pm+} \ 0 \ \ldots \right. \tag{10}$$

$$\left. Q_2^D \sum_{u\in L} b_u^s \tilde{k}_{u(4N_1+1)}^{\pm+} \ \ldots \ Q_2^D \sum_{u\in L} b_u^s \tilde{k}_{u(4N_1+2N_2)}^{\pm+} \ \ldots \ 0 \right)^T$$

in which the summation is carried out only over the set of natural oscillations of resonators coupled to the transmission line:

$$Q_t^D \sum_{u\in L} b_u^s \tilde{k}_{ut}^{\pm+} = Q_1^D \sum_{u=1}^{2N_1} b_u^s \tilde{k}_{ut}^{\pm+} + Q_2^D \sum_{u=4N_1+1}^{4N_1+2N_2} b_u^s \tilde{k}_{ut}^{\pm+} . \tag{11}$$

The calculations performed show the possibility of obtaining smooth scattering characteristics on filters of this type (Fig. 2, b, c), which may have wider rejection bands.

IV. Scattering on different add/drop filters of detuned DRs

We study the scattering on several add/drop filters [1, 6 - 8] detuned by frequencies and possible options for constructing diplexers based on them, optimized for the frequency characteristics of the S-matrix.

Let's first consider a pair of laterally coupled add/drop filters built on a cascade connection of resonators with whispering gallery oscillations (Fig. 3, a).

In general, define transmission coefficients between 1 and v port of the structure:

$$T_{1v}(\omega) = T_v^0 + \sum_{s=1}^{M} \left(\sum_{u=1}^{L} b_u^s c_u^{g\pm} \right) a^u(\omega) =$$

$$= \delta_{2v} - \frac{1}{B(\omega)} \sum_{s=1}^{M} B_s^{\pm 1v}(\omega) , \tag{12}$$

where T_v^0 is the transmission coefficient without DRs; the scattering matrix coefficients: $S_{v1}(\omega) = 20\lg|T_{1v}(\omega)|$. The $B(\omega)$ function still has the form (8). We again define the $B_s^{1v\pm}(\omega)$ dependencies as the determinant of the matrix $B(\omega)$, in which the s-th column is replaced by a column vector $\mathbf{V}_s^{1v}(\omega)$, determined by the coupling between the filter resonators and the v-th transmission line.

For the add/drop filter shown in Fig. 3, a:

$$\mathbf{V}_s^{11} = \left(Q_1^D \sum_{u\in L_1} b_u^s \tilde{k}_{u1}^{-+} \ \ Q_1^D \sum_{u\in L_1} b_u^s \tilde{k}_{u2}^{-+} \ \ 0 \ \ldots \right.$$

$$\left. Q_2^D \sum_{u\in L_1} b_u^s \tilde{k}_{u(2N_1+1)}^{-+} \ \ Q_2^D \sum_{u\in L_1} b_u^s \tilde{k}_{u(2N_1+2)}^{-+} \ \ 0 \ \ldots \ 0 \right)^T ;$$

$$\mathbf{V}_s^{12} = \left(Q_1^D \sum_{u\in L_1} b_u^s \tilde{k}_{u1}^{++} \ \ Q_1^D \sum_{u\in L_1} b_u^s \tilde{k}_{u2}^{++} \ \ 0 \ \ldots \right.$$

$$\left. Q_2^D \sum_{u\in L_1} b_u^s \tilde{k}_{u(2N_1+1)}^{++} \ \ Q_2^D \sum_{u\in L_1} b_u^s \tilde{k}_{u(2N_1+2)}^{++} \ \ 0 \ \ldots \ 0 \right)^T ; \tag{13}$$

$$\mathbf{V}_s^{13} = \left(Q_1^D \sum_{u\in L_2} b_u^s \tilde{k}_{u1}^{++} \ \ Q_1^D \sum_{u\in L_2} b_u^s \tilde{k}_{u2}^{++} \ \ 0 \ \ldots \right.$$

$$\left. Q_2^D \sum_{u\in L_2} b_u^s \tilde{k}_{u(2N_1+1)}^{++} \ \ Q_2^D \sum_{u\in L_2} b_u^s \tilde{k}_{u(2N_1+2)}^{++} \ \ 0 \ \ldots \ 0 \right)^T$$

$$\mathbf{V}_s^{14} = \left(Q_1^D \sum_{u\in L_2} b_u^s \tilde{k}_{u1}^{-+} \ \ Q_1^D \sum_{u\in L_2} b_u^s \tilde{k}_{u2}^{-+} \ \ 0 \ \ldots . \right.$$

$$\left. Q_2^D \sum_{u\in L_2} b_u^s \tilde{k}_{u(2N_1+1)}^{-+} \ \ Q_2^D \sum_{u\in L_2} b_u^s \tilde{k}_{u(2N_1+2)}^{-+} \ \ 0 \ \ldots \ 0 \right)^T$$

Where formally:

$$\sum_{u\in L_1} b_u^s \tilde{k}_{ur}^{\pm+} = b_1^s \tilde{k}_{1r}^{\pm+} + b_2^s \tilde{k}_{2r}^{\pm+} +$$

$$+ b_{2N_1+1}^s \tilde{k}_{(2N_1+1),r}^{\pm+} + b_{2N_1+2}^s \tilde{k}_{(2N_1+2),r}^{\pm+} ;$$

$$\sum_{u\in L_2} b_u^s \tilde{k}_{ur}^{\pm+} = b_{2N_1-1}^s \tilde{k}_{(2N_1-1),r}^{\pm+} + b_{2N_1}^s \tilde{k}_{(2N_1),r}^{\pm+} +$$

$$+ b_{2(N_1+N_2)-1}^s \tilde{k}_{[2(N_1+N_2)-1],r}^{\pm+} + b_{2(N_1+N_2)}^s \tilde{k}_{[2(N_1+N_2)],r}^{\pm+} .$$

The coupling matrix K (2) of the resonators for the filter shown in Fig. 3, a, we represented in the form:

$$K = \Big\| i(\tilde{k} + \tilde{k}_1\delta_{s1} + \tilde{k}_2\delta_{s2} + \tilde{k}_{2N_1-1}\delta_{s(2N_1-1)} + \tilde{k}_{2N_1}\delta_{s(2N_1)} +$$

$$+ \tilde{k}_{2N_1+1}\delta_{s(2N_1+1)} + \tilde{k}_{2N_1+2}\delta_{s(2N_1+2)} +$$

$$+ \tilde{k}_{2N_1+2N_2-1}\delta_{s(2N_1+2N_2-1)} + \tilde{k}_{2N_1+2N_2}\delta_{s(2N_1+2N_2)})\delta_{st} +$$

$$+ \kappa_{st}(1-\delta_{st}) \Big\| \qquad (14)$$

Where $\tilde{k}_1 = \tilde{k}_1^{1e}$ - is the coupling coefficient of the even oscillations of the 1-st microresonator of the 1 filter; $\tilde{k}_2 = \tilde{k}_1^{1o}$ - is the coupling coefficient of the odd oscillations of the of the 1-st microresonator of the 1 filter with the transmission line 1-2 (Fig. 3, a); $\tilde{k}_{2N_1-1} = \tilde{k}_{N_1}^{1e}$ - is the coupling coefficient of the N_1-th microresonator of the 1 filter with the transmission line 3-4 also on an even oscillation; $\tilde{k}_{2N_1} = \tilde{k}_{N_1}^{1o}$ - is the coupling coefficient of the N_1-th microresonator of the 1 filter with the transmission line 3-4 on an odd oscillation; $\tilde{k}_{2N_1+1} = \tilde{k}_1^{2e}$ - is the coupling coefficient of the even oscillations of the 1-st microresonator of the 2 filter; $\tilde{k}_{2N_1+2} = \tilde{k}_1^{2o}$ - is the coupling coefficient of the odd oscillations of the of the 1-st microresonator of the 2 filter with the transmission line 1-2 (Fig. 3, a); $\tilde{k}_{2N_1+2N_2-1} = \tilde{k}_{N_2}^{2e}$ - is the coupling coefficient of the N_2-th microresonator of the 2 filter with the transmission line 3-4 on an even oscillation; $\tilde{k}_{2N_1+2N_2} = \tilde{k}_{N_2}^{2o}$ - is the coupling coefficient of the N_2-th microresonator of the 2 filter with the transmission line 3-4 on an odd oscillation;

$\tilde{k}$ is the coupling coefficient of the resonator with open space; $\kappa_{st} = \kappa_{12}^{e,o} = \kappa_{21}^{e,o}$ - mutual coupling coefficient of adjacent resonators: if $|s-t| = 2$ and $\kappa_{st} = 0$ in other cases ($s,t = 1,2,...,2(N_1+N_2)$).

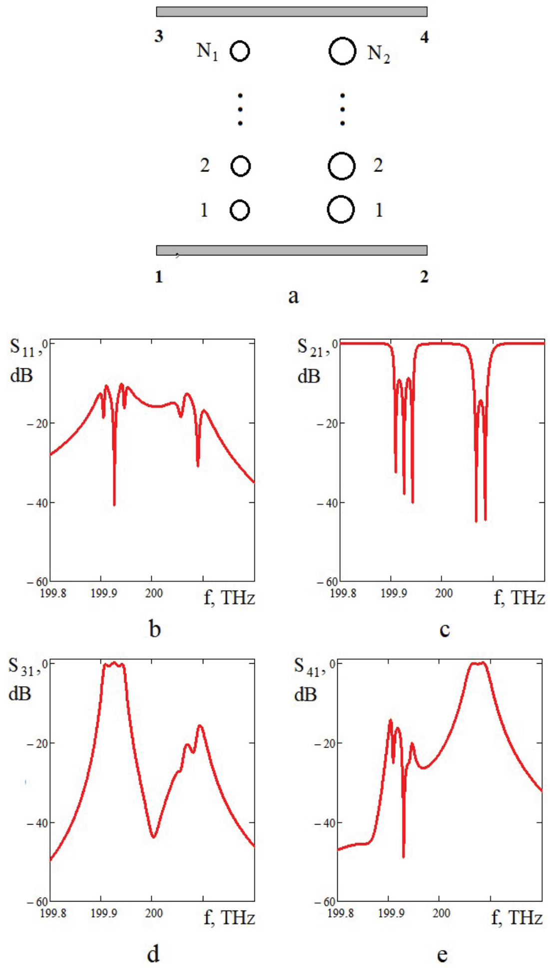

Fig. 3. Two different laterally coupled add/drop filters (a). Scattering characteristics (b - e) of filters (a): $N_1 = 3$; $N_2 = 2$; $f_0 = 200$ THz; $df = \pm 0{,}075$ THz. The distance between filters: $\Gamma\Delta z_{1,2} = 31\pi/2$. DR coupling coefficients with a transmission line: for the even mode: $\tilde{k}_s^{1,2e} = 1{,}5\cdot 10^{-4}$; for odd mode of the first filter: $\tilde{k}_s^{1o} = 8\cdot 10^{-5}$; of the second filter: $\tilde{k}_s^{2o} = 10^{-4}$ coupling coefficients of the DRs with open space: $\tilde{k} = 10^{-7}$; coupling coefficients between neighbouring resonators of the first filter: for even mode: $k_{12}^{1e} = 1{,}4\cdot 10^{-4}$; for odd mode: $k_{12}^{1o} = -1{,}4\cdot 10^{-4}$; of the second filter: for even mode: $k_{12}^{2e} = 1{,}5\cdot 10^{-4}$; for odd mode: $k_{12}^{2o} = -1{,}5\cdot 10^{-4}$.

A proposed scattering model allows us to simultaneously tune the characteristics of several add/drop filters, controlling the transmission and reflection characteristics of the resonator system in the line.

Let us assume that the natural frequencies of the partial resonators of the first filter are $f_1 = 199{,}925$ THz; and the second filter $f_2 = 200{,}075$ THz. The quality factor of the dielectric of all resonators is the same: $Q^D = 10^6$. The mutual coupling coefficients between adjacent resonators are the same: $k^g_{s(s+1)} = k^g_{12}$.

Fig. 3, b – e shows the frequency characteristics of the scattering matrix on two bandpass add/drop filters made of two systems of detuned resonators.

It is also evident from the obtained data that in this design the use of two frequency-detuned filters with a number of resonators of different parity, allows the channels to be separated with a minimum number of transmission lines used.

The attenuation between the diplexer channels can be increased by adding a complementary line to the structure (Fig. 4, a). In this case, the analytical expressions for the vectors $\mathbf{V}_s^{11}$; $\mathbf{V}_s^{12}$ coincide with (13), while

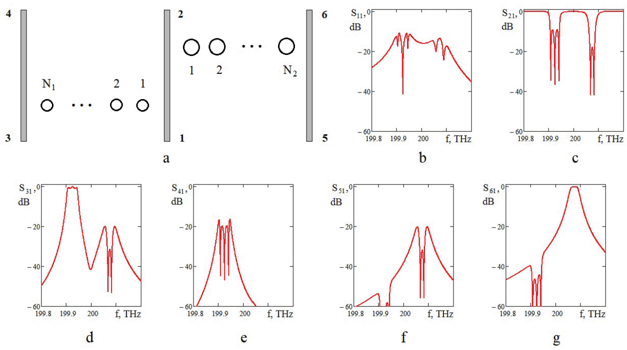

Fig. 4. Diplexer on two different bandpass add/drop filters (a). Scattering characteristics (b - e) of filters (b - g). Coupling coefficients between neighbouring resonators of the second filter: for even mode: $k_{12}^{2e} = 1{,}4\cdot 10^{-4}$; for odd mode: $k_{12}^{2o} = -1{,}4\cdot 10^{-4}$. The remaining parameters of the resonators are as in Fig. 3.

$$\mathbf{V}_s^{13} = \left(Q_1^D \sum_{u\in L_2} b_u^s \tilde{k}_{u1}^{++} \quad Q_1^D \sum_{u\in L_2} b_u^s \tilde{k}_{u2}^{++} \quad 0 \quad \ldots \quad 0 \right)^T; \tag{15}$$

$$\mathbf{V}_s^{14} = \left(Q_1^D \sum_{u\in L_2} b_u^s \tilde{k}_{u1}^{-+} \quad Q_1^D \sum_{u\in L_2} b_u^s \tilde{k}_{u2}^{-+} \quad 0 \quad \ldots \quad 0 \right)^T;$$

$$\mathbf{V}_s^{15} = \left(0 \quad \ldots \quad 0 \quad Q_2^D \sum_{u\in L_2} b_u^s \tilde{k}_{u(2N_1+1)}^{++} \quad Q_2^D \sum_{u\in L_2} b_u^s \tilde{k}_{u(2N_1+2)}^{++} \quad 0 \quad \ldots \quad 0 \right)^T$$

$$\mathbf{V}_s^{16} = \left(0 \quad \ldots \quad 0 \quad Q_2^D \sum_{u\in L_2} b_u^s \tilde{k}_{u(2N_1+1)}^{-+} \quad Q_2^D \sum_{u\in L_2} b_u^s \tilde{k}_{u(2N_1+2)}^{-+} \quad 0 \quad \ldots \quad 0 \right)^T.$$

The coupling matrix K (2) of the microresonators for the structure shown in Fig. 4, a we presented in the form (14), by eliminating the coupling between the filter resonators along the propagating waves of the line.

In a similar way, one can calculate the frequency characteristics of scattering at two frequencies detuned parallel-coupled add/drop filters.

In the case of parallel arrangement of filters (Fig. 5, a), the general solution of the scattering problem also has the form (8)-(9), in which:

$$\mathbf{V}_s^{11} = \left(Q_1^D \sum\nolimits_{u=1}^{M} b_u^s \tilde{k}_{u1}^{-+} \quad Q_1^D \sum\nolimits_{u=1}^{M} b_u^s \tilde{k}_{u2}^{-+} \quad \ldots \quad \ldots \quad Q_2^D \sum\nolimits_{u=1}^{M} b_u^s \tilde{k}_{u(M-1)}^{-+} \quad Q_2^D \sum\nolimits_{u=1}^{M} b_u^s \tilde{k}_{uM}^{-+} \right)^T;$$

$$\mathbf{V}_s^{12} = \left(Q_1^D \sum\nolimits_{u=1}^{M} b_u^s \tilde{k}_{u1}^{++} \quad Q_1^D \sum\nolimits_{u=1}^{M} b_u^s \tilde{k}_{u2}^{++} \quad \ldots \quad \ldots \quad Q_2^D \sum\nolimits_{u=1}^{M} b_u^s \tilde{k}_{u(M-1)}^{++} \quad Q_2^D \sum\nolimits_{u=1}^{M} b_u^s \tilde{k}_{uM}^{++} \right)^T$$

$$\mathbf{V}_s^{13} = \mathbf{V}_s^{12}; \qquad \mathbf{V}_s^{14} = \mathbf{V}_s^{11}. \tag{16}$$

The coupling matrix K (2), shown in Fig. 6, and in the case of the same coupling of the resonators with the transmission lines, we represented in the form:

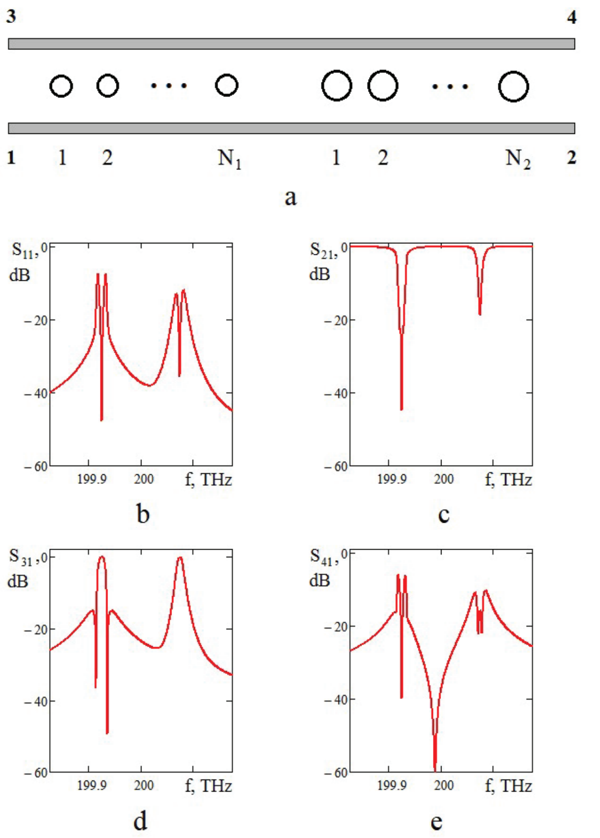

Fig. 5. Two different parallel-coupled add/drop filters (a) in a transmission line. Scattering characteristics (b - e) for $N_1 = 3$; $N_2 = 2$; $f_0 = 200$ THz; $df = \pm 0,075$ THz. The distance between adjacent resonators $\Gamma\Delta z_{s,s+1} = 31\pi/2$; the distance between nearest filter resonators: $\Gamma\Delta z_{1,2} = 62\pi/2$. DRs coupling coefficients with a transmission line: for even mode of the first filter: $\tilde{k}_s^{1e} = 3\cdot10^{-5}$; the second filter: $\tilde{k}_s^{2e} = 4\cdot10^{-5}$; for the odd mode of the filters: $\tilde{k}_s^{1o} = \tilde{k}_s^{2o} = 2\cdot10^{-5}$; coupling coefficients resonators with open space: $\tilde{k} = 10^{-7}$; coupling coefficients between neighbouring resonators of the first filter: for even mode: $k_{12}^{1e} = 8\cdot10^{-6}$; for odd mode: $k_{12}^{1o} = -4\cdot10^{-5}$; of the second filter: for even mode: $k_{12}^{2e} = 2\cdot10^{-5}$; for odd mode: $k_{12}^{2o} = -2\cdot10^{-6}$.

$$K = \left\| i(\tilde{k} + 2\sum_{v=1}^{2N_1} \tilde{k}_v^{1g}\delta_{sv} + 2\sum_{v=2N_1+1}^{M} \tilde{k}_v^{2g}\delta_{sv})\delta_{st} + \kappa_{st}(1-\delta_{st}) \right\| \quad (17)$$

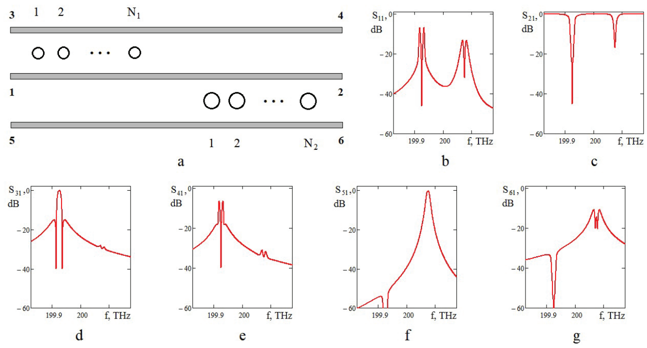

Fig. 6. Diplexer on two different parallel-coupled add/drop filters (a). Scattering characteristics (b - e) for $N_1 = 3$; $N_2 = 2$; $f_0 = 200$ THz; $df = \pm 0,075$ THz. The distance between adjacent microresonators $\Gamma\Delta z_{s,s+1} = 31\pi/2$; the distance between nearest filter resonators: $\Gamma\Delta z_{1,2} = 61\pi/2$. Microresonator coupling coefficients with a transmission line: for even mode of the first filter: $\tilde{k}_s^{1e} = 3\cdot10^{-5}$; the second filter: $\tilde{k}_s^{2e} = 4\cdot10^{-5}$; for the odd mode of the filters: $\tilde{k}_s^{1o} = \tilde{k}_s^{2o} = 2\cdot10^{-5}$; coupling coefficients of the resonators with open space: $\tilde{k} = 10^{-7}$; coupling coefficients between neighbouring resonators of the first filter: for even mode: $k_{12}^{1e} = 8\cdot10^{-6}$; for odd mode: $k_{12}^{1o} = -4\cdot10^{-5}$; of the second filter: for even mode: $k_{12}^{2e} = 2,4\cdot10^{-5}$; for odd mode: $k_{12}^{2o} = -2\cdot10^{-6}$.

Fig. 5 shows the frequency dependences of the scattering matrix of two frequency-detuned parallel-coupled add/drop filters, obtained using the relations (2) (8)-(9), (16).

A diplexer built on the basis of parallel-coupled resonators is obtained by supplement additional transmission line (Fig. 6, a) and “separating” the filters. The necessary analytical relations for the scattering matrix are obtained from (9), taking into account that the vectors $\mathbf{V}_s^{11}$; $\mathbf{V}_s^{12}$; in this case formally coincide with (16), while

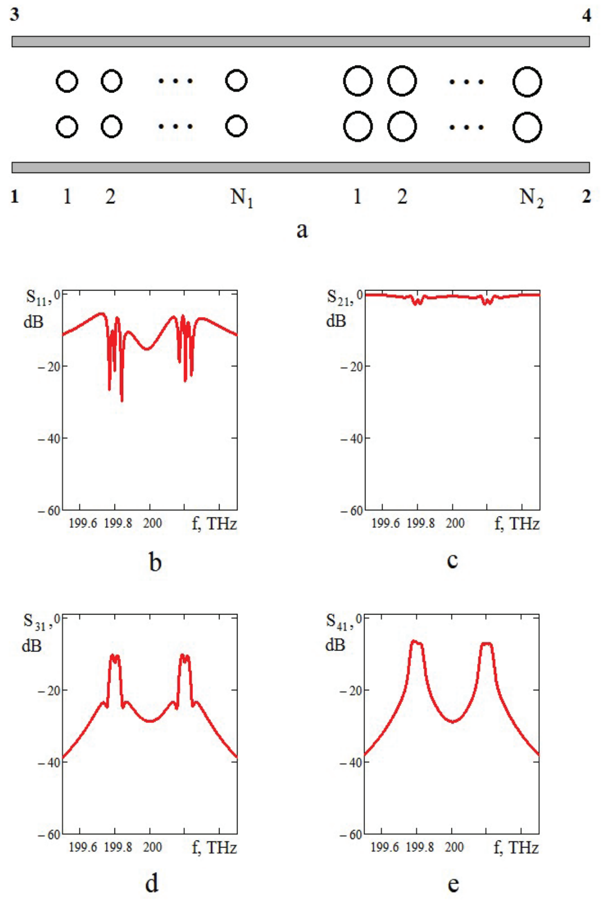

Fig. 7. Different twisted double-channel SCISSORs (a) in a transmission line. Scattering characteristics (b - e) for $N_1 = 2$; $N_2 = 2$; $f_0 = 200$ THz; $df = \pm 0{,}2$ THz. The distance between adjacent resonators $\Gamma \Delta z_{s,s+1} = 32\pi/2$; the distance between nearest filter DRs: $\Gamma \Delta z_{1,2} = 64\pi/2$. Microresonators coupling coefficients with a transmission line: for even mode: $\tilde{k}_s^{1e} = \tilde{k}_s^{2e} = 5 \cdot 10^{-4}$; for the odd mode of the filters: $\tilde{k}_s^{1o} = \tilde{k}_s^{2o} = 2 \cdot 10^{-4}$; coupling coefficients resonators with open space: $\tilde{k} = 10^{-8}$; coupling coefficients between neighbouring resonators of the filters: for even mode: $k_{12}^{1e} = k_{12}^{2e} = 3 \cdot 10^{-6}$; for odd mode: $k_{12}^{1o} = k_{12}^{2o} = -1 \cdot 10^{-6}$.

$$\mathbf{V}_s^{13} = \left(Q_1^D \sum\nolimits_{u=1}^{M} b_u^s \tilde{k}_{u1}^{++} \quad Q_1^D \sum\nolimits_{u=1}^{M} b_u^s \tilde{k}_{u2}^{++} \quad \dots \quad 0 \quad 0 \right)^T;$$

$$\mathbf{V}_s^{14} = \left(Q_1^D \sum\nolimits_{u=1}^{M} b_u^s \tilde{k}_{u1}^{-+} \quad Q_1^D \sum\nolimits_{u=1}^{M} b_u^s \tilde{k}_{u2}^{-+} \quad \dots \quad 0 \quad 0 \right)^T;$$

$$\mathbf{V}_s^{15} = \left(0 \quad 0 \quad \dots \quad Q_2^D \sum\nolimits_{u=1}^{M} b_u^s \tilde{k}_{u(M-1)}^{++} \quad Q_2^D \sum\nolimits_{u=1}^{M} b_u^s \tilde{k}_{uM}^{++} \right)^T; \tag{18}$$

$$\mathbf{V}_s^{16} = \left(0 \quad 0 \quad \dots \quad Q_2^D \sum\nolimits_{u=1}^{M} b_u^s \tilde{k}_{u(M-1)}^{-+} \quad Q_2^D \sum\nolimits_{u=1}^{M} b_u^s \tilde{k}_{uM}^{-+} \right)^T.$$

In the coupling matrix K (17) we also exclude the coupling between the resonators of filters 1-2 on the propagating waves of the line.

Fig. 6 shows the example of the scattering characteristics calculating of a diplexer built on two bandpass filters. Simultaneous optimization of filters also allows us to control the maximum attenuation between channels as well as other scattering parameters (Fig. 6, b - g).

The most complex case arises when describing scattering on frequency-detuned filters, known as twisted double-channel SCISSORs (side-coupled integrated spaced sequence of optical resonators). In the case of parallel placement of filters between two lines (Fig. 7, a), the vectors $\mathbf{V}_s^{11}$; $\mathbf{V}_s^{12}$; take a form similar to (10), (11)

$$\mathbf{V}_s^{11} = \Big(Q_1^D \sum_{u \in L} b_u^s \tilde{k}_{u1}^{-+} \quad \dots \quad Q_1^D \sum_{u \in L} b_u^s \tilde{k}_{u(2N_1)}^{-+} \quad 0 \quad \dots$$
$$Q_2^D \sum_{u \in L} b_u^s \tilde{k}_{u(4N_1+1)}^{-+} \quad \dots \quad Q_2^D \sum_{u \in L} b_u^s \tilde{k}_{u(4N_1+2N_2)}^{-+} \quad \dots \quad 0 \Big)^T \tag{19}$$

$$\mathbf{V}_s^{12} = \Big(Q_1^D \sum_{u \in L} b_u^s \tilde{k}_{u1}^{++} \quad \dots \quad Q_1^D \sum_{u \in L} b_u^s \tilde{k}_{u(2N_1)}^{++} \quad 0 \quad \dots$$
$$Q_2^D \sum_{u \in L} b_u^s \tilde{k}_{u(4N_1+1)}^{++} \quad \dots \quad Q_2^D \sum_{u \in L} b_u^s \tilde{k}_{u(4N_1+2N_2)}^{++} \quad \dots \quad 0 \Big)^T$$

while $\mathbf{V}_s^{13}$; $\mathbf{V}_s^{14}$ are defined by equalities:

$$\mathbf{V}_s^{13} = \Big(Q_1^D \sum_{u \in J} b_u^s \tilde{k}_{u1}^{++} \quad \dots \quad Q_1^D \sum_{u \in J} b_u^s \tilde{k}_{u(2N_1)}^{++} \quad 0 \quad \dots$$
$$Q_2^D \sum_{u \in J} b_u^s \tilde{k}_{u(4N_1+1)}^{++} \quad \dots \quad Q_2^D \sum_{u \in J} b_u^s \tilde{k}_{u(4N_1+2N_2)}^{++} \quad \dots \quad 0 \Big)^T$$

$$\mathbf{V}_s^{14} = \Big(Q_1^D \sum_{u\in J} b_u^s \tilde{k}_{u1}^{-+} \;\; \ldots \;\; Q_1^D \sum_{u\in J} b_u^s \tilde{k}_{u(2N_1)}^{-+} \;\; 0 \;\; \ldots$$

$$Q_2^D \sum_{u\in J} b_u^s \tilde{k}_{u(4N_1+1)}^{-+} \;\; \ldots \;\; Q_2^D \sum_{u\in J} b_u^s \tilde{k}_{u(4N_1+2N_2)}^{-+} \;\; \ldots \;\; 0 \Big)^T \quad (20)$$

where also formally:

$$\sum_{u\in J} b_u^s \tilde{k}_{ut}^{\pm+} = \sum_{u=2N_1+1}^{4N_1} b_u^s \tilde{k}_{ut}^{\pm+} + \sum_{u=4N_1+2N_2}^{4N_1+4N_2} b_u^s \tilde{k}_{ut}^{\pm+}$$

for a symmetrical arrangement of resonators relative to the lines 1-2 and 3-4.

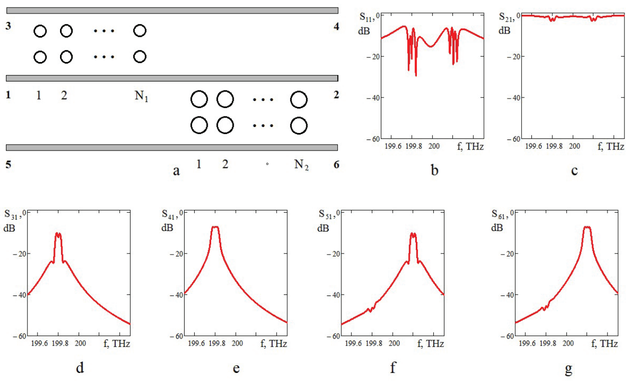

Fig. 8. Diplexer on two different twisted double-channel CISSORs (a). Scattering characteristics (b - g) of the diplexer. Parameters of the filters are as in Fig. 7.

The coupling matrix K (2) has the form:

$$K = \left\| i(\tilde{k} + \sum_{v=1}^{4N_1} \tilde{k}_v^{1g}\delta_{sv} + \sum_{v=4N_1+1}^{4N_1+4N_2} \tilde{k}_v^{2g}\delta_{sv})\delta_{st} + \kappa_{st}(1-\delta_{st}) \right\| . \quad (21)$$

Adding a third line to construct the diplexer (Fig. 8, a) redefines the vectors:

$$\mathbf{V}_s^{13} = \Big(Q_1^D \sum_{u=2N_1+1}^{4N_1} b_u^s \tilde{k}_{u1}^{++} \;\; \ldots$$

$$\ldots \;\; Q_1^D \sum_{u=2N_1+1}^{4N_1} b_u^s \tilde{k}_{u(2N_1)}^{++} \;\; 0 \;\; \ldots \;\; 0 \Big)^T ;$$

$$\mathbf{V}_s^{14} = \Big(Q_1^D \sum_{u=2N_1+1}^{4N_1} b_u^s \tilde{k}_{u1}^{-+} \;\; \ldots$$

$$\ldots \;\; Q_1^D \sum_{u=2N_1+1}^{4N_1} b_u^s \tilde{k}_{u(2N_1)}^{-+} \;\; 0 \;\; \ldots \;\; 0 \Big)^T ; \quad (22)$$

$$\mathbf{V}_s^{15} = \Big(0 \;\; \ldots \;\; 0 \;\; Q_2^D \sum_{u=4N_1+2N_2}^{4N_1+4N_2} b_u^s \tilde{k}_{u(4N_1+1)}^{-+} \;\; \ldots$$

$$\ldots \;\; Q_2^D \sum_{u=4N_1+2N_2}^{4N_1+4N_2} b_u^s \tilde{k}_{u(4N_1+2N_2)}^{-+} \;\; 0 \;\; \ldots \;\; 0 \Big)^T ;$$

$$\mathbf{V}_s^{16} = \Big(0 \;\; \ldots \;\; 0 \;\; Q_2^D \sum_{u=4N_1+2N_2}^{4N_1+4N_2} b_u^s \tilde{k}_{u(4N_1+1)}^{++} \;\; \ldots$$

$$\ldots \;\; Q_2^D \sum_{u=4N_1+2N_2}^{4N_1+4N_2} b_u^s \tilde{k}_{u(4N_1+2N_2)}^{++} \;\; 0 \;\; \ldots \;\; 0 \Big)^T$$

and changes the structure of the K matrix (21).

The use of a two-dimensional lattice for constructing a diplexer (Fig. 7, 8, a) increases the steepness of the side slopes of the frequency dependences of scattering (Fig. 7, d – e; Fig. 8, e - g), while, at the same time, leading to a decrease in the scattering asymmetry between adjacent ports (Fig. 7, 8, d – e; f - g).

Fig. 9 shows the scattering characteristics of the considered type diplexers, “optimized” according to the condition of minimum bandwidth of the filters. The obtained results demonstrate a noticeable dependence of the frequency characteristics of scattering on the choice of filter types.

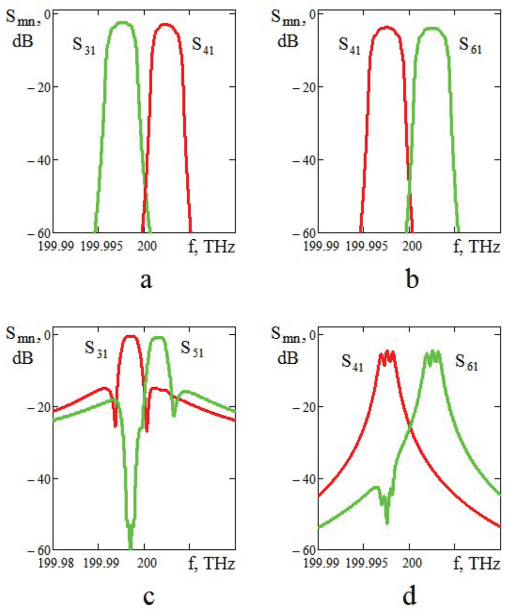

Fig. 9. "Optimized" scattering characteristics of different diplexers: (a) on two laterally coupled add/drop filters (Fig. 3, a); ($N_1 = 7$; $N_2 = 8$ DRs; $\Delta f = 50$ GHz); (b) on two add/drop filters in different transmission lines (Fig. 4, a); ($N_1 = N_2 = 8$ DRs; $\Delta f = 50$ GHz); (c) on two parallel-coupled add/drop filters (Fig. 6, a); ($N_1 = N_2 = 3$ DRs; $\Delta f = 60$ GHz); (d) on two different twisted double-channel SCISSORs (Fig. 8, a); ($N_1 = N_2 = 4$ DRs; $\Delta f = 50$ GHz).

V. Conclusion

A general theory of scattering of electromagnetic waves by systems of detuned in frequency dielectric resonators, proposed in [27], opens up new possibilities for constructing correct models of complex systems of bandpass and notch filters suitable for the optimization of multiplexers. The proposed electromagnetic models of frequency-detuned add/drop filters, constructed based on the use of DRs with degenerate whispering gallery oscillations, confirm the main features of the scattering characteristics studied earlier by using direct numerical solutions of Maxwell's equations for similar structures.

The conducted research allows significantly accelerating the design and optimization of scattering characteristics of modern optical communication systems using technology WDM.

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Оптимізація матриці розсіювання частотно-розстроєних фільтрів введення/виводу для мультиплексорів, побудованих на системах оптичних діелектричних резонаторів з коливаннями шепочучої галереї

Проблематика. Суттєве підвищення швидкості передачі інформації в волоконно-отичних мережах звязку визначається жорсткими вимогами, що пред'являються до елементної бази приймально-передавальних пристроїв. Одної із важливих складових частин таких пристроїв є диплексери, побудовані на смугових фільтрах, яки часто виконуються на діелектричних резонаторах з коливаннями шепочучої галереї. Розрахунок та оптимізація параметрів багатоланкових смугових фільтрів та диплексерів, побудованих на їх основі, неможливо без подальшого розвитку теорії їх проектування. Розвиток теорії диплексерів сьогодні часто базується на електродинамічному моделюванні яке ґрунтується на попередніх розрахунках параметрів розсіювання фільтрів в різноманітних лініях передачі складної топології зв’язків.

Мета досліджень. Метою даного дослідження є побудова електродинамічних моделей розсіювання хвиль на багатозв’язних структурах ДР з виродженими типами власних коливань МШГ, які містять декілька частотно-розстроєних смугових або режекторних фільтрів, розташованих в одній або кількох лініях передачі. Для вирішення проблеми розсіювання ми використовуємо систему рівнянь, отриману з теорії збурень для рівнянь Максвелла [24], модифіковану для опису полів DR з коливаннями шепочучої галереї в лінії передачі. Побудова таких рішень ускладнюється тим, що кожен із часткових оптичних резонаторів фільтрів у разі збудження в ньому азимутально неоднорідної ШГМ має, як правило, два вироджених типи власних коливань. Крім того, кожен такий тип коливань характеризується різними комплексними значеннями коефіцієнтів зв'язку з лінією, простором, а також з іншими резонаторами. Остання обставина призводить до того, що системи рівнянь для амплітуд власних і вимушених коливань резонаторів подвоюються. Знаки коефіцієнтів взаємного зв’язку зазвичай різні, це призводить до того, що поведінку ДР в системі стає складніше передбачити, тому другою метою цієї роботи є дослідження закономірностей характеристик розсіювання хвиль ліній передачи. на системах частотно-розстроєних ДР з виродженими типами коливань з можливістю побудови мультиплексорів для сучасних оптичних систем зв'язку.

Методика реалізації. Для розрахунку та аналізу матриць розсіювання використовуються методи технічної електродинаміки. Кінцевим результатом є отримання нових аналітичних рівнянь та формул для нових складних структур зв’язаних діелектричних резонаторів з коливаннями шепочучої галереї.

Результати досліджень. Розглянуто частотні залежності матриць розсіювання на складних структурах частотно-розстроюваних фільтрів, побудованих на сполучених оптичних діелектричних резонаторах (ДР) з коливаннями шепочучої галереї, розташованих в одній або кількох лініях передачі. Побудовано електромагнітні моделі смугових і фільтрів введення/виводу, які складаються з різноманітних оптичних резонаторів з виродженими типами власних коливань. Наведено загальні аналітичні вирази коефіцієнтів матриць для побудови систем рівнянь, що описують зв'язані коливання, а також вимушені коливання резонаторів у випадках їх використання в оптичних фільтрах з послідовним і паралельним розташуванням. Знайдено загальні аналітичні рішення для поля розсіювання на частотно-розстроєних резонаторах, розташованих у різних оптичних лініях передачі. Наведено приклади розрахунку частотних залежностей матриці розсіювання для найбільш цікавих структур, що складаються з двох різночастотних розстроєних фільтрів. Розраховано частотні дисперсійні характеристики декількох типів пристроїв, які складаються з двох режекторних фільтрів з різними смугами загородження, виконаних на розстроєних ДР в одній лінії передачі. Можливості запропонованого методу продемонстровано на прикладі розрахунку характеристик розсіювання відомих типів диплексерів, побудованих на основі використання двох фільтрів введення/виводу з різними частотними смугами пропускання. Частотні залежності матриць розсіювання двох найпоширеніших типів пристроїв, розташованих паралельно між двома або чотирма регулярними лініями передачі фільтрів введення/виводу з різною кількістю резонаторів; фільтри введення/виводу з боковими зв'язками; паралельно з'єднані фільтри введення/виводу; скручена двоканальна з бічним зв'язком інтегрована просторова послідовність резонаторів. Побудовано нові моделі розсіювання диплексерів, що складаються з оптичних резонаторів різної топології з’єднання: послідовного, паралельного з використанням фільтрів введення/виводу з боковими зв'язками; паралельно з'єднані фільтри введення/виводу; скручена двоканальна з бічним зв'язком інтегрована просторова послідовність резонаторів.

Розраховано частотні залежності матриці розсіювання диплексера. Проведено порівняння характеристик розроблених диплексерів, отриманих на основі досліджуваних фільтрів різних типів.

Висновки. Розширено теорія конструювання диплексерів яке відбувається на одночасної оптимізації матриці розсіювання декількох фільтрів, побудованих на складних системах діелектричних резонаторів з виродженими типами коливань шепочучої галереї. Побудована методика розрахунку та знайдено нові аналітичні співвідношення для коефіцієнтів матриці розсіювання оптичних диплексерів різних видів.

Ключові слова: *розсіювання; діелектричний резонатор; матриця розсіювання; режекторний фільтр; фільтри введення/виводу з боковими зв'язками; паралельно з'єднані фільтри введення/виводу; скручена двоканальна з бічним зв'язком інтегрована просторова послідовність резонаторів; диплексер.*