

# RESONATORS OF MECHANICALLY TUNABLE WAVEGUIDE FILTERS FOR TROPOSPHERIC COMMUNICATION

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The class of resonators which allows building mechanically tunable waveguide bandpass filters with near constant bandwidth in overall a wide tuning frequency range has been investigated. The concept comprising in a successful combining inductive and capacitive discontinuities to obtain near constant quality factor of resonators in wide frequency variation range is implemented. It is proved that the most technologically advanced solution can be obtained by using the waveguide connections containing double ridged sections for compensation of phase shifts in resonators appearing because of frequency variation. Adequate mathematical models of the resonators based on generalized scattering matrix approach are obtained. The diffraction problems for doubled discontinuities, such as doubled ridged sections in rectangular waveguide, have been solved by the integral equation method. The calculation of resonance frequencies and quality factors of resonators are carried out in wide dimension range variations. Due to near constant quality factors over wide tuning frequency range, the developed waveguide resonators open new possibilities in realization of high efficient and cost effective frequency tunable filters of transmit tropospheric stations and various telecommunication systems for tropospheric communication.

## Introduction

The growth of the complexity of problems to be solved by modern systems of information transmission has stimulated the development of tropospheric stations for organization of ultra long communication operating in centimeter range [1–2]. Functional principle of such stations is based on the phenomenon of electromagnetic energy reradiation in electrically inhomogeneous troposphere. Because of the low intensity of tropospheric irregularities, the average signal power at the tropospheric communication is very low and rapidly decreases with growth of distance. To mediate the influence of signal fading on the quality of tropospheric communication, several methods of improving the transmission are employed. For this purpose, the transmission and reception for the same message on multiple carrier frequencies are widely used. Furthermore, the transmitters of sufficiently high power are applied in order to compensate the signal losses due to fading phenomenon. Despite the great advances in the implementation of high quality ultra long communication, the problem of finding the ways for further improvement of tropospheric station characteristics remains highly relevant. The necessity of ensuring the proper communication between objects, located at large distances from each other, nominates creating the tropospheric stations with frequency tuning in a number of urgent tasks. The coverage of steady ultra long communication is achieved by employment of appropriate equipment also with tuning the frequency. The feature of such tropospheric station is the presence of bandpass filters, which pass band

moves along the frequency range under tuning. In its transmitting block, the waveguide filters, intended on transfer of significant power levels, are applied. Waveguide filters turning mechanically by metal rods imposed into resonators through the side wall of waveguide are most widely used [3, 4]. Such a technical solution of filter tuning problem provides a good linearity of resonator frequency dependencies as a function of the depth of metal rod plunge into the cavity.

The main disadvantage of existing waveguide mechanically tunable filters, based on traditional discontinuities in the form of inductive and capacitive elements designed in [4], is a significant change of their pass band width, when resonant frequencies of cavities are tuned. Filters with inductive discontinuities are characterized by severe narrowing of pass band width under tuning in the range of lower operating frequencies with simultaneous increase of insertion loss on central frequency. Filters with capacitive elements have an inverse dependence of pass band width changes on frequency while its slope is less than for filters with inductive discontinuities. The main disadvantage that prevents the use of capacitive discontinuities in waveguide mechanically tunable filters is a low level of transferred power due to small gaps between edges of diaphragm or rods.

To overcome the mentioned disadvantages of existing waveguide mechanically tunable filters, a new approach to resonators design consisting in combination of inductive and capacitive discontinuities is proposed in [3]. A successful combination of conductivity values of these discontinuities allows significantly improving diapason

properties of waveguide mechanically tunable bandpass filter and achieving near constant pass band width in wide range of tuning without appreciable change of insertion loss. The proposed approach is illustrated by the design example of narrow-band waveguide mechanically tunable bandpass filter with quarter-wave coupling between resonators. This filter is performed on the finite thickness plate symmetrically situated along rectangular waveguide parallel to its narrow walls.

The key building blocks of filter structure, investigated in [3], are waveguide mechanically tunable resonators formed by cascade connections of inductive and capacitive discontinuities. The inductive discontinuity is realized as a partition separating the standard rectangular waveguide on two identical below cutoff regions. The capacitive discontinuity is formed by a section of double ridge waveguide of finite length. By combining these key building blocks in various combinations differing only by dimensions as well as by adding the tunable element, one can obtain the separate variants of tunable filter constructions.

As a tunable element, a round rod inserting into resonator cavity through the side wall of the rectangular waveguide is entirely applied. Such technical solution decreases the incline of calibration response of the resonator and increases its linearity. The equivalent representation of mechanically tunable waveguide resonator with round rod as a tunable element is shown in Fig. 1. The presence of tunable element inevitably breaks the resonator symmetry with respect to vertical plane. Due to absence of resonator symmetry in vertical plane, the theoretical investigation of such combined structure becomes extremely complicated. Therefore, in [3] all elements of resonator were calculated separately without the tunable element which characteristics were determined by approximate analysis according to [4].

The mathematical model of mechanically tunable resonators in the form of generalized scattering matrix of key building block waveguide structure was developed for designing the filter. The generalized scattering matrix of composite connection was determined by combining of generalized scattering matrices of inductive and capacitive discontinuities through the section of transmission line between them. An algorithm of the generalized scattering matrix calculation was built taking into account longitudinal symmetry planes. As a result, the estimated models of inductive and capacitive discontinuity structures were transformed to the forms shown in Fig 1. Taking into account the longitudinal symmetry planes allowed simplifying a task on a calculation stage of coupling coefficients of connected waveguide eigenmodes.

The required relations for the calculation of generalized scattering matrices were obtained by integral equations method [5, 6] in assumption of ideal conductivity of metal walls of waveguides and discontinuities. A distinctive feature of this problem is three-dimensional nature of discontinuities, which requires the use of numerical methods for calculating of eigenmodes of double ridged waveguide to be connected with rectangular waveguide. For this purpose, the method of partial regions taking into account peculiarity of the electromagnetic fields behavior on the rectangular edge [7, 8] was employed.

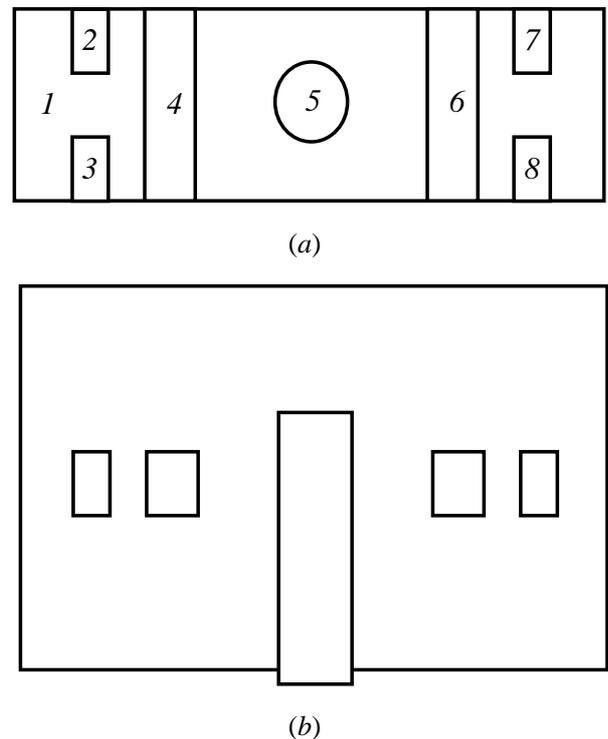


Fig. 1. Schematic representation of front view without side wall (a) and top view without upper wall (b) of waveguide mechanically tunable resonator: (1) the section of uniform rectangular waveguide; (2), (3) the upper and lower posts of double ridged capacitive discontinuity; (4) the finite thickness plate divided the uniform rectangular waveguide on two equal below cutoff rectangular waveguides; (5) the round rod inserted into resonator cavity through the side wall changing the resonance frequency at its moving; (6) the plate with equivalent inductive conductivity the same as (4); (7), (8) the upper and lower posts of double ridged waveguide section the same as (2), (3).

A characteristic feature of the solution obtain in [3] was using the single mode approximation of fields in gap region between ridges. As a result, characteristic parameters of certain resonators and filter a whole were found approximately. Experimental investigations of filters calculated on approximate technique show the distortion of frequency responses under tuning. This has resulted in the necessity to adjust the resonant frequen-

cies of resonators. For this purpose, a special technique of the resonant frequencies adjustment of manufactured samples was developed. In such a way, if the approximate technique of calculation is used, to obtain required characteristics of tunable filters, the experimental completion of their produced samples is needed. The experimental completion of produced mechanically tunable waveguide filters considerably complicates their industrial production. Taking into account the importance of the task solution of such filters production that satisfy the modern requirements, considerable practical interest is refinement of resonator calculation technique as composite parts of tunable filter.

**Statement of the problem**

The purpose of this paper is the advance of existing design technique of mechanically tunable waveguide resonators, which allows simplifying the experimental adjustments of produced samples as well as the calculation and theoretical research of adjustment characteristics ensuring the near constant quality factors of resonators in wide frequency range.

To obtain more correct solution of mechanically tunable waveguide resonator problem then that obtained in [3], it is necessary to take into consideration the presence of tunable round rod. To simplify the numerical analysis of entire structure asymmetric with respect to vertical plane, we implement the approach proposed in [9]. The essence of this approach consisted in replacement of complicated structure comprising a dielectric insert by the simple one with purely metallic discontinuities. When applying to the structure shown in Fig. 1, this original idea can be implemented in order to replace asymmetric tunable element by its symmetric virtual analog. Such virtual structure can be performed as finite length section of double ridged waveguide. Then, the equivalent structure to be investigated takes the configuration shown in Fig. 2. A transition from asymmetric tunable resonator structure to its symmetric virtual analog crucially simplifies the problem solution, reduces the computer resources and increases the accuracy of computations. The designing tunable resonators in rigorous formulation assumes finding generalized scattering matrices of their waveguide components and taking into consideration the entire set of eigenmodes of double ridged waveguide in gap region.

**Scattering matrices of double ridged waveguide section of finite length**

To create effective calculating algorithms of mechanically tunable waveguide resonator components, we take into account their mirror symmetry with respect to a plane going through the middle of the structure

perpendicularly to its longitudinal axes. The advantage of this approach is evident in the case when calculating generalized scattering matrices of longitudinally complicated waveguide structures with nearly placed discontinuities.

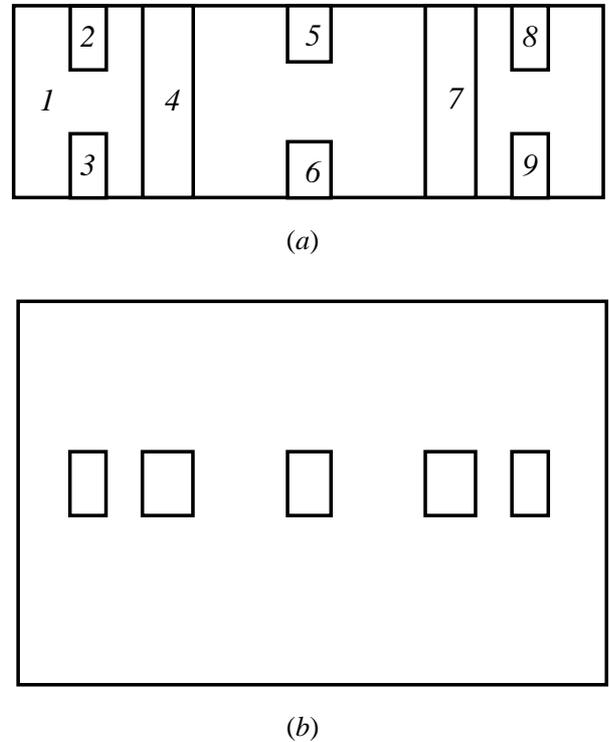


Fig. 2. Schematic representation of front view without side wall (a) and top view without upper wall (b) of waveguide mechanically tunable resonator with virtual tuning element: (1) the section of uniform rectangular waveguide; (2, 3) the upper and lower posts of double ridged waveguide section with capacitive conductivity; (4) the finite thickness plate with equivalent inductive conductivity; (5, 6) the upper and lower posts of double ridged waveguide section with capacitive conductivity equivalent to the round rod inserted into resonator cavity through the side wall; (7, 8, 9) the designations corresponding to the configuration denoted by marks (6), (7) and (8) of Fig. 1.

Using integral equation method, we can utilize the following system for calculation of generalized scattering matrices of symmetrical resonator components [6]

$$\begin{aligned} \sum_v \sum_k Y_{vk}^{(1)} \Psi_{vk}^{(1)} P_{vk}^{(11)} + \sum_v \sum_k U [VP_{vk}^{(12)} - P_{vk}^{(22)}] &= \\ &= 2\delta_{w1} Y_{pm}^{(1)} \Psi_{pm}^{(1)} ; \\ \sum_v \sum_k U [VP_{vk}^{(12)} - P_{vk}^{(22)}] + \sum_v \sum_k Y_{vk}^{(3)} \Psi_{vk}^{(2)} P_{vk}^{(23)} &= \\ &= 2\delta_{w2} Y_{qn}^{(3)} \Psi_{qn}^{(3)} ; \end{aligned} \tag{1}$$

$$P_{vk}^{(il)} = \int_{s_i} \mathbf{E}^{(i)} \Psi_{vk}^{(l)} ds; U = Y_{vk}^{(2)} \Psi_{vk}^{(2)} / \sinh \gamma_{vk} t;$$

$$V = \cosh \gamma_{vk} t,$$

where  $m=1,2,\dots,M_p$ ;  $n=1,2,\dots,N_q$ ;  $k=1,2,\dots,K_v^{(l)}$ ;  $l=1,2,3$ ;  $\mathbf{E}^{(i)}$  is unknown tangential electric field in  $i$ th ( $i=1,2$ ) coupling window;  $\Psi_{vk}^{(l)}$  is orthonormalized vector eigenfunction of  $k$ th mode in  $l$ th partial region of transverse-electric ( $v=1$ ) or transverse-magnetic ( $v=2$ ) types;  $Y_{vk}^{(l)}$  is corresponding to its admittance;  $\gamma_{vk}$  is propagation coefficient of  $k$ th mode of transverse-electric ( $v=1$ ) or transverse-magnetic ( $v=2$ ) types in coupling waveguide;  $\delta_{w1}$ ,  $\delta_{w2}$  are Kronecker symbols;  $w=1$ , if the diffraction problem is considered for the case of incidence  $M_p$  transverse-electric ( $p=1$ ) or transverse-magnetic ( $p=2$ ) electromagnetic waves from the left side;  $w=2$ , if  $N_q$  electromagnetic waves of transverse-electric ( $q=1$ ) or transverse-magnetic ( $q=2$ ) types are incident from the right side;  $K_v^{(l)}$  is the number of modes of transverse-electric ( $v=1$ ) or transverse-magnetic ( $v=2$ ) types which are taken into account in  $l$ th waveguide;  $s_i$  is the area of  $i$ th coupling window;  $t$  is the symmetric resonator component length.

In the considered case of the symmetrical structure, the tangential electric and magnetic fields on both sides of doubled discontinuity are identical. As a result, the system of integral equations (1) takes the form

$$\sum_v \sum_k Y_{vk}^{(1)} \Psi_{vk}^{(1)} P_{vk}^{(11)} + \sum_v \sum_k U [V P_{vk}^{(12)} - P_{vk}^{(22)}] = 2Y_{pm}^{(1)} \Psi_{pm}^{(1)}; \quad (2)$$

$$\sum_v \sum_k U [V P_{vk}^{(12)} - P_{vk}^{(22)}] + \sum_v \sum_k Y_{vk}^{(3)} \Psi_{vk}^{(2)} P_{vk}^{(23)} = 0.$$

Taking into account the symmetry properties, the expressions (2) can be reduced to two independent systems of integral equations relative to sums and differences of tangential electric fields in coupling windows:

$$\sum_v \sum_k Y_{vk}^{(1)} \Psi_{vk}^{(1)} P_{vk}^{(11)} + \sum_v \sum_k Q_\xi Y_{vk}^{(2)} \Psi_{vk}^{(2)} P_{vk}^{(12)} = 2Y_{pm}^{(1)} \Psi_{pm}^{(1)};$$

$$P_{vk}^{(11)} = \int_{s_1} \mathbf{F}_1 \Psi_{vk}^{(1)} ds; P_{vk}^{(12)} = \int_{s_1} \mathbf{F}_2 \Psi_{vk}^{(2)} ds; \quad (3)$$

$$\mathbf{F}_1 = \mathbf{E}_1 + \mathbf{E}_2; \mathbf{F}_2 = \mathbf{E}_1 - \mathbf{E}_2;$$

$$Q_1 = \tanh(\gamma_{vk} t / 2); Q_2 = \coth(\gamma_{vk} t / 2);$$

$$m=1,2,\dots,M_p; k=1,2,\dots,K_v; p=1,2; v=1,2.$$

Consider a segment of double ridge waveguide which one quarter part of its cross section is shown in Fig. 3. The considered doubled discontinuity represents

two symmetrical posts in E-plane of rectangular waveguide placed symmetrically on top and bottom walls.

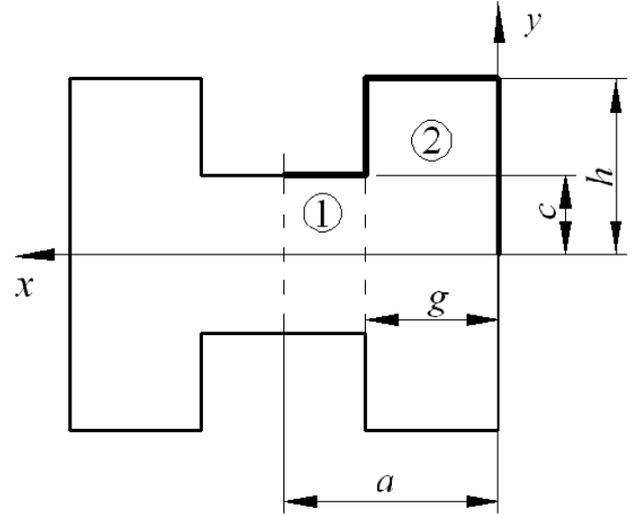


Fig. 3. Double ridged waveguide cross-section and coordinate system for the entire resonator configuration.

To solve (3) for the considered case at  $\xi=1,2$ , we apply the Galerkin's method as it has been done in [6]. Represent the summary ( $\xi=1$ ) and differential ( $\xi=2$ ) tangential electric fields for every mode incident on the double ridged waveguide discontinuity by the expansion into series of orthonormalized vector eigenfunctions of the coupling waveguide. Then, these approximating fields for symmetrical double ridged waveguide discontinuity of finite length in rectangular waveguide shown in Fig. 2 can be written as

$$\mathbf{F}_\xi = \sum_\mu \sum_h C_{\mu h}^{(i)} \Phi_{\mu h}^{(2)}, \quad (4)$$

where  $h=1,2,\dots,H_\mu^{(1)}$ ;  $\Phi_{\mu h}^{(2)}$  are orthonormalized vector coordinate functions of transverse-electric ( $\mu=1$ ) or transverse-magnetic ( $\mu=2$ ) types in coupling double ridged waveguide section;  $C_{\mu h}^{(i)}$  are unknown expansion coefficients;  $H_\mu^{(i)}$  is the number of approximating functions of transverse-electric ( $\mu=1$ ) or transverse-magnetic ( $\mu=2$ ) types in coupling section.

Substituting (4) into (3), taking into account the orthogonality of eigenfunctions in connected waveguides and performing transformation in accordance with Galerkin's method, we obtain two systems of linear algebraic equations corresponding to even ( $\xi=1$ ) and odd ( $\xi=2$ ) interpretations of doubled discontinuity excitation. In this way, the desired two systems of linear algebraic equations for determination of generalized scattering matrix of the symmetrical double ridged waveguide discontinuity of finite length in rectangular waveguide can be presented in the form:

$$\sum_{\mu} \sum_h C_{\mu h}^{(1)} \left[ \sum_v \sum_k Y_{vk}^{(1)} \eta_{vk}^{(11uv)} \eta_{hk}^{(11\mu v)} + Y_{vh}^{(2)} Q_{\xi} \delta_{u\mu} \delta_{vh} \right] = 2Y_{pm}^{(1)} \eta_{vm}^{(11up)}, \quad (5)$$

$$u = 1, v = 1, 2, \dots, H_1^{(1)}; u = 2, v = 1, 2, \dots, H_2^{(1)};$$

$$p = 1, m = 1, 2, \dots, M_1; p = 2, m = 1, 2, \dots, M_2;$$

$$\eta_{hk}^{(21\mu v)} = \int_{s_1} \Phi_{\mu h}^{(2)} \Psi_{vk}^{(1)} ds. \quad (6)$$

The expression (6) represents the coupling coefficients of double ridged waveguide window coordinate functions and vector eigenfunctions of rectangular waveguide. It defines the accordance between mathematical formulation of internal boundary problem and real double ridged waveguide to rectangular waveguide structure.

The key moment in solution of considered boundary problem is the definition of vector eigenfunctions of joined waveguides. For the rectangular waveguide, they are known. According to coordinate system, shown in Fig. 3, the scalar eigenfunctions for transverse-electric and transverse-magnetic modes in the rectangular waveguide representing the upper part of the overall cross section of considered structure due its symmetry with respect to horizontal plane can be written as

$$\varphi_{uv} = A_{uv} \cos \alpha_u x \cos \beta_v y; \quad (7)$$

$$\phi_{uv} = B_{uv} \sin \alpha_u x \sin \beta_v y, \quad (8)$$

where  $\alpha_u = u\pi / (2a)$ ;  $\beta_v = v\pi / h$ ;  $2a$  denotes the width of partial rectangular waveguide;  $A_{uv}$ ,  $B_{uv}$  are expansion coefficients determined by normalization condition.

The transverse components of electric fields in rectangular waveguide can be defined in accordance with equalities

$$-\mathbf{z} \times \mathbf{u} \text{grad} \varphi = -\mathbf{y} \partial \varphi / \partial x + \mathbf{x} \partial \varphi / \partial y; \quad (9)$$

$$\mathbf{u} \text{grad} \phi = \mathbf{x} \partial \phi / \partial x + \mathbf{y} \partial \phi / \partial y, \quad (10)$$

where  $\varphi$ ,  $\phi$  are the scalar eigenfunctions for transverse-electric and transverse-magnetic modes, respectively;  $\mathbf{u}$  is an unity vector;  $\mathbf{x}$ ,  $\mathbf{y}$ ,  $\mathbf{z}$  are the unity vectors directed along  $x$ ,  $y$  and  $z$  axes, respectively.

Using expressions (7)–(10), the transverse components of orthonormalized vector eigenfunctions of rectangular waveguide can be written as

$$\begin{aligned} \Psi_{1k}^{(1)} &= -\mathbf{x} A_{uv} \beta_v \cos \alpha_u x \sin \beta_v y + \\ &+ \mathbf{y} A_{uv} \alpha_u \sin \alpha_u x \cos \beta_v y; \end{aligned} \quad (11)$$

$$\begin{aligned} \Psi_{2k}^{(1)} &= \mathbf{x} B_{uv} \alpha_u \cos \alpha_u x \sin \beta_v y + \\ &+ \mathbf{y} B_{uv} \beta_v \sin \alpha_u x \cos \beta_v y, \end{aligned} \quad (12)$$

where the mode sequence number  $k$  is defined with taking into account the indices  $u$  and  $v$ .

### Solution of internal boundary problem for uniform double ridged waveguide

To determine vector eigenfunctions of double ridged waveguide, it is necessary to apply numerical methods of internal boundary problem solution. As has been shown in [7, 8], the suitable approach for solving of this complicated problem is the partial region method with taking into account the edge field singularity. To simplify the solution of this problem, it should be taken into account the symmetry of double ridged waveguide cross section relative to vertical and horizontal planes. In this case, it is convenient to consider one quarter part of cross section as shown in Fig. 3. Based on excitation conditions of double ridged waveguide section, it is advisable to suppose that there are a magnetic wall in vertical symmetry plane of the waveguide and an electric wall in horizontal symmetry plane. Whereas the tasks of the cutoff mode numbers and electromagnetic fields definition are solved in [7, 8], the solution sequence will be described below in abbreviated form according to designations shown in Fig 3.

According to [7] the solution of Helmholtz's equation for transverse-electric modes in partial regions 1 and 2 can be written as follows

$$\Psi_1 = \sum_m A_m \sin \alpha_m (a - x) \cos p_m y; \quad (13)$$

$$\Psi_2 = \sum_m B_m \cos \beta_m x \cos q_m y, \quad (14)$$

where  $\alpha_m = (\kappa_{\mu}^2 - p_m^2)^{\Phi}$ ;  $\beta_m = (\kappa_{\mu}^2 - q_m^2)^{\Phi}$ ;  $p_m = m\pi / c$ ;  $q_m = m\pi / h$ ;  $\kappa_{\mu}$  is the cutoff wavenumber for  $\mu$  th mode;  $\Psi_1$ ,  $\Psi_2$  are components of scalar eigenfunction of double ridged waveguide;  $A_m$  and  $B_m$  are the unknown coefficients;  $m = 0, 1, 2, \dots, M_1 - 1$  for region 1;  $M_1$  is the number of terms in series (13);  $m = 0, 1, 2, \dots, M_2 - 1$  for region 2;  $M_2$  is number of terms in series (14);  $\Phi = 1/2$ .

The unknown coefficients in (13) and (14) can be found through the electric field value  $E_y$  on the common boundary of partial regions:

$$A_m = 2\varepsilon_m / (c\alpha_m \cos \alpha_m b) \int_0^c E_y \cos p_m y dy; \quad (15)$$

$$B_m = 2\varepsilon_m / (h\beta_m \sin \beta_m g) \int_0^c E_y \cos q_m y dy, \quad (16)$$

where  $b = a - g$ ;  $\varepsilon_m = 1/2$  if  $m = 0$  otherwise  $\varepsilon_m = 1$ .

To define the unknown tangential electric field in coupling window, the following homogeneous system of linear algebraic equations has been obtained [8]:

$$\sum_i X_i [\sum_m 2U_i U_j \varepsilon_m / \alpha_m \tan \alpha_m b - \sum_m 2V_i V_j c / h \varepsilon_m / \beta_m \cot \beta_m g] = 0; \quad (17)$$

$$U_i = (-1)^i \pi \Gamma(2i + 2\lambda) J_{2i+\lambda}(m\pi) / [(2i)! \Gamma(\lambda) (2m\pi)^\lambda];$$

$$V_i = (-1)^i \pi \Gamma(2i + 2\lambda) J_{2i+\lambda}(q_m c) / [(2i)! \Gamma(\lambda) (2q_m c)^\lambda].$$

Here  $\lambda = 1/6$ ;  $\Gamma(2i + 2\lambda)$ ,  $\Gamma(\lambda)$  are the gamma functions;  $J_{2i+\lambda}(m\pi)$ ,  $J_{2i+\lambda}(q_m c)$  are the Bessel's functions of the first kind;  $X_i$  are the unknown expansion coefficients of the electric field in coupling window;  $i = 0, 1, \dots, I-1$ ;  $j = 0, 1, \dots, I-1$ ;  $I$  is the number of expansion terms;  $U_j$ ,  $V_j$  quantities are obtained from  $U_i$ ,  $V_i$  by replacement of indexes so that we obtain

$$U_j = (-1)^j \pi \Gamma(2j + 2\lambda) J_{2j+\lambda}(m\pi) / [(2j)! \Gamma(\lambda) (2m\pi)^\lambda];$$

$$V_j = (-1)^j \pi \Gamma(2j + 2\lambda) J_{2j+\lambda}(q_m c) / [(2j)! \Gamma(\lambda) (2q_m c)^\lambda].$$

A condition of nontrivial solution of (17) is equality to zero of its determinant. This equality represents the characteristic equation for computing the cutoff mode numbers of transverse-electric modes of double-ridged waveguide. To find the determinant zeros, we utilize the dichotomy method using procedure, which allows avoiding the appearance in solution of spurious roots. This procedure is based on excluding from the consideration the values of characteristic equation approaching the infinity. These values are defined by the following equations:

$$\cos \alpha_m b = 0; \quad \sin \beta_m g = 0. \quad (18)$$

The computed results indicate that the zeros and breaks of the determinant of linear algebraic system (17) can be situated very close. Hence at the computer realization of the dichotomy method, the bypass of breaks points through using expressions (18) yet not ensure an absence of spurious roots of dispersive equation. It should be noted that the appearance in solution of spurious roots would inevitably lead to knowingly erroneous definition of scattering matrix of double ridged to rectangular waveguides junction when solving the general problem. Therefore, the searching ways of errorless solution of internal boundary eigenvalue problem is a key and substantial task in the theory of double ridged waveguide junctions.

To obtain the correct solution for double ridged section in the form of two posts symmetrically placed in

rectangular waveguide we employ the computer algorithm based on a functional for cutoff numbers which in considered case takes the form [8]:

$$\kappa_\mu^2 = \int_s (\text{grad} \Psi_\mu)^2 ds / \int_s \Psi_\mu^2 ds, \quad (19)$$

where  $\Psi_\mu$  is the  $\mu$  th mode scalar eigenfunction of the double ridged waveguide;  $s$  denotes the square of its cross section.

The equality (19) is fulfilled if and only if  $\kappa$  will be the cutoff mode number, that is  $\kappa = \kappa_\mu$ . To apply the expression (19) for calculation of the cutoff mode numbers of double ridged waveguide, the transversal components of the overall fields are defined in accordance with differential operator:

$$-\mathbf{z} \times \mathbf{u} \text{grad} \Psi = -\mathbf{y} \partial \Psi / \partial x + \mathbf{x} \partial \Psi / \partial y, \quad (20)$$

where  $\mathbf{u}$  is a unity vector;  $\mathbf{x}$ ,  $\mathbf{y}$ ,  $\mathbf{z}$  are the unity vectors directed along  $x$ ,  $y$  and  $z$  axes, respectively.

To perform the computations in accordance with (20), it is necessary to solve the homogeneous system of linear algebraic equations. As a result, the unknown coefficients  $X_i$  and the distributions of tangential components of electrical fields in coupling window for various transverse-electric modes are obtained. At the computation of the coefficients under unknowns, the estimation algorithm of Bessel's functions of fractional indexes based on recursive procedure operating in direction from high indexes to small ones are applied. This algorithm allows achieving a high accuracy of the Bessel's function computation which is defined entirely by capability of computer used.

As the unknown coefficients  $X_i$  and the distributions of tangential components of electrical fields in coupling window on the common boundary of partial regions for various transverse-electric modes have been obtained the amplitude coefficients in (15) and (16) acquire the following form:

$$A_m = 2\varepsilon_m \delta_m / (\alpha_m \cos \alpha_m b); \quad (21)$$

$$B_m = 2\varepsilon_m c \sigma_m / (h \beta_m \sin \beta_m g); \quad (22)$$

$$\delta_m = \sum_i X_i U_i; \quad \sigma_m = \sum_i X_i V_i.$$

When the coefficients  $A_m$  and  $B_m$  in (21), (22) are evaluated, the denominator in (19) is defined by the expression

$$\begin{aligned} \int_s \Psi_\mu^2 ds &= \sum_m A_m^2 \int_s (\sin \alpha_m b)^2 ds + \sum_m B_m^2 \int_s (\cos \beta_m x)^2 ds = \\ &= \sum_m 2\varepsilon_m b c \delta_m^2 / \alpha_m^2 \times \end{aligned}$$

$$\begin{aligned} & \times [1 / (\cos \alpha_m b)^2 - (\tan \alpha_m b) / (\alpha_m b)] + \\ & + \sum_m 2\epsilon_m g c^2 \sigma_m^2 / (h\beta_m^2) \times \\ & \times [1 / (\sin \beta_m g)^2 + (\cot \beta_m g) / (\beta_m g)]. \quad (23) \end{aligned}$$

To determine the numerator in (19), the components of (20) in partial regions of double ridged waveguide section are defined as

$$\begin{aligned} -\mathbf{z} \times \mathbf{u} \text{grad} \psi_1 &= \mathbf{y} \sum_m A_m \alpha_m \cos \alpha_m (a-x) \cos p_m y - \\ & - \mathbf{x} \sum_m A_m p_m \sin \alpha_m (a-x) \sin p_m y; \quad (24) \end{aligned}$$

$$\begin{aligned} -\mathbf{z} \times \mathbf{u} \text{grad} \psi_2 &= \mathbf{y} \sum_m B_m \beta_m \sin \beta_m x \cos q_m y - \\ & - \mathbf{x} \sum_m B_m q_m \cos \beta_m x \sin q_m y. \quad (25) \end{aligned}$$

Using relations (24) and (25), the numerator in (19) is calculated as follows:

$$\begin{aligned} \int_s (\text{grad} \psi_\mu)^2 ds &= \sum_m 2\epsilon_m b c \delta_m^2 / \alpha_m^2 \times \\ & \times [(\alpha_m^2 + p_m^2) / (\cos \alpha_m b)^2 + (\alpha_m^2 - \\ & - p_m^2) (\tan \alpha_m b) / (\alpha_m b)] + \\ & + \sum_m 2\epsilon_m g c^2 \sigma_m^2 / (h\beta_m^2) [(q_m^2 + \beta_m^2) / (\sin \beta_m g)^2 + \\ & + (q_m^2 - \beta_m^2) (\cot \beta_m g) / (\beta_m g)]. \quad (26) \end{aligned}$$

If  $\kappa_\mu^2 - p_m^2 < 0$  or  $\kappa_\mu^2 - q_m^2 < 0$ , the corresponding trigonometric functions in expressions (23) and (26) are replaced by hyperbolic ones. Despite the presence of imaginary components in (23) and (26), their overall values are always real.

Consider briefly the computational algorithm for finding cutoff mode numbers of transverse-magnetic modes based on technique [7, 8]. According to [7] scalar eigenfunctions of partial regions can be written as follows

$$\xi_1 = \sum_n A_n \cos \alpha_n (a-x) \sin p_n y; \quad (27)$$

$$\xi_2 = \sum_n B_n \sin \beta_n x \sin q_n y. \quad (28)$$

where  $\alpha_n = (\zeta_v^2 - p_n^2)^\vartheta$ ;  $\beta_n = (\zeta_v^2 - q_n^2)^\vartheta$ ;  $p_n = n\pi / c$ ;  $q_n = n\pi / h$ ;  $\zeta_v$  is the cutoff mode number for  $v$  th mode;  $A_n$  and  $B_n$  are the unknown coefficients;  $n=1, 2, \dots, N_1$  for region 1;  $N_1$  is the number of terms in series (27);  $n=1, 2, \dots, N_2$  for region 2;  $N_2$  is number of terms in series (28).

Similar to the previous case, the homogeneous system of linear algebraic equations relatively to unknown

expansion coefficients of tangential electric field on the boundary of partial regions of double ridged waveguide is defined as

$$\begin{aligned} & \sum_i X_i [\sum_n 2U_i U_j \alpha_n \tan \alpha_n b - \\ & - \sum_n 2V_i V_j c / h\beta_n \cot \beta_n g] = 0; \quad (29) \end{aligned}$$

$$U_i = (-1)^i \pi [\Gamma(\tau) J_\zeta(n\pi)] / [(2i+1)! \Gamma(\vartheta) (2n\pi)^\vartheta];$$

$$V_i = (-1)^i \pi [\Gamma(\tau) J_\zeta(q_n c)] / [(2i+1)! \Gamma(\vartheta) (2q_n c)^\vartheta].$$

Here  $\tau = 2i + 2\vartheta + 1$ ;  $\zeta = 2i + \vartheta + 1$ ;  $\vartheta = 7/6$ ;  $\Gamma(\tau)$ ,  $\Gamma(\vartheta)$  denote the gamma functions;  $J_\zeta(n\pi)$ ,  $J_\zeta(q_n c)$  are the Bessel's functions of the first kind;  $X_i$  are the expansion coefficients of the electric field in coupling window;  $i=0, 1, \dots, I-1$ ;  $j=0, 1, \dots, I-1$ ;  $I$  is the number of expansion terms;  $U_j$ ,  $V_j$  quantities are obtained from  $U_i$ ,  $V_i$  by replacement of indexes so that

$$U_j = (-1)^j \pi [\Gamma(\tau) J_\zeta(n\pi)] / [(2j+1)! \Gamma(\vartheta) (2n\pi)^\vartheta];$$

$$V_j = (-1)^j \pi [\Gamma(\tau) J_\zeta(q_n c)] / [(2j+1)! \Gamma(\vartheta) (2q_n c)^\vartheta].$$

The resolution algorithm of system (29) is the same as (17). The points on the mode number axis in which there are breaks of determinant of (29) and which should be excluded from the calculation are define by solving of the following equations:

$$\cos \alpha_n b = 0; \quad \sin \beta_n g = 0.$$

As the unknown coefficients  $X_i$  have been found from solution (29) and the distribution of tangential components of electrical fields in coupling window on the common boundary of partial regions for various transverse-magnetic modes have been obtained, the amplitude coefficients in (27) and (28) get the form:

$$A_n = 2\delta_n / \cos \alpha_n b; \quad (30)$$

$$B_n = 2c\sigma_n / (h \sin \beta_n g); \quad (31)$$

$$\delta_n = \sum_i X_i U_i; \quad \sigma_n = \sum_i X_i V_i,$$

where the values  $U_i$  and  $V_i$  are determined by expressions (29).

Consider briefly a sequence of calculation associated with the use of the functional for cutoff mode numbers which in the case of transverse-magnetic modes takes the form [8]

$$\zeta_v^2 = \int_s (\text{grad} \xi_v)^2 ds / \int_s \xi_v^2 ds, \quad (32)$$

where  $\xi_v$  represents the  $v$  th mode scalar eigenfunction of the double ridged waveguide;  $s$  is the square of waveguide cross section.

As in the previous case, for application of the expression (32) to calculation algorithm of the cutoff mode numbers of double ridged waveguide, we must define the transverse components of the overall fields in accordance with differential operator:

$$\mathbf{u}\text{grad}\xi = \mathbf{x}\partial\xi / \partial x + \mathbf{y}\partial\xi / \partial y, \quad (33)$$

where  $\mathbf{u}$ ,  $\mathbf{x}$ ,  $\mathbf{y}$  are the unity vectors the same as in expression (20).

According to (33), we obtain the following expression for transverse components of the electric fields:

$$\begin{aligned} \mathbf{u}\text{grad}\xi_1 = & \mathbf{x}\sum_n A_n \alpha_n \sin \alpha_n (a-x) \sin p_n y + \\ & + \mathbf{y}\sum_n A_n p_n \cos \alpha_n (a-x) \cos p_n y; \end{aligned} \quad (34)$$

$$\begin{aligned} \mathbf{u}\text{grad}\xi_2 = & \mathbf{x}\sum_n B_n \beta_n \cos \beta_n x \sin q_n y + \\ & + \mathbf{y}\sum_n B_n q_n \sin \beta_n x \cos q_n y. \end{aligned} \quad (35)$$

Using (34), (35) and evaluating the integrals similarly to previous consideration, we obtain

$$\begin{aligned} \int_s \xi_v^2 ds = & \sum_n 2bc\delta_n^2 \times \\ & \times [1 / (\cos \alpha_n b)^2 + (\tan \alpha_n b) / (\alpha_n b)] + \\ & + \sum_n 2gc^2\sigma_n^2 / h \times \\ & \times [1 / (\sin \beta_n g)^2 - (\cot \beta_n g) / (\beta_n g)]. \end{aligned} \quad (36)$$

Combining (34), (35) and evaluating the integrals, we get

$$\begin{aligned} \int_s (\text{grad}\xi_v)^2 ds = & \sum_n 2bc\delta_n^2 \times \\ & \times [(p_n^2 + \alpha_n^2) / (\cos \alpha_n b)^2 + (p_n^2 - \alpha_n^2)(\tan \alpha_n b) / (\alpha_n b)] + \\ & + \sum_n 2gc^2\sigma_n^2 / h [(\beta_n^2 + q_n^2) / (\sin \beta_n g)^2 + \\ & + (\beta_n^2 - q_n^2)(\cot \beta_n g) / (\beta_n g)]. \end{aligned} \quad (37)$$

As well as in (23), (26) if  $\zeta_v^2 - p_n^2 < 0$  or  $\zeta_v^2 - q_n^2 < 0$ , the corresponding trigonometric functions in expressions (36) and (37) should be replaced by hyperbolic ones. Despite the presence of imaginary components in (36) and (37), their overall values are always real.

According to received relations, the computation of cutoff frequencies and electromagnetic fields in double ridged waveguide section were carried out for funda-

mental mode and first higher transverse-electric and transverse-magnetic modes.

Based on computed results, the evaluation of broadband properties as well as electric and magnetic fields images of the double ridged waveguide section as a component of tunable filter were investigated in wide range of its cross-section dimensions. The results obtained were applied for designing the transition structure from double ridged section to rectangular waveguide.

### Definition of coupling coefficients

To solve the scattering electromagnetic modes problem by the section of double ridged waveguide, we must previously define the coefficients (6) coupling the parameters of mathematical and physical models of double ridged waveguide section. Using the designations of eigenfunctions for transverse-electric and transverse-magnetic modes in the double ridged and rectangular waveguides, we can write the extended expressions of (6) as follows:

$$\eta_{hk}^{(2111)} = \int_{s_1} \Phi_{1h}^{(2)} \Psi_{1k}^{(1)} ds; \quad (38)$$

$$\eta_{hk}^{(2112)} = \int_{s_1} \Phi_{1h}^{(2)} \Psi_{2k}^{(1)} ds; \quad (39)$$

$$\eta_{hk}^{(2121)} = \int_{s_1} \Phi_{2h}^{(2)} \Psi_{1k}^{(1)} ds; \quad (40)$$

$$\eta_{hk}^{(2122)} = \int_{s_1} \Phi_{2h}^{(2)} \Psi_{2k}^{(1)} ds. \quad (41)$$

Consider in details the evaluation of integral (38) comprising the product of transverse-electric eigenfunctions of the double ridged and rectangular waveguides. Represent the expression (38) in the extended form

$$\begin{aligned} \int_{s_1} \Phi_{1h}^{(2)} \Psi_{1k}^{(1)} ds = & \int_{s_{11}} [\mathbf{x}\Phi_{1hx}^{(21)} + \mathbf{y}\Phi_{1hy}^{(21)}] \cdot [\mathbf{x}\Psi_{1kx}^{(1)} + \mathbf{y}\Psi_{1ky}^{(1)}] ds_1 + \\ & + \int_{s_{12}} [\mathbf{x}\Phi_{1hx}^{(22)} + \mathbf{y}\Phi_{1hy}^{(22)}] \cdot [\mathbf{x}\Psi_{1kx}^{(1)} + \mathbf{y}\Psi_{1ky}^{(1)}] ds_2, \end{aligned} \quad (42)$$

where  $\Phi_{1hx}^{(21)}$ ,  $\Phi_{1hy}^{(21)}$  denote  $x$  and  $y$  components of function  $\Phi$  in first partial region of double ridge waveguide;  $\Phi_{1hx}^{(22)}$ ,  $\Phi_{1hy}^{(22)}$  are  $x$  and  $y$  components of function  $\Phi$  in second partial region of double ridge waveguide;  $s_1$ ,  $s_2$  define the cross section squares of first and second partial region of double ridge waveguide.

Taking into account the properties of scalar multiplication of two vectors, we get

$$\int_{s_1} \Phi_{1h}^{(2)} \Psi_{1k}^{(1)} ds = \int_{y_{11} x_{11}} [\Phi_{1hx}^{(21)} \Psi_{1kx}^{(1)} + \Phi_{1hy}^{(21)} \Psi_{1ky}^{(1)}] dx dy +$$

$$+ \int_{y_{12}} \int_{x_{12}} [\Phi_{1hx}^{(22)} \Psi_{1kx}^{(1)} + \Phi_{1hy}^{(22)} \Psi_{1ky}^{(1)}] dx dy. \quad (43)$$

Substituting the transversal components of vector eigenfunctions (34, 35) and (11, 12) into (43) as well as establishing the limits of integration in accordance with Fig. 3 and adding the normalization factors, we obtain

$$\begin{aligned} \int_{s_1} \Phi_{1h}^{(2)} \Psi_{1k}^{(1)} ds = N_h^{(2)} N_k^{(1)} [\beta_v \sum_m A_m p_m \times \\ \times \int_g^a \sin \alpha_m (a-x) \cos \alpha_u x dx \int_0^c \sin p_m y \sin \beta_v y dy + \\ + \alpha_u \sum_m A_m \alpha_m \int_g^a \cos \alpha_m (a-x) \sin \alpha_u x dx \times \\ \times \int_0^c \cos p_m y \cos \beta_v y dy + \\ + \beta_v \sum_m B_m q_m \int_0^g \cos \beta_m x \cos \alpha_u x dx \times \\ \times \int_0^h \sin q_m y \sin \beta_v y dy + \\ + \alpha_u \sum_m B_m \beta_m \int_0^g \sin \beta_m x \sin \alpha_u x dx \times \\ \times \int_0^h \cos q_m y \cos \beta_v y dy], \end{aligned} \quad (44)$$

where  $N_h^{(2)}$ ,  $N_k^{(1)}$  are the normalization factors.

To calculate (44), therein integrals along coordinates  $x$  and  $y$  should be derived. Consider successively the evaluation of single integrals in [44] based on tables of integrals from trigonometric functions [10]. Using [10] and performing corresponding transformation, we get the following expressions for these integrals:

$$\begin{aligned} \int_g^a \sin \alpha_m (a-x) \cos \alpha_u x dx &= [\alpha_u \sin \alpha_m b \sin \alpha_u g - \\ &- \alpha_m \cos \alpha_m b \cos \alpha_u g] / (\alpha_m^2 - \alpha_u^2); \\ \int_g^a \cos \alpha_m (a-x) \sin \alpha_u x dx &= [\alpha_u \cos \alpha_m b \cos \alpha_u g - \\ &- \alpha_m \sin \alpha_m b \sin \alpha_u g] / (\alpha_u^2 - \alpha_m^2); \\ \int_0^g \cos \beta_m x \cos \alpha_u x dx &= [\beta_m \sin \beta_m g \cos \alpha_u g - \\ &- \alpha_u \cos \beta_m g \sin \alpha_u g] / (\beta_m^2 - \alpha_u^2); \\ \int_0^g \sin \beta_m x \sin \alpha_u x dx &= [\alpha_u \sin \beta_m g \cos \alpha_u g - \end{aligned}$$

$$- \beta_m \cos \beta_m g \sin \alpha_u g] / (\beta_m^2 - \alpha_u^2);$$

$$\int_0^c \sin p_m y \sin \beta_v y dy = c_1;$$

$$c_1 = -(-1)^m \sin \beta_v c [p_m / (p_m^2 - \beta_v^2)], \quad p_m \neq \beta_v;$$

$$c_1 = c/2, \quad p_m = \beta_v \neq 0;$$

$$c_1 = 0, \quad p_m = \beta_v = 0;$$

$$\int_0^c \cos p_m y \cos \beta_v y dy = c_2;$$

$$c_2 = -(-1)^m \sin \beta_v c [\beta_v / (p_m^2 - \beta_v^2)], \quad p_m \neq \beta_v;$$

$$c_2 = c/2, \quad p_m = \beta_v \neq 0;$$

$$c_2 = c, \quad p_m = \beta_v = 0;$$

$$\int_0^h \sin q_m y \sin \beta_v y dy = h_1, \quad q_m = \beta_v;$$

$$h_1 = 0, \quad q_m \neq \beta_v$$

$$h_1 = h/2, \quad p_m = \beta_v \neq 0;$$

$$h_1 = 0, \quad p_m = \beta_v = 0;$$

$$\int_0^h \cos q_m y \cos \beta_v y dy = h_2, \quad q_m = \beta_v;$$

$$h_2 = 0, \quad q_m \neq \beta_v$$

$$h_2 = h/2, \quad p_m = \beta_v \neq 0;$$

$$h_2 = h, \quad p_m = \beta_v = 0;$$

Substituting the relations (43) and (44) together with values of single integrals into (38), we obtain the expression for computing the coupling coefficients between transverse-electric modes of double ridged and rectangular waveguides as follows:

$$\begin{aligned} \eta_{hk}^{(2111)} &= N_h^{(2)} N_k^{(1)} \{ \sum_m 2\epsilon_m T_{1m} / (\alpha_m^2 - \alpha_u^2) \times \\ &\times [p_m \beta_v (\cos \alpha_u g - \alpha_u / \alpha_m \tan \alpha_m b \sin \alpha_u g) c_1 + \\ &+ \alpha_u (\alpha_u \cos \alpha_u g - \alpha_m \tan \alpha_m b \sin \alpha_u g) c_2] - \\ &- 2\delta_{mv} c / h \epsilon_v T_{2m} / (q_m^2 - \beta_v^2) [q_m \beta_v (\cos \alpha_u g - \\ &- \alpha_u / \beta_m \cot \beta_m g \sin \alpha_u g) h_1 + \alpha_u (\alpha_u \cos \alpha_u g - \\ &- \beta_m \cot \beta_m g \sin \alpha_u g) h_2] \}, \end{aligned}$$

where  $T_{1m}$ ,  $T_{2m}$  are the frequency-independent coefficients derived by using relations (21) and (22);  $\delta_{mv}$  is

the Kronecker symbol. The remaining coupling coefficients (39)–(41) are found by similar way. At that, the coupling coefficient (39) equals zero as for other waveguide junctions.

### Numerical results

To investigate the considered waveguide resonators to be applied for designing the tunable bandpass filters, the FORTRAN program was developed. To verify the computational algorithm using the obtained relations, the resonance curve of ridged section in rectangular waveguide with dimensions presented in [11] was calculated. The obtained results are in good agreement with the experimental and theoretical data of [11].

The main characteristics of waveguide tunable resonators were calculated by using the developed program. Some results of these resonators investigation are depicted in Figures 4–7.

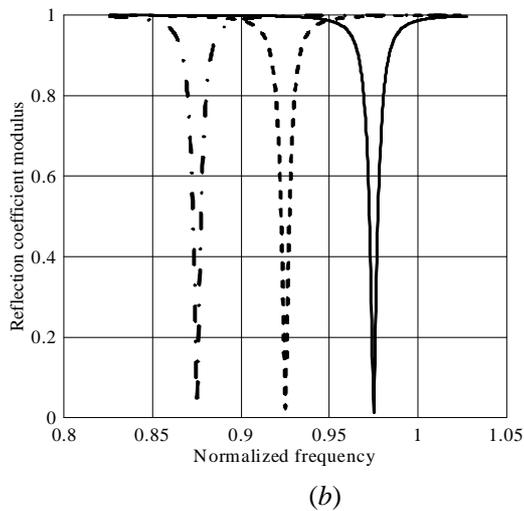
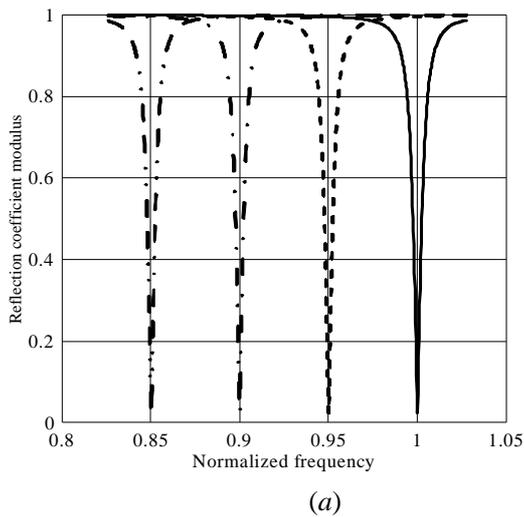


Fig. 4. Responses of the reflection coefficient modulus of the resonator with the correcting capacitive elements versus the normalized frequency under tuning the resonator on the variety of resonance frequencies.

Figures 4 and 5 illustrate the phenomenon of the resonance curve movement at the frequency tuning. These figures display the behavior of the reflection coefficient modulus responses as a function of normalized frequency  $f_{norm} = f / f_{high}$  under tuning the resonator on the variety of resonance frequencies, where  $f_{high}$  denotes the higher frequency of tuning range and  $f$  is the operating frequency. For comparison, the characteristics  $f_{norm} = f / f_{high}$  of the same tunable resonator but without the correcting capacitive posts are presented in Fig 5.

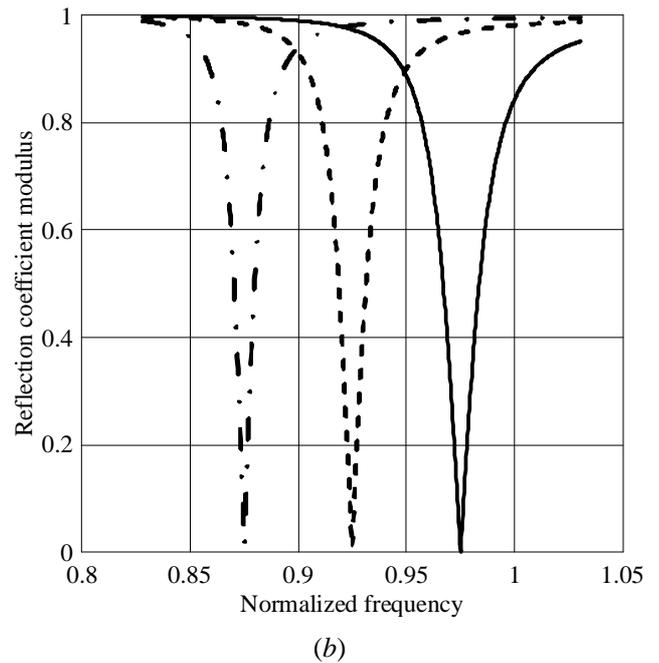
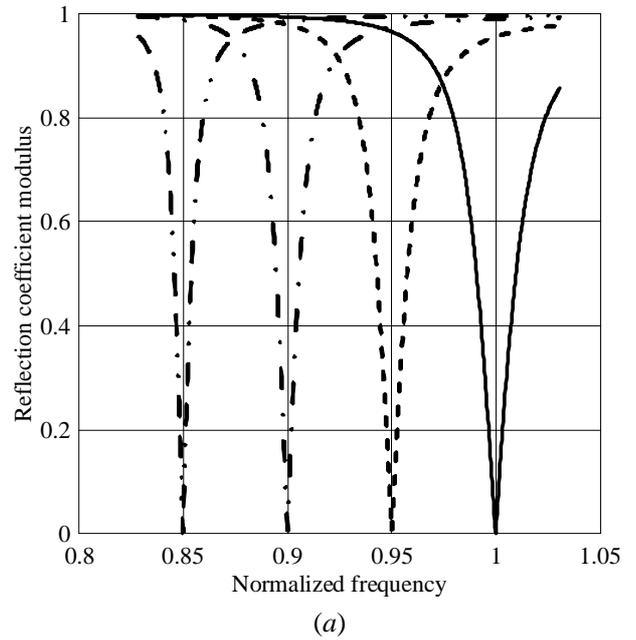


Fig. 5. Characteristics of the tunable resonator without the correcting capacitive elements on the variety of resonance frequencies.

Figures 4 and 5 display characteristics of the tunable resonators for the cases of their optimal dimensions with and without the elements correcting the phase shifts conditioned due the frequency tuning. Fig. 4a and Fig. 5a illustrate scenarios of the resonance curves movement for four normalized tuning frequencies:  $f_{norm} = 1$ ,  $f_{norm} = 0.95$ ,  $f_{norm} = 0.9$ ,  $f_{norm} = 0.85$  by solid lines, dashed lines, dash-dotted lines and dash-dot-dot lines, respectively. Fig. 4b and Fig. 5b show the similar characteristics for three normalized tuning frequencies:  $f_{norm} = 0.975$ ,  $f_{norm} = 0.925$ ,  $f_{norm} = 0.875$  denoted by solid, dashed and dash-dotted lines, respectively. As follows from Fig. 4, the form of the resonance curve does not altered in overall tuning range achieving the value 15 percent.

Fig. 5 displays the responses of the reflection coefficient modulus of the resonator without correcting elements as a function of normalized frequency under tuning the resonator on the variety of resonance frequencies. This case corresponds to the resonator on inductive discontinuities forming by two finite thickness strips disposed in vertical symmetry plane of the rectangular waveguide. It can be seen that the form of the resonance curve is noticeably changed in considered tuning range. The resonance curve width is considerable decreased at the variation of operating frequency into lower region of tuning range.

Fig. 6 shows the variation of resonator loaded quality factor ( $Q$ -factor) of optimal dimensioned resonator with correcting elements in dependence on the tuning frequency. Fig. 7 illustrates the analogous response of the resonator without correcting elements.

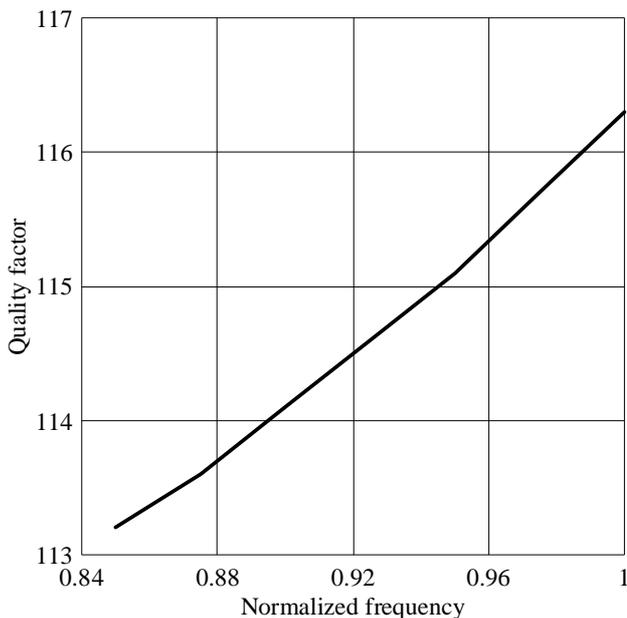


Fig. 6. Loaded quality factor of the resonator with correcting elements versus the normalized frequency.

As follows from Fig. 7, the value of quality factor of the resonator without correcting elements (on inductive strips) has significant dependence from the tuning frequency. These quality factor values on both ends of tuning range are differed almost twice. This circumstance prevents the establishment of the waveguide tuning filters with good performances.

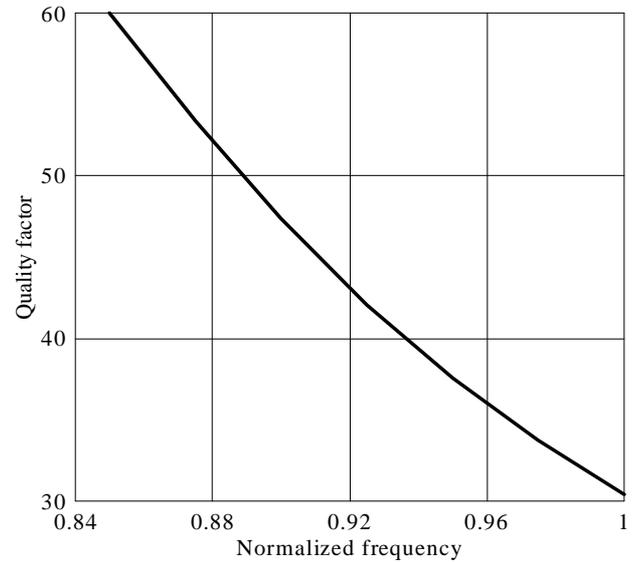


Fig. 7. Loaded quality factor of the resonator without correcting elements versus the normalized frequency.

The equipment of such resonators by two correcting sections disposed on its both ends as shown in Figures 1 and 2 allows obtaining the desired inclination of quality factor responses within the tuning range. To obtain nearly constant bandwidth of waveguide filter in tuning process, the quality factors of each resonator should be not changed in overall tuning range. The quality factor response of optimally designed resonator is depicted in Fig. 6. It should be noted that required responses of reflection coefficient and quality factor of resonator can be obtained at the specific ratios of its geometric parameters.

### Conclusion

Based on rigorous approach, the class of resonators which allow building mechanically tunable waveguide bandpass filters with near constant bandwidth in a wide tuning frequency range has been extensively investigated. By the successful combining inductive and capacitive discontinuities, the near constant quality factor of resonators in wide frequency variation range is achieved.

It is proved that the most technologically advanced solution can be obtain by using the waveguide connections containing double ridged posts for compensation of phase shifts in resonators appearing due to frequency variation. As a tunable element, a round rod inserting

into resonator cavity through the side wall of the rectangular waveguide was considered. Such technical solution decreases the inclination of calibration response of the resonator and increases its linearity. The presence of tunable element inevitably breaks the resonator symmetry with respect to vertical plane. Due to absence of resonator symmetry in vertical plane, the theoretical investigation of such combined structure becomes extremely complicated. To simplify the numerical analysis of entire structure asymmetric with respect to vertical plane, the idea proposed in [9] was implemented. This original idea was modified in order to replace asymmetric tunable element by its symmetric virtual analog.

A transition from asymmetric tunable resonator structure to its symmetric virtual analog crucially simplifies the problem solution, reduces the computer resources and increases the accuracy of computations. To create effective calculating algorithms of mechanically tunable waveguide resonator components, their mirror symmetry with respect to a plane going through the middle of the structure perpendicularly to its longitudinal axes is taken into account. Adequate mathematical models of the resonator components based on generalized scattering matrix approach are obtained. The diffraction problems for doubled discontinuities such as doubled ridged posts in rectangular waveguide have been solved by the integral equation method.

To determine vector eigenfunctions of double ridged waveguide, the numerical methods of internal boundary problem solution have been applied. The suitable approach for solving of this complicated problem was obtained by the partial region method with taking into account the edge field singularity. To simplify the solution of this problem, the symmetry of double ridged waveguide cross section relative to vertical and horizontal planes was taken into account.

To investigate the considered waveguide resonators to be applied for designing the tunable bandpass filters, the FORTRAN program was developed. The computational algorithm using the obtained relations was verified by the comparison of calculated results of the resonance curve of ridged section in rectangular waveguide with the experimental and theoretical data presented in [11]. The calculation of resonance frequencies and loaded quality factors of resonators are carried out in wide dimension range variations.

Despite the great advances in the implementation of high quality ultra long communication, the problem of finding the ways for further improvement of tropospheric station characteristics remains highly relevant. This circumstance necessitates the further investigations of waveguide resonators as the components of mechan-

ically tunable bandpass filters. To improve significantly the characteristics of mechanically tunable waveguide filters of transmit tropospheric stations operating at high power levels, the following problem should be necessarily solved. It primarily concerns the researches of the various constructions of waveguide tunable resonators with the purpose of achieving the high linearity of their calibration characteristics representing the dependence of resonant frequency from the immersion depth of tuning metallic rod into resonator cavity. Others problems are connected with the necessity of extension of tuning range and improvement of the stop band properties of tunable resonators. In this respect, the developed waveguide resonators open new possibilities in realization of high efficient and cost effective mechanically tunable filters for various transmit systems of tropospheric communication.

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