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SCATTERING MATRICES OF LONGITUDINALLY COMPLICATED WAVEGUIDE STRUCTURES

Petro Ya. Stepanenko

National Technical University of Ukraine “KPI”, Kyiv, Ukraine

An approach to the calculation of longitudinally complicated waveguide structures is developed based on integral equation method. A problem of electromagnetic wave diffraction on the complicated structure consisting of a great number of connected waveguide sections has been solved in rigorous formulation. The expressions for its generalized scattering matrices which order is limited only by capabilities of computation technology are obtained. A designing technique for calculation of waveguide connections symmetrical with respect to the transverse plane passing through the middle of the structure perpendicularly to its longitudinal axis has been proposed. This technique is based on the solution of two simultaneous systems of linear algebraic equations with complex coefficients. The mathematical model of the symmetrical waveguide structure in the form of two simultaneous systems of linear algebraic equations with real coefficients is derived for estimation of single mode scattering matrix. The effectiveness of the proposed approach is illustrated by example of definition of mathematical models for calculation of scattering matrices of diaphragm of finite thickness and resonant enlargement between two circular waveguides. Proposed and known techniques for calculation of scattering matrices of these structures regarding the cost of computer time are compared. For the correct providing of this comparative analysis, the expressions for obtaining generalized scattering matrix of two circular waveguides junction are derived as a special case of scattering matrix of doubled discontinuity. It is shown that the proposed design technique based on the solution of simultaneous systems of linear algebraic equations which number is equaled to a number of discontinuities in waveguide structure ensures significant cost savings of computer time in comparison with the known approach based on the consecutive combining of generalized scattering matrices of separate junctions.

Introduction

In telecommunication wireless cellular networks, the antenna systems are widely used for transmitting and receiving of information in high frequency band. [1]. Designing of necessary characteristics of transmitting signals as well as processing of received signals are carried out by means of appropriate microwave equipment [2]. Mostly, this equipment consists of devices based on metallic waveguides which have considerably lower losses in comparison with other structures especially in the high range of centimeter band. In most cases, these devices represent a combination of transverse junctions of metallic waveguides [3]. An arrangement of these connections in consecutive order is equivalent to physical model of longitudinally complicated waveguide structure. To obtain high-quality characteristics of such structure, all its elements must be carefully chosen based on a rigorous solution of corresponding electromagnetic problems [3, 4].

Modern methods of analysis of longitudinally irregular waveguide structures rely on a decomposition principle of complicated device onto simple regions. As a result, a multistep physical model presenting a cascade combination of waveguide segments is derived. The discontinuities which cause scattering of electromagnetic waves that propagate along the structure appear on the connections of waveguide sections. In some cases, like for deriving filtering as well as phase shifting properties, such discontinuities are created with the express purpose. In any case, geometrical parameters of discontinuities are subject to careful selection in order to obtain desired characteristics of all microwave device performed on the mentioned longitudinally irregular waveguide structure.

To achieve best results, physical parameters of such structures are founded using the optimization methods. Optimization methods which essentially allow bypassing the problem of local minima are widely used. Such optimization methods used for designing of filters, phase shifters, etc, are based on different modifications of approach called as the evolution strategy [5]. Application of the evolution strategy method guarantees finding a global minimum of the objective function. This evolution strategy method has been successfully applied to designing of a great number of complicated waveguide structures [6, 7].

As shown in [6—8], the computation of frequency characteristic of the parameter under optimization is carried out many times. The number of objective function calculations for complicated broadband devices can reach tens and even hundreds of thousands. Therefore, application of optimization methods based on the evolution strategy approach is associated with a lot of computer time even when high-performance computing facilities are used. Furthermore, use of efficient optimization methods is usually followed by the calculation of objective function with high accuracy. Since expending of computer time at the expense of optimization itself can be considered as fixed, and they actually cannot be reduced, choosing the effective method of the objective function calculation that mostly reduces to designation of device frequency characteristics is of great importance.

Thus, when using optimization methods that ensure the guaranteed finding a global minimum of the objective function owing to a large number of iterations, it is desirable to have effective algorithms providing high calculation accuracy at low computing time expenses. These algorithms can be constructed on the basis of electromagnetic problems solution by the integral equations method.

There are a considerable number of the integral equations method modifications. For instance, in [8], the simplest case has been considered, where only principal mode propagation was taken into account in both input and output waveguides. This method has been successfully used for designing of band-pass filters on thick inductive irises in rectangular waveguide. As a result, a simple scattering matrix of the filter has been obtained.

It is shown that solving the problem by using coupled integral equations can significantly reduce the computing time expenses as compared to the method based on combination of generalized scattering matrices of separate waveguide junctions. It is interesting to investigate the coupled integral equations method for general case when a large number of electromagnetic waves may exist in input as well as in output waveguides. Solving this simultaneous system of integral equations allows calculating the generalized scattering matrix of overall multistep waveguide structure without determining the scattering matrices of separate discontinuities.

Statement of the problem

The purpose of this paper is the development of a variant of integral equations method initiated in [9] concerning the solution of diffraction problem of electromagnetic waves on longitudinally complicated waveguide structures, obtaining of appropriate mathematical models in the form of scattering matrices, as well as their applications to the numerical computation of waveguide elements specifically such as a diaphragm of finite thickness and a resonant enlargement between two circular waveguides.

Solution of internal boundary problem

To obtain the correct solution for longitudinally complicated waveguide structure with closely spaced discontinuities, a great number of higher order modes should be taken into account in each waveguide section. If these calculations are carried out by estimating and combining generalized scattering matrices of separate junctions, to obtain the correct solution, it is necessary to find the scattering matrices of high order. Therefore, it is interesting to investigate the approach based on simultaneous solution of coupled integral equations to find generalized scattering matrix of overall multistep waveguide structure without determining scattering matrices of separate discontinuities.

Consider a section of longitudinally complicated waveguide structure containing u segments of regular transmission lines of different cross sections which are placed between two semi-infinite waveguides and formed $r = u + 1$ discontinuities at that these segments have lengths κ_k, where $k = 1,2,...,u$. We believe that all metal surfaces of waveguides are perfectly conducting and waveguides are filled with homogeneous isotropic mediums without losses.

Assume that M_1 transverse-electric modes TE_m and M_2 transverse-magnetic modes TM_m are incident alternately on the structure from the left waveguide, as well as N_1 transverse-electric modes TE_n and N_2 transverse-magnetic modes TM_n are incident alternately on the structure from the right waveguide. The modes numbering in waveguides will be produced by single index in order of critical wave number increase.

We can write the transverse components of electric fields for first waveguide in the plane of the first junction as follows:

$$\mathbf{E}^{(1)} = (1 + R_{pm})\mathbf{\Psi}^{(1)}_{pm} + \sum_{\mu}\sum_{\alpha} A_{\mu\alpha}\mathbf{\Psi}^{(1)}_{\mu\alpha}\zeta^{(1)}_{\mu\alpha} ; \quad (1)$$

$$\zeta^{(1)}_{\mu\alpha} = (1 - \delta_{m\alpha}\delta_{p\mu}),$$

where $\mathbf{E}^{(1)}$ is unknown tangential electric field in first coupling window; R_{pm} is reflection coefficient in the case when the m th transverse-electric ($p = 1$) or transverse-magnetic ($p = 2$) mode is incident on the structure from the side of the left semi-infinite waveguide; $A_{\mu\alpha}$ are the expansion coefficients of tangential electric field in first waveguide being at the same time transformation coefficients of m th incident mode into α th reflected mode of transverse-electric ($\mu = 1$) and transverse-magnetic ($\mu = 2$) types; $\mathbf{\Psi}^{(1)}_{\mu\alpha}$ are orthonormalized vector eigenfunctions of first waveguide; $\delta_{m\alpha}$, $\delta_{p\mu}$ are Kronecker symbols.

Analogous relation can be obtained for the case of modes incidence on the structure from the side of right semi-infinite waveguide

$$\mathbf{E}^{(\sigma)} = (1+R_{qn})\mathbf{\Psi}_{qn}^{(L)} + \sum_{\nu}\sum_{\beta} B_{\nu\beta}\mathbf{\Psi}_{\nu\beta}^{(L)}\zeta_{\nu\beta}^{(L)} \; ; \qquad (2)$$

$$\zeta_{\nu\beta}^{(L)} = (1-\delta_{n\beta}\delta_{q\nu})\,; \; \sigma = L-1,$$

where $\mathbf{E}^{(\sigma)}$ is unknown tangential electric field in last coupling window; R_{qn} is reflection coefficient if the nth transverse-electric ($q=1$) or transverse-magnetic ($q=2$) mode is incident on the structure from the right side; $B_{\nu\beta}$ are the expansion coefficients of tangential electric field in last waveguide being at the same time transformation coefficients of nth incident mode into βth reflected mode of transverse-electric ($\nu=1$) and transverse-magnetic ($\nu=2$) types; $\mathbf{\Psi}_{\nu\beta}^{(L)}$ are orthonormalized vector eigenfunctions of last waveguide; $\delta_{n\beta}$, $\delta_{q\nu}$ are Kronecker symbols.

Based on relation (1) and (2), we can construct generalized scattering matrix in the form of separate blocks. Scalar multiplying (1) and (2) on the systems of orthonormalized vector eigenfunctions of first and last waveguides and integrating the resulting expressions on the areas of coupling windows, we obtain the relations for calculating electric field scattering matrix elements

$$R_{pm} = \int_{s_1} \mathbf{E}^{(1)}\mathbf{\Psi}_{pm}^{(1)} ds - 1\,; \; (3) \qquad A_{\mu\alpha} = \int_{s_1} \mathbf{E}^{(1)}\mathbf{\Psi}_{\mu\alpha}^{(1)} ds \quad (4)$$

$$R_{qn} = \int_{s_\sigma} \mathbf{E}^{(\sigma)}\mathbf{\Psi}_{qn}^{(L)} ds - 1\,; \; (5) \qquad B_{\nu\beta} = \int_{s_\sigma} \mathbf{E}^{(\sigma)}\mathbf{\Psi}_{\nu\beta}^{(L)} ds\,, \; (6)$$

where s_1, s_σ are the areas of first and last coupling windows.

Assume that distributions of tangential electric fields in first and last coupling windows have been found by the solution of appropriate electromagnetic problems for all cases of waveguide eigenmodes incidence on the inhomogeneous structure. Then, using relations (3)—(6), we can find numerical values of generalized scattering matrix elements.

Consider the case when eigenmodes of the first waveguide are incident on the structure from the left side. Using relations (3) and (4), we find a block submatrix of the reflection coefficients. In this case, the relation (3) defines own reflection coefficients which numerical values are disposed along the main diagonal of the block submatrix.

The equation (4) allows calculating mutual elements of the block submatrix disposed symmetrically relative to its main diagonal. These elements are transformation coefficients of incident modes into reflected modes. The relation (6) defines transmission coefficients of modes with their transformations on the discontinuities of longitudinally inhomogeneous waveguide structure. As a result, the rectangular block submatrix is received in contrast to the square submatrix of reflection coefficients. These two submatrices form the left half of the generalized scattering matrix of all waveguide structure. Considering the case when eigenmodes of the right waveguide are alternatively incident on the structure, we also obtain two submatrices forming the right half of the generalized scattering matrix.

To find unknown tangential electric fields $\mathbf{E}^{(1)}$ and $\mathbf{E}^{(\sigma)}$, it is necessary to solve the electromagnetic problem taking into account the interior composition of the waveguide structure. The availability of discontinuities causes the appearance of scattered electromagnetic fields in sections of the waveguide structure under consideration. These scattered fields can be represented as the superposition of transmitted and reflected propagating modes, as well as of the infinite number of higher order evanescent modes. Then, the tangential electric and magnetic fields in each kth interior waveguide section can be written as the sums of the incident and reflected modes

$$\mathbf{E}^{(k)} = \sum_{\mu}\sum_{\alpha} \mathbf{\Psi}_{\mu\alpha}^{(k)}(C_{\mu\alpha}e^{-\gamma_{\mu\alpha}z} + D_{\mu\alpha}e^{\gamma_{\mu\alpha}z})\,; \qquad (7)$$

$$-\mathbf{z}\times\mathbf{H}^{(k)} = \sum_{\mu}\sum_{\alpha} Y_{\mu\alpha}^{(k)}\mathbf{\Psi}_{\mu\alpha}^{(k)}(-C_{\mu\alpha}e^{-\gamma_{\mu\alpha}z} + D_{\mu\alpha}e^{\gamma_{\mu\alpha}z})\,, \; (8)$$

where $\mathbf{\Psi}_{\mu\alpha}^{(k)}$ is orthonormalized vector αth eigenfunction of kth waveguide of transverse-electric ($\mu=1$) or transverse-magnetic ($\mu=2$) types; $Y_{\mu\alpha}^{(k)}$ are corresponding admittances of eigenmodes; $\gamma_{\mu\alpha}$ are their propagation coefficients; $C_{\mu\alpha}$, $D_{\mu\alpha}$ are unknown coefficients of field expansions; $\mathbf{z}$ is an unit vector in the direction of axis z of waveguide structure.

Applying the orthogonality conditions to the equations of type (7), we express unknown coefficients $C_{\mu\alpha}$ and $D_{\mu\alpha}$ for each section through tangential components of electric fields in adjacent coupling windows. Substituting these coefficients $C_{\mu\alpha}$ and $D_{\mu\alpha}$ into expressions for tangential components of magnetic fields of type (8) and matching the resulting fields of corresponding subregions, we obtain the following complex system of integral equations with respect to tangential components of electric fields in the coupling windows:

$$\sum_i Y_i^{(1)}\mathbf{\Psi}_i^{(1)} D_{101} + \sum_i Y_i^{(2)}\mathbf{\Psi}_i^{(2)}[\cosh\gamma_i^{(2)}\kappa_1 D_{102} -$$

$$-D_{212}]/\sinh\gamma_i^{(2)}\kappa_1 = 2\delta_{w1}Y_{pm}^{(1)}\mathbf{\Psi}_{pm}^{(1)}\,;$$

$$\sum_i Y_i^{(t)}\mathbf{\Psi}_i^{(t)}[\cosh\gamma_i^{(t)}\kappa_e D_{tet} - D_{egt}]/\sinh\gamma_i^{(t)}\kappa_e +$$

$$+\sum_i Y_i^{(f)}\mathbf{\Psi}_i^{(f)}[\cosh\gamma_i^{(f)}\kappa_t D_{tef} - D_{ftf}]/\sinh\gamma_i^{(f)}\kappa_t = 0$$

$$t = 2,3,...,u \qquad (9)$$

$$\sum_i Y_i^{(r)}\mathbf{\Psi}_i^{(r)}[\cosh\gamma_i^{(r)}\kappa_u D_{rur} - D_{udr}]/\sinh\gamma_i^{(r)}\kappa_u +$$

$$+\sum_i Y_i^{(h)}\mathbf{\Psi}_i^{(h)} D_{ruh} = 2\delta_{w2} Y_{qn}^{(h)}\mathbf{\Psi}_{qn}^{(h)},$$

$$w = 1,\ p = l,\ m = 1,2,...,M_l;$$

$$w = 2,\ q = l,\ n = 1,2,...,N_l;$$

$$D_{\tau et} = \int_{s_\tau} \mathbf{E}(\sum_{k=0}^{e}\kappa_k)\mathbf{\Psi}_i^{(t)} ds;\ \kappa_0 = 0.$$

Here i, l are generalized summation indices indicating the identity of values to transverse-electric or transverse-magnetic modes introduced for fields description simplicity; $h = r+1$ is the number of waveguide sections in considered complicated structure; $r = u+1$ is the number of discontinuities; $d = u-1$; $g = e-1$; $e = t-1$; $f = t+1$.

To solve the system of integral equations (9), we use the Galerkin's method. For this, we expand the unknown electric fields into series of orthonormalized coordinate functions of the coupling windows

$$\mathbf{E}^{(t)} = \sum_{j=1}^{J_t} C_j^{(t)}\mathbf{\Phi}_j^{(t)}, \qquad (10)$$

where t is the number of partial regions interfaces; j is generalized summation index indicating the identity of values to transverse-electric or transverse-magnetic modes; J_t is the number of expansion terms which are taken into account.

Substituting (10) into (9) and performing transformation in accordance with Galerkin's method we reduce the boundary problem to the system of linear algebraic equations for the complex expansion coefficients $C_j^{(t)}$

$$\sum_{j=1}^{J_t} C_j^{(1)}[\sum_i Y_i^{(1)}\eta_{vi}^{(11)}\eta_{ji}^{(11)} + \sum_i Y_i^{(2)}\eta_{vi}^{(12)}\eta_{ji}^{(12)}\coth\gamma_i^{(2)}\kappa_1] -$$

$$-\sum_{j=1}^{J_t} C_j^{(2)}\sum_i Y_i^{(2)}\eta_{vi}^{(12)}\eta_{ji}^{(22)}/\sinh\gamma_i^{(2)}\kappa_1 = 2\delta_{w1}Y_{pm}^{(1)}\eta_{vm}^{(11)};$$

$$-\sum_{j=1}^{J_t} C_j^{(e)}\sum_i Y_i^{(t)}\eta_{vi}^{(tt)}\eta_{ji}^{(et)}/\sinh\gamma_i^{(t)}\kappa_e +$$

$$+\sum_{j=1}^{J_t} C_j^{(t)}[\sum_i Y_i^{(t)}\eta_{vi}^{(tt)}\eta_{ji}^{(tt)}\coth\gamma_i^{(t)}\kappa_e +$$

$$+\sum_i Y_i^{(f)}\eta_{vi}^{(tf)}\eta_{ji}^{(tf)}\coth\gamma_i^{(f)}\kappa_t] -$$

$$-\sum_{j=1}^{J_t} C_j^{(f)}\sum_i Y_i^{(f)}\eta_{vi}^{(tf)}\eta_{ji}^{(ff)}/\sinh\gamma_i^{(f)}\kappa_1 = 0$$

$$t = 2,3,...,u \qquad (11)$$

$$-\sum_{j=1}^{J_t} C_j^{(u)}\sum_i Y_i^{(r)}\eta_{vi}^{(rr)}\eta_{ji}^{(ur)}/\sinh\gamma_i^{(r)}\kappa_u +$$

$$+\sum_{j=1}^{J_t} C_j^{(r)}[\sum_i Y_i^{(h)}\eta_{vi}^{(rh)}\eta_{ji}^{(rh)} + \sum_i Y_i^{(r)}\eta_{vi}^{(rr)}\eta_{ji}^{(rr)}\coth\gamma_i^{(r)}\kappa_u] =$$

$$= 2\delta_{w2}Y_{qn}^{(h)}\eta_{vn}^{(rh)};$$

$$v = 1,2,...,J_t,$$

$$w = 1,\ p = l,\ m = 1,2,...,M_l;$$

$$w = 2,\ q = l,\ n = 1,2,...,N_l.$$

As follows from (11), the obtained system has the same matrix of coefficients at unknowns and different right parts corresponding to electromagnetic problem solution under the alternative incidence on the inhomogeneous structure of eigenmodes of both the left and right waveguides. Such configuration of the system (11) simplifies finding its numerical solution.

The coupling coefficients of windows coordinate functions and vector eigenfunctions of waveguides are defined by the expression

$$\eta_{ji}^{(tf)} = \int_{s_t}\mathbf{\Phi}_j^{(t)}\mathbf{\Psi}_i^{(f)} ds. \qquad (12)$$

The system of linear algebraic equations (11) is suitable for calculating of modal scattering matrices of longitudinally complicated waveguide structure of general form with a great number of junctions. Depending upon the shapes and dimensions of connected waveguide sections some coupling coefficients can be equaled to unity or zero.

Because the system (11) is modified according to the configuration of the structure, the mathematical model based on simultaneous solution of the coupled integral equations (9) yields in universality to the method which is founded on the combination of generalized scattering matrices of separate junctions. However, this mathematical model has significant advantages with respect to counting rate especially in the case of closely spaced discontinuities strongly interacting in the higher order modes.

Generalized scattering matrices of symmetrical waveguide structures

The advantage of the approach discussed above is particularly evident in the case when calculating generalized scattering matrices of longitudinally complicated waveguide structures that are symmetrical with respect to a plane going through the middle of the structure perpendicularly to its longitudinal axes. Examples of symmetrical waveguide structures are band-pass and band-stop filters with discontinuities of finite thickness.

Most often, the scattering matrix of the filter is calculated by successive combining of the generalized scattering matrices of elementary basic discontinuities [7]. Scattering matrices of these discontinuities are high order enough because a large number of eigenmodes in diaphragm aperture should be taken into account to obtain accurate results. Therefore, the algorithms based on direct combining of generalized scattering matrices are uneconomical with respect to computer time expenses. The considered approach based on simultaneous solution of the coupled integral equations (9) allows creating more effective algorithms.

Assume that the distributions of tangential electric fields in coupling windows equidistant from the symmetry plane of longitudinally complicated waveguide structure are identical and vector eigenfunctions in corresponding regions are the same. Adding and subtracting k and $r-k+1$ ($k=1,2,...,b$; $b=r/2$) equations of the system (9) at $p=l$ for every $n=1,2,...,N_l$, we obtain two independent systems of integral equations with respect to sums and differences of tangential electric fields in coupling windows placed equidistant from the symmetry plane of the waveguide structure

$$\sum_i Y_i^{(1)}\mathbf{\Psi}_i^{(1)}F_{101}+\sum_i Y_i^{(2)}\mathbf{\Psi}_i^{(2)}[\cosh\gamma_i^{(2)}\kappa_1 F_{102}-$$
$$-F_{212}]/\sinh\gamma_i^{(2)}\kappa_1=2Y_{pn}^{(1)}\mathbf{\Psi}_{pn}^{(1)};$$
$$\sum_i Y_i^{(t)}\mathbf{\Psi}_i^{(t)}[\cosh\gamma_i^{(t)}\kappa_e F_{tet}-F_{egt}]/\sinh\gamma_i^{(t)}\kappa_e+$$
$$+\sum_i Y_i^{(f)}\mathbf{\Psi}_i^{(f)}[\cosh\gamma_i^{(f)}\kappa_t F_{tef}-F_{ftf}]/\sinh\gamma_i^{(f)}\kappa_t=0;$$
$$t=2,3,...,u \qquad (13)$$
$$\sum_i Y_i^{(b)}\mathbf{\Psi}_i^{(b)}[\cosh\gamma_i^{(b)}\kappa_u F_{bub}-F_{udb}]/\sinh\gamma_i^{(b)}\kappa_u+$$
$$+\sum_i Y_i^{(h)}\mathbf{\Psi}_i^{(h)}T_a F_{buh}=0;$$
$$F_{\tau et}=\int_{s_\tau}\mathbf{E}_a(\sum_{k=0}^{e}\kappa_k)\mathbf{\Psi}_i^{(t)}ds;$$
$$\mathbf{E}_1=\mathbf{E}(\sum_{k=0}^{t-1}\kappa_k)+\mathbf{E}(\sum_{k=0}^{r-t}\kappa_k);\ \mathbf{E}_2=\mathbf{E}(\sum_{k=0}^{t-1}\kappa_k)-\mathbf{E}(\sum_{k=0}^{r-t}\kappa_k);$$
$$T_1=\tanh[\gamma_i^{(b+1)}\kappa_b/2];\ T_1=\coth[\gamma_i^{(b+1)}\kappa_b/2],$$

where $u=b-1$; $h=b+1$; $a=1$ if a magnetic wall is placed in symmetry plane; $a=2$ if the case of electric wall is considered.

Approximating unknown electric fields $\mathbf{E}_a$ by series of orthonormalized coordinate functions of coupling windows for $p=l$ and every $n=1,2,...,N_l$, we obtain two independent systems of linear algebraic equations with respect to expansion coefficients $C_j^{(t)}$

$$\sum_{j=1}^{J_t}C_j^{(1)}[\sum_i Y_i^{(1)}\eta_{vi}^{(11)}\eta_{ji}^{(11)}+\sum_i Y_i^{(2)}\eta_{vi}^{(12)}\eta_{ji}^{(12)}\coth\gamma_i^{(2)}\kappa_1]-$$
$$-\sum_{j=1}^{J_t}C_j^{(2)}\sum_i Y_i^{(2)}\eta_{vi}^{(12)}\eta_{ji}^{(22)}/\sinh\gamma_i^{(2)}\kappa_1=2Y_{pn}^{(1)}\eta_{vn}^{(11)}$$
$$-\sum_{j=1}^{J_t}C_j^{(e)}\sum_i Y_i^{(t)}\eta_{vi}^{(tt)}\eta_{ji}^{(et)}/\sinh\gamma_i^{(t)}\kappa_e+$$
$$+\sum_{j=1}^{J_t}C_j^{(t)}[\sum_i Y_i^{(t)}\eta_{vi}^{(tt)}\eta_{ji}^{(tt)}/\coth\gamma_i^{(t)}\kappa_e+$$
$$+\sum_i Y_i^{(f)}\eta_{vi}^{(tf)}\eta_{ji}^{(tf)}\coth\gamma_i^{(f)}\kappa_t]-$$
$$-\sum_{j=1}^{J_t}C_j^{(f)}\sum_i Y_i^{(f)}\eta_{vi}^{(tf)}\eta_{ji}^{(ff)}/\sinh\gamma_i^{(f)}\kappa_t=0;$$
$$t=2,3,...,u; \qquad (14)$$
$$-\sum_{j=1}^{J_t}C_j^{(u)}\sum_i Y_i^{(b)}\eta_{vi}^{(bb)}\eta_{ji}^{(ub)}/\sinh\gamma_i^{(b)}\kappa_u+$$
$$+\sum_{j=1}^{J_t}C_j^{(b)}[\sum_i Y_i^{(b)}\eta_{vi}^{(bb)}\eta_{ji}^{(bb)}\coth\gamma_i^{(b)}\kappa_u+$$
$$+\sum_i Y_i^{(h)}\eta_{vi}^{(bh)}\eta_{ji}^{(bh)}T_a]=0$$
$$v=1,2,...,J_t,$$

where all designations are the same as in (11).

Single mode scattering matrices of symmetrical waveguide structures

To increase the design efficiency of filters on the stage of synthesis, it is reasonably to use the single mode approximation and to carry out final calculation by means of generalized scattering matrices. In these cases, it is advisable to employ single mode matrices equally with generalized scattering matrices. Assuming single mode excitation of the waveguide structure, one can obtain the mathematical model in the form of two systems of integral equations with real coefficients. This allows us to considerably decrease the computer time expenses.

When analyzing the system (13), we can observe that all equations except for the first one have pure imaginary coefficients. The presence of real parts in complex coefficients of the first equation for both $a=1,2$ is specified only by propagating modes. Therefore, by transforming the first equation for $a = 1,2$, one can reduce the boundary problem to two systems of linear algebraic equations with real coefficients. The solution of these systems allows determining single mode matrices of symmetrical waveguide structure.

Two corresponding independent systems of integral equations with real coefficients can be obtained by supposing in (13) $F_{101} = 1 + \exp(-2j\varepsilon_a)$, $j = (-1)^{1/2}$ and making the following replacement $\mathbf{E}_a = \exp(-j\varepsilon_a)\mathbf{E}_{1a}$. As a result, the first equations of the system (13) take the following form:

$$\sum_i (1-\delta_{i1}) Y_i^{(1)} \mathbf{\Psi}_i^{(1)} \int_{s_1} \mathbf{E}_{1a}(\kappa_0) \mathbf{\Psi}_i^{(1)} ds + \\ + \sum_i Y_i^{(2)} \mathbf{\Psi}_i^{(2)} [\cosh\gamma_i^{(2)}\kappa_1 \int_{s_1} \mathbf{E}_{1a}(\kappa_0) \mathbf{\Psi}_i^{(2)} ds - \\ - \int_{s_2} \mathbf{E}_{1a}(\kappa_1) \mathbf{\Psi}_i^{(2)} ds] / \sinh\gamma_i^{(2)}\kappa_1 = \\ = 2j\sin(\varepsilon_a) Y_i^{(1)} \mathbf{\Psi}_i^{(1)}; \tag{15}$$

$$\varepsilon_a = \arctan[2 / \int_{s_1} \mathbf{E}_{1a} \mathbf{\Phi}_1^{(1)} ds].$$

The remaining equations of these systems can be derived from (13) when designation $\mathbf{E}_a$ is being replaced by $\mathbf{E}_{1a}$.

Let us expand the unknown electric fields $\mathbf{E}_{1a}$ into series of orthonormalized vector coordinate functions of the coupling windows ζ_μ

$$\mathbf{E}_{1a} = \sum_\mu C_{a\mu} \zeta_\mu , \tag{16}$$

where $C_{a\mu}$ are unknown coefficients; μ is the generalized summation index indicating the identity of values to transverse-electric or transverse-magnetic modes.

Substituting (16) into (15), using the Galerkin's method and denoting $B_{a\mu} = C_{a\mu} / \sin(\varepsilon_a)$, we obtain two systems ($a = 1,2$) of linear algebraic equations with real coefficients $B_{a\mu}$ which first equations are reduced to

$$\sum_\mu B_{a\mu}^{(1)} [\sum_i (1-\delta_{i1}) Y_i^{(1)} \eta_{vi}^{(11)} \eta_{\mu i}^{(11)} + \\ + \sum_i Y_i^{(2)} \eta_{vi}^{(12)} \eta_{\mu i}^{(12)} \cot\gamma_i^{(2)}\kappa_1] - \\ - \sum_\mu B_{a\mu}^{(2)} \sum_i Y_i^{(2)} \eta_{vi}^{(12)} \eta_{\mu i}^{(22)} / \sin\gamma_i^{(2)}\kappa_1 = 2jY_i^{(1)} \eta_{v1}^{(11)} . \tag{17}$$

Remaining equations of these systems immediately follow from (14) at the replacement of $C_j^{(t)}$ by $B_{a\mu}^{(t)}$.

After solving the systems (17), the elements of single mode scattering matrix (reflection R and transmission T coefficients) can be calculated by the following relations:

$$R = \sum_\mu [B_{1\mu}^{(1)} F_1 + B_{2\mu}^{(1)} F_2] \eta_{\mu 1}^{(11)} - 1;$$

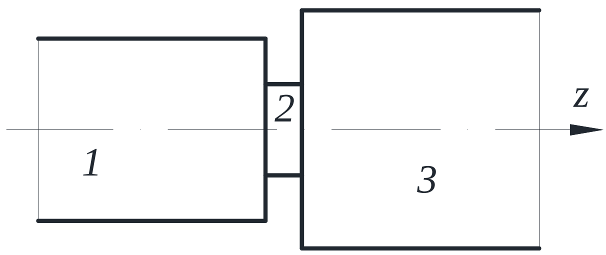
$$T = \sum_\mu [B_{1\mu}^{(1)} F_1 - B_{2\mu}^{(1)} F_2] \eta_{\mu 1}^{(11)} ;$$

$$F_a = \sin\varepsilon_a \exp(-j\varepsilon_a)/2 .$$

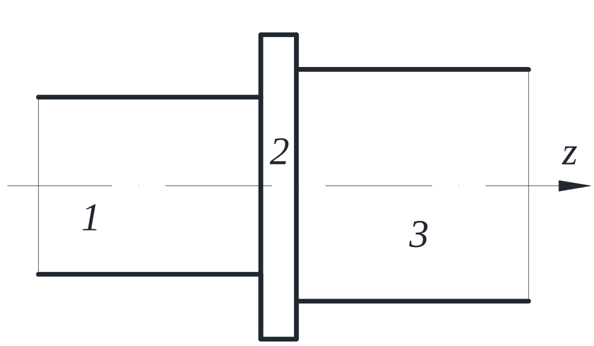
During derivation of relations for calculating generalized and single-mode scattering matrices, the views of jointed waveguides were not specified. Thus, the obtained formulas are without respect to the forms of connected waveguides cross-sections. The association of obtained mathematical relations with objective waveguide structures is performed by substituting into expressions (12) the ratios for coordinate functions of coupling windows and eigenfunctions of waveguides.

Results

Let us illustrate the effectiveness of the proposed approach on calculation examples of finite thickness diaphragm and resonant enlargement between two circular waveguides operating in the fundamental mode TE_{11}. The waveguide elements considered are shown in Fig. 1. We consider the case when two circular waveguides have different cross-sections, as well as the scenarios of symmetrical diaphragm and resonant enlargement.



(*a*)



(*b*)

Fig. 1. The diaphragm of finite thickness (*a*) and resonant enlargement (*b*) between two circular waveguides.

We will define scattering matrices of these waveguide elements by simultaneous solving of two coupled integral equations, as well as by conventional technique and then compare the computing time for these considered cases. The system of two coupled integral equations on the number of single discontinuities based on (9) can be written as

$$\sum_{\nu}\sum_{k} Y_{\nu k}^{(1)} \mathbf{\Psi}_{\nu k}^{(1)} P_{\nu k}^{(11)} + \sum_{\nu}\sum_{k} U[V P_{\nu k}^{(12)} - P_{\nu k}^{(22)}] = = 2\delta_{w1} Y_{pm}^{(1)} \mathbf{\Psi}_{pm}^{(1)} ;$$

$$\sum_{\nu}\sum_{k} U[V P_{\nu k}^{(12)} - P_{\nu k}^{(22)}] + \sum_{\nu}\sum_{k} Y_{\nu k}^{(3)} \mathbf{\Psi}_{\nu k}^{(2)} P_{\nu k}^{(23)} = = 2\delta_{w2} Y_{qn}^{(3)} \mathbf{\Psi}_{qn}^{(3)} ; \quad (18)$$

$$P_{\nu k}^{(il)} = \int_{s_i} \mathbf{E}^{(i)} \mathbf{\Psi}_{\nu k}^{(l)} ds ; \; U = Y_{\nu k}^{(2)} \mathbf{\Psi}_{\nu k}^{(2)} / \sinh \gamma_{\nu k} t ;$$

$$V = \cosh \gamma_{\nu k} t ,$$

where $m = 1,2,...,M_p$; $n = 1,2,...,N_q$; $k = 1,2,...,K_{\nu}^{(l)}$; $l = 1,2,3$; $\mathbf{E}^{(i)}$ is unknown tangential electric field in ith ($i = 1,2$) coupling window; $\mathbf{\Psi}_{\nu k}^{(l)}$ is orthonormalized vector eigenfunction of k th mode in l th partial region of transverse-electric ($\nu = 1$) or transverse-magnetic ($\nu = 2$) types; $Y_{\nu k}^{(l)}$ is corresponding to its admittance; $\gamma_{\nu k}$ is propagation coefficient of k th mode of transverse-electric ($\nu = 1$) or transverse-magnetic ($\nu = 2$) types in coupling waveguide; δ_{w1}, δ_{w2} are Kronecker symbols; $w = 1$, if the diffraction problem is considered for case of the incidence M_p transverse-electric ($p = 1$) or transverse-magnetic ($p = 2$) electromagnetic waves from the left side; $w = 2$, if N_q electromagnetic waves of transverse-electric ($q = 1$) or transverse-magnetic ($q = 2$) types are incident from the right side; $K_{\nu}^{(l)}$ is the number of modes of transverse-electric ($\nu = 1$) or transverse-magnetic ($\nu = 2$) types which are taken into account in l th waveguide; s_i is the area of i th coupling window; t is the diaphragm thickness or the resonant enlargement length.

To solve (18), we apply Galerkin's method as it has been done previously. To this end, we approximate the unknown tangential electric fields $\mathbf{E}^{(i)}$ by a series of orthonormalized vector eigenfunctions of coupling waveguide

$$\mathbf{E}^{(i)} = \sum_{\mu}\sum_{h} C_{\mu h}^{(i)} \mathbf{\Phi}_{\mu h}^{(i)} , \quad (19)$$

where $h = 1,2,...,H_{\mu}^{(i)}$; $\mathbf{\Phi}_{\mu h}^{(i)}$ are orthonormalized vector coordinate functions of transverse-electric ($\mu = 1$) or transverse-magnetic ($\mu = 2$) types in i th coupling window; $C_{\mu h}^{(i)}$ are unknown expansion coefficients; $H_{\mu}^{(i)}$ is the number of approximating functions of transverse-electric ($\mu = 1$) or transverse-magnetic ($\mu = 2$) types in i th coupling window.

Substituting (19) into (18) and performing transformations in accordance with Galerkin's method we obtain the system of linear algebraic equations for the complex expansion coefficients $C_{\mu h}^{(i)}$

$$\sum_{\mu}\sum_{h} C_{\mu h}^{(1)} [\sum_{\nu}\sum_{k} Y_{\nu k}^{(1)} \eta_{\nu k}^{(11u\nu)} \eta_{hk}^{(11\mu\nu)} + +\sum_{\nu}\sum_{k} Y_{\nu k}^{(2)} \eta_{\nu k}^{(12u\nu)} \eta_{hk}^{(12\mu\nu)} \coth\gamma_{\nu k} t] - -\sum_{\mu}\sum_{h} C_{\mu h}^{(2)} \sum_{\nu}\sum_{k} Y_{\nu k}^{(2)} \eta_{\nu k}^{(12u\nu)} \eta_{hk}^{(22\mu\nu)} / \sinh\gamma_{\nu k} t = = 2\delta_{w1} Y_{pm}^{(1)} \eta_{\nu m}^{(11up)} ;$$

$$u = 1 , \; \nu = 1,2,...,H_1^{(1)} ; \; u = 2 , \; \nu = 1,2,...,H_2^{(1)} ;$$

$$w = 1 , \; p = 1 , \; m = 1,2,...,M_1 ; \; p = 2 , \; m = 1,2,...,M_2 ; \quad (20)$$

$$-\sum_{\mu}\sum_{h} C_{\mu h}^{(1)} \sum_{\nu}\sum_{k} Y_{\nu k}^{(2)} \eta_{\nu k}^{(12u\nu)} \eta_{hk}^{(22\mu\nu)} / \sinh\gamma_{\nu k} t + +\sum_{\mu}\sum_{h} C_{\mu h}^{(2)} [\sum_{\nu}\sum_{k} Y_{\nu k}^{(2)} \eta_{\nu k}^{(22u\nu)} \eta_{hk}^{(22\mu\nu)} \coth\gamma_{\nu k} t +\sum_{\nu}\sum_{k} Y_{\nu k}^{(3)} \eta_{\nu k}^{(23u\nu)} \eta_{hk}^{(23\mu\nu)}] = = 2\delta_{w2} Y_{qn}^{(3)} \eta_{\nu n}^{(23uq)} ;$$

$$u = 1 , \; \nu = 1,2,...,H_1^{(2)} ; \; u = 2 , \; \nu = 1,2,...,H_2^{(2)} ;$$

$$w = 2 , \; q = 1 , \; n = 1,2,...,N_1 ; \; q = 2 , \; n = 1,2,...,N_2 ;$$

$$\eta_{hk}^{(il\mu\nu)} = \int_{s_i} \mathbf{\Phi}_{\mu h}^{(i)} \mathbf{\Psi}_{\nu k}^{(l)} ds . \quad (21)$$

The expression (21) represents the coupling coefficient of h th coordinate function of μ th type in i th coupling window and k th vector eigenfunction of ν th type in l th waveguide. The indexes $\mu = 1$ and $\nu = 1$ designate the identity of values to transverse-electric modes. The indexes $\mu = 2$ and $\nu = 2$ characterize the values relating to the fields of transverse-magnetic type.

The relations (20) and (21) represent the problem solution regarding doubled discontinuity of general form. As applied to structures shown in Fig. 1, the system of linear algebraic equations (20) is simplified because certain coefficients (21) are equaled to unity. Because for diaphragm of finite thickness $\eta_{hk}^{(12\mu\nu)} = 1$, $\eta_{\nu k}^{(12u\nu)} = 1$, $\eta_{hk}^{(22\mu\nu)} = 1$, $\eta_{\nu k}^{(22u\nu)} = 1$, these coefficients are replaced by Kronecker symbols $\delta_{u\mu}$, δ_{vh}, and the system (20) is reduced to the form:

$$\sum_\mu\sum_h C^{(1)}_{\mu h}[\sum_\nu\sum_k Y^{(1)}_{\nu k}\eta^{(11u\nu)}_{\nu k}\eta^{(11\mu\nu)}_{hk} + +Y^{(2)}_{\nu h}\delta_{u\mu}\delta_{vh}\coth\gamma_{\nu h}t] - -\sum_\mu\sum_h C^{(2)}_{\mu h}Y^{(2)}_{\mu h}\delta_{u\mu}\delta_{vh}/\sinh\gamma_{\mu h}t = 2\delta_{w1}Y^{(1)}_{pm}\eta^{(11up)}_{vm};$$

$u=1$, $v=1,2,...,H^{(1)}_1$; $u=2$, $v=1,2,...,H^{(1)}_2$;

$w=1$, $p=1$, $m=1,2,...,M_1$; $p=2$, $m=1,2,...,M_2$; (22)

$$-\sum_\mu\sum_h C^{(1)}_{\mu h}Y^{(2)}_{\mu h}\delta_{u\mu}\delta_{vh}/\sinh\gamma_{\mu h}t + +\sum_\mu\sum_h C^{(2)}_{\mu h}[Y^{(2)}_{\nu h}\delta_{u\mu}\delta_{vh}\coth\gamma_{\nu h}t + +\sum_\nu\sum_k Y^{(3)}_{\nu k}\eta^{(23u\nu)}_{\nu k}\eta^{(23\mu\nu)}_{hk}] = 2\delta_{w2}Y^{(3)}_{qn}\eta^{(23uq)}_{vn};$$

$u=1$, $v=1,2,...,H^{(2)}_1$; $u=2$, $v=1,2,...,H^{(2)}_2$;

$w=2$, $q=1$, $n=1,2,...,N_1$; $q=2$, $n=1,2,...,N_2$.

The relation between the system (22) and objective waveguide structures are performed by the coefficients (21). Substituting the expressions for vector eigenfunctions of circular waveguide into (21) and estimating the integrals, we obtain the following relations for coupling coefficients:

$$\eta^{(il\mu\nu)}_{hk} = \pi/2V^{(i)}_{\mu h}V^{(l)}_{\nu k}\sigma^{(i)}_{\mu h}\sigma^{(l)}_{\nu k}/(\xi^2_{\mu h}-\xi^2_{\nu k})\times$$
$$\times\alpha J_1(\alpha)[J_0(\beta)-\varsigma J_2(\beta)]-\beta J_1(\beta)[J_0(\alpha)-\varsigma J_2(\alpha)];$$
$$\xi_{\mu h}=\sigma^i_{\mu h};\ \xi_{\nu k}=\sigma^l_{\nu k};\ \alpha=\sigma^{(i)}_{\mu h}a_2;\ \beta=\sigma^l_{\nu k}a_2,$$

where $\varsigma=(-1)^{\mu+\nu}$; $V^{(i)}_{\mu h}$, $V^{(l)}_{\nu k}$ are the normalizing coefficients; $J_\chi(\alpha)$, $J_\chi(\beta)$ are the Bessel functions of χ-th order; r_2 is the radius of the diaphragm window. The normalizing coefficients are defined by the following relations:

$$V^{(i)}_{\mu h}=[\int_{s_i}\mathbf{\Phi}^{(i)}_{\mu h}\mathbf{\Phi}^{(i)}_{\mu h}ds]^{-1/2};\ V^{(l)}_{\nu k}=[\int_{s_i}\mathbf{\Psi}^{(l)}_{\nu k}\mathbf{\Psi}^{(l)}_{\nu k}ds]^{-1/2}.$$

Consider the case of the resonant enlargement between two circular waveguides shown in Fig. 1*b*. In this case, the coupling coefficients $\eta^{(11u\nu)}_{\nu k}$, $\eta^{(11\mu\nu)}_{hk}$, $\eta^{(23u\nu)}_{\nu k}$, $\eta^{(23\mu\nu)}_{hk}$, as well as the coupling coefficients $\eta^{(11up)}_{vm}$ and $\eta^{(23uq)}_{vn}$ are equaled to unity. As a result, these coefficients are replaced by Kronecker symbols $\delta_{u\mu}$, δ_{vh}, δ_{up}, δ_{vm}, δ_{uq}, δ_{vn}, and the system (20) is simplified to the form:

$$\sum_\mu\sum_h C^{(1)}_{\mu h}[Y^{(1)}_{\mu h}\delta_{u\mu}\delta_{vh} + +\sum_\nu\sum_k Y^{(2)}_{\nu k}\eta^{(12u\nu)}_{\nu k}\eta^{(12\mu\nu)}_{hk}\coth\gamma_{\nu k}t] - -\sum_\mu\sum_h C^{(2)}_{\mu h}\sum_\nu\sum_k Y^{(2)}_{\nu k}\eta^{(12u\nu)}_{\nu k}\eta^{(22\mu\nu)}_{hk}/\sinh\gamma_{\nu k}t = =2\delta_{w1}Y^{(1)}_{pm}\delta_{up}\delta_{vm};$$

$u=1$, $v=1,2,...,H^{(1)}_1$; $u=2$, $v=1,2,...,H^{(1)}_2$;

$w=1$, $p=1$, $m=1,2,...,M_1$; $p=2$, $m=1,2,...,M_2$; (23)

$$-\sum_\mu\sum_h C^{(1)}_{\mu h}\sum_\nu\sum_k Y^{(2)}_{\nu k}\eta^{(12u\nu)}_{\nu k}\eta^{(22\mu\nu)}_{hk}/\sinh\gamma_{\nu k}t + +\sum_\mu\sum_h C^{(2)}_{\mu h}[\sum_\nu\sum_k Y^{(2)}_{\nu k}\eta^{(22u\nu)}_{\nu k}\eta^{(22\mu\nu)}_{hk}\coth\gamma_{\nu k}t +Y^{(3)}_{\mu h}\delta_{u\mu}\delta_{vh}] = 2\delta_{w2}Y^{(3)}_{qn}\delta_{uq}\delta_{vn};$$

$u=1$, $v=1,2,...,H^{(2)}_1$; $u=2$, $v=1,2,...,H^{(2)}_2$;

$w=2$, $q=1$, $n=1,2,...,N_1$; $q=2$, $n=1,2,...,N_2$.

Consider now the symmetrical structures, where the radii of input and output waveguides are equal: $r_1=r_3$. In this case, the tangential electric and magnetic fields on both sides of doubled discontinuity are identical. As a result, the system of integral equations (18) takes the form

$$\sum_\nu\sum_k Y^{(1)}_{\nu k}\mathbf{\Psi}^{(1)}_{\nu k}P^{(11)}_{\nu k} + \sum_\nu\sum_k U[VP^{(12)}_{\nu k}-P^{(22)}_{\nu k}] = 2Y^{(1)}_{pm}\mathbf{\Psi}^{(1)}_{pm};$$

(24)

$$\sum_\nu\sum_k U[VP^{(12)}_{\nu k}-P^{(22)}_{\nu k}] + \sum_\nu\sum_k Y^{(3)}_{\nu k}\mathbf{\Psi}^{(2)}_{\nu k}P^{(23)}_{\nu k} = 0.$$

Take into account the mirror symmetry of doubled discontinuity with respect to a plane going through the middle of the structure perpendicularly to its longitudinal axes. Then, the expressions (24) one can reduce to two independent systems of integral equations relative to sums and differences of tangential electric fields in coupling windows. Form the sum and difference of first and second integral equations in (24) for every mode incident on doubled discontinuity from the left side. This corresponds to the placement of magnetic ($\xi=1$) and electric ($\xi=2$) walls in the symmetry plane of doubled discontinuity. Carrying out the transformation, we find

$$\sum_\nu\sum_k Y^{(1)}_{\nu k}\mathbf{\Psi}^{(1)}_{\nu k}P^{(11)}_{\nu k} + \sum_\nu\sum_k Q_\xi Y^{(2)}_{\nu k}\mathbf{\Psi}^{(2)}_{\nu k}P^{(12)}_{\nu k} = 2Y^{(1)}_{pm}\mathbf{\Psi}^{(1)}_{pm};$$

$$P^{(11)}_{\nu k}=\int_{s_1}\mathbf{F}_1\mathbf{\Psi}^{(1)}_{\nu k}ds;\ P^{(12)}_{\nu k}=\int_{s_1}\mathbf{F}_2\mathbf{\Psi}^{(2)}_{\nu k}ds; \quad (25)$$

$$\mathbf{F}_1 = \mathbf{E}_1 + \mathbf{E}_2\,;\ \mathbf{F}_2 = \mathbf{E}_1 - \mathbf{E}_2\,;$$

$$Q_1 = \tanh(\gamma_{\nu k} t/2)\,;\ Q_2 = \coth(\gamma_{\nu k} t/2)\,;$$

$$m = 1,2,...,M_p\,;\ k = 1,2,...,K_\nu\,;\ p = 1,2\,;\ \nu = 1,2\,.$$

To solve (25) at $\xi = 1,2$, we apply the Galerkin's method. Represent the summary ($\xi = 1$) and differential ($\xi = 2$) tangential electric fields for every mode incident on the discontinuity by the expansion into series of orthonormalized vector eigenfunctions of the coupling waveguide. Then, these approximating fields for symmetrical diaphragm of finite thickness in circular waveguide (Fig. 1*a*) and symmetrical enlargement of circular waveguide (Fig. 1*b*) can be written as

$$\mathbf{F}_\xi = \sum_\mu \sum_h C^{(1)}_{\mu h} \mathbf{\Phi}^{(2)}_{\mu h}\,;\ \mathbf{F}_\xi = \sum_\mu \sum_h C^{(1)}_{\mu h} \mathbf{\Phi}^{(1)}_{\mu h}\,. \qquad (26)$$

Substituting (26) into (25), taking into account the orthogonality of eigenfunctions in connected waveguides and performing transformation in accordance with Galerkin's method, we obtain two systems of linear algebraic equations corresponding to even ($\xi = 1$) and odd ($\xi = 2$) interpretations of each doubled discontinuity excitation. In this way, the desired two systems of linear algebraic equations for determination of generalized scattering matrix of the symmetrical diaphragm of finite thickness in circular waveguide can be present in the form:

$$\sum_\mu \sum_h C^{(1)}_{\mu h} [\sum_\nu \sum_k Y^{(1)}_{\nu k} \eta^{(11u\nu)}_{\nu k} \eta^{(11\mu\nu)}_{hk} + $$
$$+ Y^{(2)}_{\nu h} Q_\xi \delta_{u\mu} \delta_{\nu h}] = 2 Y^{(1)}_{pm} \eta^{(11up)}_{\nu m}\,, \qquad (27)$$

where all designations are the same as in (22).

Performing similar transformation for the symmetrical enlargement of circular waveguide, we obtain two ($\xi = 1,2$) following systems of linear algebraic equations:

$$\sum_\mu \sum_h C^{(1)}_{\mu h} [Y^{(2)}_{\nu h} Q_\xi \delta_{u\mu} \delta_{\nu h} + $$
$$+ \sum_\nu \sum_k Y^{(2)}_{\nu k} \eta^{(12u\nu)}_{\nu k} \eta^{(12\mu\nu)}_{hk}] = 2 Y^{(1)}_{pm} \delta_{up} \delta_{\nu m}\,, \qquad (28)$$

where all designations are the same as in (23).

One can see that for every incident mode, the systems of linear algebraic equations (27) and (28) have the same matrices of coefficients at unknowns. They differ only in the matrices of right parts which number equals a half of order of generalized scattering matrix of considered waveguide structure. Therefore, to solve (27) and (28), it is advisable to use the subprogram of solution of linear algebraic equations with multiple right parts. Solving each of the systems (27), (28) at $\xi = 1,2$, we find the sums $\mathbf{F}_1$ and differences $\mathbf{F}_2$ of tangential electric fields in first and second coupling windows of corresponding discontinuity. Using complex coefficients $C^{(1)}_{\mu h}$ found by solving systems (27), (28) and expressions for $\mathbf{F}_1$, $\mathbf{F}_2$, we define the distributions of tangential components of electric fields in coupling windows. Then, the elements of generalized scattering matrix can be obtained according to relations (3)—(6).

Therefore, taking into account the mirror symmetry of doubled discontinuity allows us to solve two systems of linear algebraic equations of order $H^{(1)}_1 + H^{(1)}_2$ with M right parts instead of one system of order $2[H^{(1)}_1 + H^{(1)}_2]$. As will be shown below, this provides a significant gain in computing time with the same calculation accuracy.

At the synthesis of several microwave devices such as filters in the operating frequency range of circular waveguide, it is expedient to use the single mode matrices. In this case, the computing time can be further significantly reduced due to transition from the solution (27), (28) with complex coefficients to the solution of corresponding systems of linear algebraic equations with real coefficients.

Supposing in (24) $P^{(11)}_{\nu k} = 1 + \exp(-2 j b_\xi)$ and $M_1 = 1$, we obtain two ($\xi = 1,2$) integral equations with real coefficients

$$\sum_\nu \sum_k (1 - \delta_{1\nu} \delta_{1k}) Y^{(1)}_{\nu k} \mathbf{\Psi}^{(1)}_{\nu k} G^{(11)}_{\nu k} + \sum_\nu \sum_k Q_\xi Y^{(2)}_{\nu k} \mathbf{\Psi}^{(2)}_{\nu k} G^{(12)}_{\nu k} = $$
$$= 2 j Y^{(1)}_1 \mathbf{\Psi}^{(1)}_1 \qquad (29)$$

$$G^{(11)}_{\nu k} = \int_{s_1} \mathbf{E}_\xi \mathbf{\Psi}^{(1)}_{\nu k} ds\,;\ G^{(12)}_{\nu k} = \int_{s_1} \mathbf{E}_\xi \mathbf{\Psi}^{(2)}_{\nu k} ds$$

$$\mathbf{E}_\xi = \mathbf{F}_\xi \exp(j b_\xi)/\sin b_\xi\,;\ b_\xi = \arctan(2 / \int_{s_1} \mathbf{E}_\xi \mathbf{\Psi}^{(1)}_{11} ds)\,.$$

To solve (29) for each of discontinuities shown in Fig. 1, we use the representation analogous to (26)

$$\mathbf{E}_\xi = \sum_\mu \sum_h D^{(1)}_{\mu h} \mathbf{\Phi}^{(2)}_{\mu h}\,;\ \mathbf{E}_\xi = \sum_\mu \sum_h D^{(1)}_{\mu h} \mathbf{\Phi}^{(1)}_{\mu h}\,, \qquad (30)$$

where $D^{(1)}_{\mu h}$ are real coefficients, and $\mathbf{\Phi}^{(1)}_{\mu h}$, $\mathbf{\Phi}^{(2)}_{\mu h}$ are vector eigenfunctions the same as in (26).

Substituting (30) into (29) and performing transformation in accordance with Galerkin's method, we obtain the following systems of linear algebraic equations for diaphragm of finite thickness and resonant enlargement between two circular waveguides, respectively

$$\sum_\mu \sum_h D^{(1)}_{\mu h} [\sum_\nu \sum_k (1 - \delta_{1\nu} \delta_{1k}) Y^{(1)}_{\nu k} \eta^{(11u\nu)}_{\nu k} \eta^{(11\mu\nu)}_{hk} + $$
$$+ Y^{(2)}_{\nu h} Q_\xi \delta_{u\mu} \delta_{\nu h}] = 2 j Y^{(1)}_{11} \eta^{(1111)}_{\nu 1}\,; \qquad (31)$$

$$u=1,\ v=1,2,...,H_1^{(1)};\ u=2,\ v=1,2,...,H_2^{(1)}.$$

$$\sum_{\mu}\sum_{h} D_{\mu h}^{(1)}[Y_{vh}^{(1)}\delta_{u\mu}\delta_{vh} + +\sum_{\nu}\sum_{k} Y_{\nu k}^{(2)} Q_{\xi}\eta_{\nu k}^{(12u\nu)}\eta_{hk}^{(12\mu\nu)}] = 2jY_{11}^{(1)}\delta_{u1}\delta_{v1}; \quad (32)$$

$$u=1,\ v=1,2,...,H_1^{(2)};\ u=2,\ v=1,2,...,H_2^{(2)}.$$

Let us compare expenses for the computing time spent on computation of generalized scattering matrices and single mode scattering matrices of considered doubled discontinuities using the proposed approach based on simultaneous solution of the system of two coupled integral equations and by means of known technique. For correct comparison, it is necessary to obtain the expressions for generalized scattering matrix of separate junction between two circular waveguides. The required solution of electromagnetic problem for single junction can be obtained as a special case of doubled discontinuity considering only its half. Carrying out the corresponding transformations, we obtain the following system of linear algebraic equations for the unknown complex expansion coefficients of tangential electric field in coupling window by orthonormalized vector eigenfunctions of circular waveguide of smaller cross-section:

$$\sum_{\mu}\sum_{h} C_{\mu h}^{(1)}[Y_{\mu h}^{(1)}\delta_{u\mu}\delta_{vh} + +\sum_{\nu}\sum_{k} Y_{\nu k}^{(2)}\eta_{\nu k}^{(12u\nu)}\eta_{hk}^{(12\mu\nu)}] = \quad (33)$$

$$= 2\delta_{w1}Y_{pm}^{(1)}\delta_{up}\delta_{vm} + 2\delta_{w2}Y_{qn}^{(2)}\eta_{vn}^{(12uq)};$$

$$u=1,\ v=1,2,...,H_1^{(1)};\ u=2,\ v=1,2,...,H_2^{(1)};$$

$$w=1,\ p=1,\ m=1,2,...,M_1;\ p=2,\ m=1,2,...,M_2;$$

$$w=2,\ q=1,\ n=1,2,...,N_1;\ q=2,\ n=1,2,...,N_2,$$

where all designations are the same as in (27).

The presence of frequency-independent coupling coefficients in the systems of linear algebraic equations is the specificity of the obtained solution. For invariable dimension of structure, these coefficients may be determined one time and be stored in the computer memory. This largely accelerates the computation of frequency characteristics of longitudinally inhomogeneous waveguide structure.

All systems of linear algebraic equations obtained have the same matrices of coefficients under unknowns and differ only in right parts. Therefore, the subprogram of solution of the systems of linear algebraic equations with several right parts is used to numerically realize the obtained mathematical models. When calculating the coefficients of these systems for propagating modes, the hyperbolic functions are replaced by trigonometric ones.

The mathematical models obtained are realized as a complex of FORTRAN programs. Calculations of a great number of longitudinally inhomogeneous waveguide structures were performed by using this program. The results of these calculations show the advantage of developed approach as compared with the widely used method based on combination of the generalized scattering matrices of discontinuities and waveguide sections between them. To compare the developed and known algorithms against the computing time, the calculations of doubled discontinuities shown in Fig. 1 have been performed at the following dimensions ratio of the structures: $r_1 = 10$ mm; $r_2/r_1 = 0.5$; $r_3/r_1 = 1.25$ for diaphragm of finite thickness and $r_1 = 10$ mm; $r_2/r_1 = 1.5$; $r_3/r_1 = 1.25$ for resonant enlargement. The length of coupling region *2* between waveguides *1* and *3* was taken equal to $t = 2$ mm. The calculations were performed at the frequency 11 GHz. Twenty transverse-electric and twenty transverse-magnetic modes were taken into account in input waveguide. The numbers of considered modes in waveguides *2* and *3* were determined in accordance with chosen dimensions ratios.

To obtain the correct comparative results against computing time for two considered approaches relying on high productivity computer, the hundred thousand iterations were used at computation. The critical mode numbers of waveguides and coupling coefficients of eigenmodes were computed separately and stored in the computer memory. Therefore, the main computer time expenses were connected with computation of scattering matrices. The obtained results allow estimating the efficiency of proposed approach in comparison with the known technique.

By using known algorithms, the results of generalized scattering matrices computation were obtained within 10 min 15 sec for diaphragm of finite thickness (Fig. 1*a*) and 18 min 40 sec for resonant enlargement (Fig. 1*b*). Using the proposed approach, the complete computing time of the diaphragm of finite thickness (Fig. 1*a*) according to (20) constitutes 4 min 10 sec. Further significant reduction of computing time is observed for the resonant enlargement between two circular waveguides (Fig. 1*b*) because in this case there is no need to compute generalized scattering matrices of separate discontinuities and their combination. Therefore, the computer performed hundred thousand iterations within 5 min 25 sec. By using the proposed approach, the gain in computing time under calculation of structures shown in Fig. 1 is 2.5 and 3.5 times, respectively.

Even more significant gain in computing time is obtained for the case of symmetrical doubled discontinuities when $r_3 = r_1$. When using the relation (27), the expenses of computing time for this case were 1 min 50 sec. At computation of the resonant enlargement between two circular waveguides (Fig. 1*b*) according to relation (28), the corresponding execution time of one hundred thousand iterations was 2 min 35 sec.

A distinctive feature of the calculation algorithm of single mode scattering matrices of considered doubled discontinuities in accordance with relations (31), (32) is negligible small computing time. One hundred thousand iterations is performed only for 20—30 sec at computation of the single mode scattering matrices for the considered cases of the diaphragm of finite thickness (Fig. 1*a*) and resonant enlargement between two circular waveguides (Fig. 1*b*).

The results of performed comparison illustrate the efficiency of proposed approach to calculate and optimize longitudinally complicated waveguide structures and various devices on their basis. This approach can be successfully applied to calculation of more complicated waveguide structures when numerical methods should be involved to find eigenmodes of connected waveguides.

Conclusion

In the rigorous formulation, the diffraction problems of transverse-electric and transverse-magnetic modes on the longitudinally complicated waveguide structures are solved. Based on the integral equation method, An approach to calculation of the longitudinally complicated waveguide structures is developed. The expressions for calculating generalized scattering matrices of complicated structures consisting of many connected waveguide sections are derived. A technique for calculating waveguide longitudinally complicated connections that are symmetrical with respect to a plane going through the middle of the structure perpendicularly to its longitudinal axes is proposed. This problem is reduced to two independent systems of integral equations relative to sums and differences of tangential electric fields in coupling windows. The formulas for description of symmetrical waveguide structure by single mode scattering matrix in the form of two independent systems of linear algebraic equations with real coefficients are derived.

The efficiency of developed approach was illustrated by the example of establishing the mathematical models of diaphragm of finite thickness and resonant enlargement between two circular waveguides. The proposed and known calculation techniques in respect to computing time expenses are compared. To perform comparative calculations, the mathematical model of separate junction between two circular waveguides is derived as a special case of generalized scattering matrices of doubled discontinuities in circular waveguide.

It is shown that proposed technique based on simultaneous solution of the systems of coupled integral equations ensures significant saving of computing time compared to the known approach based on sequential combining of generalized scattering matrices of separate junctions.

The technique proposed can be applied for the optimization of devices on the basis of longitudinally complicated waveguide structures, where the multiple calculations of frequency characteristics at finding of objective functions are required.

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