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# DETECTION OF A HARMONIC SIGNAL AGAINST THE BACKGROUND OF A NONSTATIONARY GAUSSIAN INTERFERENCE WITH COMPLEX SPECTRUM

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**Background.** Modern radar stations for various purposes operate in the conditions of interference created by the imprints of the radar signal from the background surface, from metrological formations (precipitation, clouds, etc.) and artificial radiation sources. Ensuring the operation of the radar in such difficult conditions requires the construction of adaptive signal processing algorithms that have high efficiency and maintain them when changing signal-to-noise situations.

**Objective.** The purpose of the paper is creation of an adaptive algorithm for detecting a harmonic signal against the background of spatially correlated interference and estimating its parameters.

**Methods.** Construction of a two-dimensional autoregressive model of a mixture of correlated spatial noise and harmonic signal and application of the empirical Bayesian approach to the synthesis of an adaptive algorithm for detecting and evaluating signal and noise parameters.

**Results.** A two-dimensional adaptive space-time algorithm for detecting a radar signal reflected from a moving target against the background of a space-correlated interference is synthesized. The analysis of the efficiency of the algorithm by the Monte Carlo method is carried out.

**Conclusions.** It is shown that the empirical Bayesian approach is an effective working methodology in solving the problem of detecting a harmonic signal and estimating its parameters under conditions of interference with a complex frequency spectrum under different conditions of a priori uncertainty of their parameters.

**Keywords:** harmonic signal; detection-measurement; autoregressive models; a priori uncertainty.

## Introduction

Modern radar stations for various purposes operate in the conditions of interference created by the imprints of the radar signal from the background surface, from metrological formations (precipitation, clouds, etc.) and artificial radiation sources. Ensuring radar operation in such complex conditions requires the construction of adaptive signal processing algorithms that have high efficiency and store them in difficult operating conditions when changing signal-to-noise situations both in space and time [1-6].

The problem of research and processing of useful signals [7-14] is related to the randomness of the observed processes. Signal processing theory allows obtaining optimal signal detection algorithms for parametric description of probability distribution (PDF). Autoregressive models were used in a number of works to describe signal-interference mixtures with a complex spectrum [12].

Harmonic signal (HG) is the most widely used in radar technology. The autoregressive model of the harmonic signal and the autoregressive model of the spatially correlated Gaussian noise are used in the proposed work.

In a new way, developers are increasingly paying attention to adaptive and nonparametric approaches to the synthesis of signal processing algorithms [12-14]. They are based on compatible real-time detection and measurement procedures. In this case, the a priori information has only a generalized form, such as the form of the useful signal and the type of interference. To work effectively in these conditions, signal processing channels are equipped with many tools for detection and measuring information parameters in real time in complex signal-interference conditions. In particular, this paper uses algorithms for estimating the frequency of the useful harmonic signal [15],[21],[22].

In this paper, we consider the problem of detecting of a harmonic signal and estimating its frequency under the action of a complex interference mixture of correlated Gaussian noise with an unknown variance and narrowband active interference.

To synthesize the detection-measurement algorithm, we use the empirical Bayesian method. The empirical Bayesian method consists in constructing in the first step a parametric decision test for a situation with known parameters and substitution, in the second step,

in the obtained rule values of unknown parameters of the signal-noise mixture by the estimates.

### Formulation of the problem

The radar viewing area is divided into  $n$  range separation intervals and  $m$  azimuth separation intervals. The size of the separation intervals is determined by the tactical and technical characteristics of the radar: range - duration of the probing pulse; in azimuth - the width of the pattern of antenna. The radar receiver obtains sequentially  $n$  signals reflected from  $n$  elementary of dimensional areas during one period of probing. In the next probing period,  $n$  signals are received. If the radar antenna is in azimuth scan mode, the receiver obtains signals from other dimensional areas. If the antenna does not scan, the prints will belong to the same dimensional areas. In both cases, a two-dimensional discrete field of reflected radar signals  $X$  is formed.

Consider a two-dimensional discrete field (lattice) on one coordinate we will postpone numbers of dimensional separation elements of a radar-  $j = \overline{1, n}$ , on the second - numbers of probing signals -  $i = \overline{1, m}$ . The elements of the field will be radar signals reflected from  $j$ -th - discrete range in  $i$ -th probing period. In high-resolution radar, the samples in each line will be correlated due to the dimensional correlation of the prints from the background surface. The samples in the stacks are packets of signals reflected from the same element of the radar distinction, relating to different probing signals.

In coherent Doppler radars, the reflected signals are fed to a phase detector, which generates signals proportional to the phase differences of the received signal and the coherent local oscillator signal. They may have constant amplitude, in the case of a stationary target, or, due to the Doppler effect, their amplitude may change with a frequency proportional to the radial velocity of the target relative to the radar.

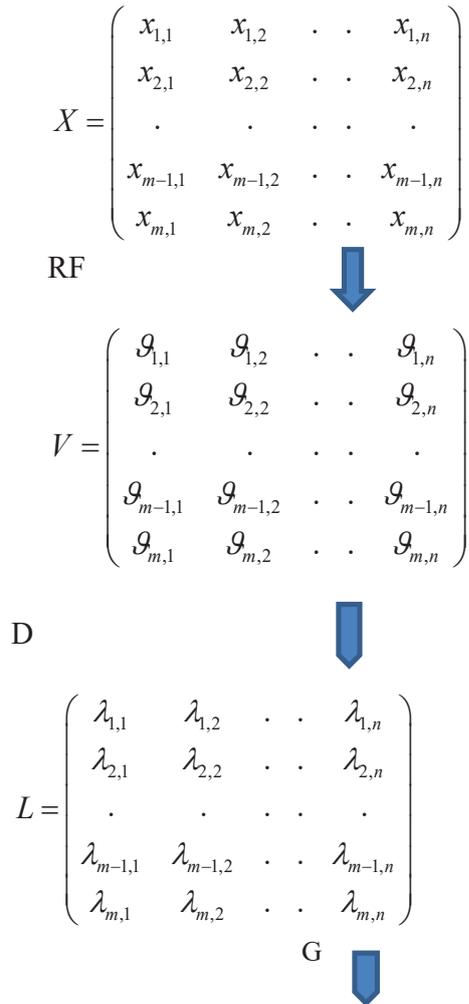
In addition, the radar may be subject to active interference, which will affect all elements of the separation.

The task is to synthesize an algorithm for detecting a signal from a moving target under conditions of such complex interference.

The two-dimensional filtering algorithm consists of four steps.

In the first step, we apply an RF notch filter to suppress the dimensional correlated interference to each row of the matrix  $X$  and obtain a matrix of differences  $V$  that contains noise, uncompensated interference, and a signal reflected from a moving target. The signals from the target can be in several elements of the column of the matrix  $V$ , which stand side by side. They form a signal packet.

We will assume that the columns are statistically independent and the problem of detecting the signal from a moving target will be solved for each column separately. As a result of applying the detection-estimation algorithm  $D$  to the matrix of differences  $V$  we obtain a matrix of values of the test statistics  $L$ . The elements of this matrix are compared with the decision threshold  $V_d$  (operator  $C$ ) and, in case of exceeding the element  $\lambda_{i,j}$  threshold  $V_d$ , decision matrix element  $\gamma_{i,j}$  takes the value 1. Otherwise it takes the value 0.



$$G = \begin{pmatrix} \gamma_{1,1} & \gamma_{1,2} & \cdot & \cdot & \gamma_{1,n} \\ \gamma_{2,1} & \gamma_{2,2} & \cdot & \cdot & \gamma_{2,n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \gamma_{m-1,1} & \gamma_{m-1,2} & \cdot & \cdot & \gamma_{m-1,n} \\ \gamma_{m,1} & \gamma_{m,2} & \cdot & \cdot & \gamma_{m,n} \end{pmatrix}$$

Algorithm for calculating test statistics  $D(X, \Theta) = \lambda_{i,j}$ , according to the Neumann-Pearson lemma, the ratio of likelihood functions also depends on apriority information about the density distributions of the probability of reflections of the matrix X for the cases of the presence of a useful signal and its absence.

In the situation with apriority uncertainty of the distributions of mixtures or uncertainty of the distribution parameters, the empirical Bayesian approach is used, which involves estimating the unknown elements of the vector of signal parameters and noise.

In the case of a decision on the presence of the target, estimates of the parameters of the signal-interference situation are displayed on the monitor and used in further calculations. A generalized block diagram of the detection-evaluation algorithm is shown in Fig.1

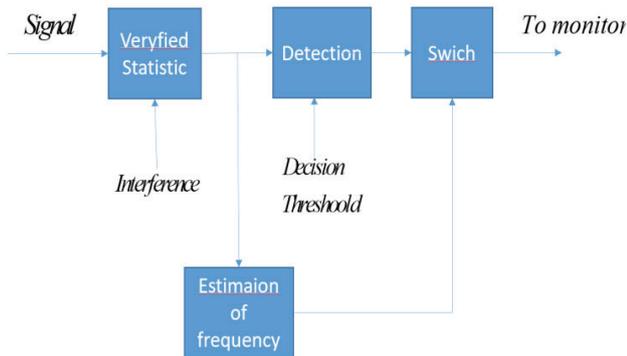


Fig.1. The structure of joint detection-measurement algorithm of HS.

### Synthesis of the adaptive notch filter algorithm of dimensional correlated noise

**Signal and interference models.** In the conditions of a priori data insufficient, the Neumann-Pearson criterion is the most suitable for the synthesis of detection algorithms [14, 16].

According to this criterion, need to set the density of the mixture of signal and noise for cases of presence and absence of signal and calculate the likelihood ratio.

$$L = \frac{f_1(X, \Theta)}{f_0(X, \Theta)} \quad (1)$$

where  $f_1(X, \Theta), f_0(X, \Theta)$  - the distribution density of radar reflections X for the presence of the signal from the target and its absence, respectively,  $\Theta$  - the vector parameters of the mixture of signal and interference.

As a model of correlated noise, we accept the Gaussian autoregressive random process.

$$x_i(t_i) = \sum_{j=1}^k a_j x_{i-j}(t_{i-j}) + \eta_i(t_i) \quad (2)$$

where  $a_1, \dots, a_k$  are autoregressive coefficients,  $\eta_i$  are normal random numbers with variance  $\sigma_3^2$  and zero mean.

The autoregressive k-order process - is such a process, the multivariate probability distribution of which can be represented by the product:

$$f(x_1, \dots, x_n, t_1, \dots, t_n) = f(x_1, t_1) \times \prod_{i=2}^n f(x_i, t_i | x_{i-1}, t_{i-1}, \dots, x_{i-\min(k,i)}, t_{i-\min(k,i)}) \quad (3)$$

where  $f(x_1, t_1)$  - unconditional density of the probabilities,  $f(x_i, t_i | x_{i-1}, t_{i-1}, \dots, x_{i-k}, t_{i-k})$  - conditional probability density of the process values in the moment  $t_i$ , under the condition that in previous  $t_{i-1}, \dots, t_{i-k}$  time points of autoregressive process had values  $x_{i-1}, \dots, x_{i-k}$ .

In the following, we will omit the notation  $t_i$  of time points and will use only the subscript at  $x_i$ . Under the zero hypothesis  $H_0$  the multivariate density of the probability distribution  $x_1, \dots, x_n$  from the Gauss'-autoregressive process is:

$$f(x_1, \dots, x_n | H_0) = \frac{1}{\sqrt{2\pi\sigma_3^2}} e^{-\frac{x_1^2}{2\sigma_3^2}} \times \prod_{i=2}^n \frac{\exp\left\{-\frac{(x_i - \sum_{j=1}^{\min(i,k)} a_j x_{i-j})^2}{2\sigma_{\min(i,k)}^2 (a_1, \dots, a_{\min(i,k)})}\right\}}{\sqrt{2\pi\sigma_{\min(i,k)}^2 (a_1, \dots, a_{\min(i,k)})}} \quad (4)$$

where  $\sigma_{\min(i,k)}^2 (a_1, \dots, a_{\min(i,k)})$  - is variance of residuals

$$\mathcal{G}_i = x_i - \sum_{j=1}^{\min(i,k)} a_j x_{i-j} \text{ on the } i\text{-th time moment, that}$$

depends on the autoregressive coefficient  $a_1, \dots, a_{\min(i,k)}$ .

Sequence of residuals (matrix(V))

$$\mathcal{G}_i = x_i - \sum_{j=1}^{\min(i,k)} a_j x_{i-j}, \quad i = \overline{1, n}$$

can be considered as the output of the notch filter, which compensates for the spatial correlated interference. At the output of such a filter a sequence of random variables with a symmetric  $2k + 1$  diagonal correlation matrix is formed. At the stage of synthesis, we will consider it diagonal and the values of the sequence - independent.

### Synthesis of the algorithm for estimating of the spatial correlated radar reflections parameters

To synthesize an adaptive notch filter, it is necessary to estimate the values of the autoregression coefficients  $a_1, \dots, a_k$ . According to the least squares method we must find values of  $a_1, \dots, a_k$  that provides minimum of function (5).

$$I = \sum_{i=1}^n (x_i - \sum_{j=1}^k a_j x_{i-j})^2 \rightarrow \min \quad (5)$$

Likelihood equations we obtain due calculation of partial derivatives of (5) on autoregressive coefficients  $a_1, \dots, a_k$ :

$$\begin{cases} a_1 \sum_{i=k+1}^n x_{i-1} x_{i-1} + \dots + a_k \sum_{i=k+1}^n x_{i-k} x_{i-1} = \sum_{i=k+1}^n x_i x_{i-1}; \\ \dots \\ a_1 \sum_{i=k+1}^n x_{i-1} x_{i-k} + \dots + a_k \sum_{i=k+1}^n x_{i-k} x_{i-k} = \sum_{i=k+1}^n x_i x_{i-k}; \end{cases} \quad (6)$$

The matrix form of (6):

$$\begin{pmatrix} \sum_{i=k+1}^n x_{i-1} x_{i-1} & \dots & \sum_{i=k+1}^n x_{i-k} x_{i-1} \\ \dots & \dots & \dots \\ \sum_{i=k+1}^n x_{i-1} x_{i-k} & \dots & \sum_{i=k+1}^n x_{i-k} x_{i-k} \end{pmatrix} \begin{pmatrix} a_1 \\ \dots \\ a_k \end{pmatrix} = \begin{pmatrix} \sum_{i=k+1}^n x_i x_{i-1} \\ \dots \\ \sum_{i=k+1}^n x_i x_{i-k} \end{pmatrix} \quad (7)$$

where (7), we can represent as

$$\mathbf{CA} = \mathbf{B}.$$

Estimates of parameters of an autoregressive model of interference are calculated by (8).

$$\mathbf{A} = \mathbf{C}^{-1} \mathbf{B} \quad (8)$$

Estimations of autoregressive parameters (8)

$$a_1^* \dots a_k^*$$

are used for building of the rejection filter to reduce spatial correlated interference and to calculate matrix V

$$\mathcal{G}_{ij} = (x_{i,j} - \sum_{q=1}^k a_q^* x_{i,j-q}), \quad i = \overline{1, m}; \quad j = \overline{1, n}.$$

### Synthesis of Doppler frequency detection-estimation algorithm

Consider the vector of values in the j-th column of matrix V (9).

$$V_j = \{\mathcal{G}_{ij}^s\}, \quad i = \overline{1, m} \quad (9)$$

Each element of the vector is the sum of the signal  $S_{ij}$  and uncompensated remnants of the interference  $\mathcal{G}_{ij}$  (10).

$$u_{ij} = S_{ij} + \mathcal{G}_{ij}, \quad i = \overline{1, m} \quad (10)$$

In the following consideration, the index j, for simplicity, will not be used, meaning that the calculations are performed with elements of one column.

We consider a useful signal  $\mathbf{S} = \{s_i\}$  of known form, but with unknown parameters. It's distorted by residuals  $\{\mathcal{G}_i\}$  of unknown power. On the base of sample (1), two hypotheses are tested. According to the hypothesis  $H_0$  – no signal – PDF  $L^{(0)}$  are determined only by the residuals variance  $\sigma_g^2$ . If there is a signal – a hypothesis  $H_1$  – a multidimensional mixture PDF  $L^{(1)}$  depends on its statistical characteristics. Synthesis of detection algorithm is carried out by likelihood ratio (LR) (11).

$$\mathcal{L}(\mathbf{X}) = L^{(1)}(\mathbf{X})/L^{(0)}(\mathbf{X}) > V_d \quad (11)$$

where  $V_d$  – a signal detection threshold that provides the required false alarm rate (FAR).

### Application of the empirical Bayesian approach

Within the outlined problem statement, the synthesis of algorithms (detectors) of the harmonic signal is solved within the framework of the empirical Bayesian method [14] by the criterion of "maximum likelihood detection" (12).

$$\mathcal{L}_M(\mathbf{X} | \hat{\theta}_s, \hat{\theta}_g) = \max_{\{\hat{\theta}_s\} \{\hat{\theta}_g\}} L^{(1)}(\mathbf{X}, \hat{\theta}_s, \hat{\theta}_g) / \max_{\{\hat{\theta}_g\}} L^{(0)}(\mathbf{X}, \hat{\theta}_s, \hat{\theta}_g) > V_d \quad (12)$$

where  $\hat{\theta}_s, \hat{\theta}_g$  – sets of estimates of unknown signal parameters and interference, respectively.

In this work we use a typical autoregressive (AR) model of sinewave signal [15]. This model allows us to reduce the problem of adaptive signal detection algorithm synthesis. A sinewave signal (10) can be also represented (13).

$$s_i = \alpha \cdot s_{i-1} - s_{i-2}, \quad i = \overline{3, n} \quad (13)$$

Where the AR parameter  $\alpha$  is related with phase shift between neighbouring samples  $\gamma$  as (14):

$$\alpha = 2 \cos(\gamma) \quad (14)$$

Normalized frequency of harmonic signal is connected with  $\gamma$  by the following relation (15).

$$\omega = \gamma / \tau \quad (15)$$

During measurements, the condition  $0 < \gamma < \pi$  of the Nyquist criterion must be assured to avoid uncertainty between parameters  $\alpha$  and  $\gamma$ . One of the features of the AR-model is implicit dependence on the amplitude and initial phase. They are defined by two first samples  $s_1, s_2$  of the sequence. Hence, only parameter  $\hat{\alpha}$  left for estimation.

In the signal discrete (column) we observe the sum of the remnants of the spatial noise and the signal (16).

$$u_i = s_i + \mathcal{G}_i, \quad i = \overline{3, n} \quad (16)$$

Using two substitutions  $s_{i-1} = u_{i-1} - \mathcal{G}_{i-1}$  and  $s_{i-2} = u_{i-2} - \mathcal{G}_{i-2}$ , we convert the last formula to the recurrent equation of the AR-model of the moving average (17).

$$u_i = \alpha u_{i-1} - u_{i-2} + (\mathcal{G}_i - \alpha \mathcal{G}_{i-1} + \mathcal{G}_{i-2}) = \alpha u_{i-1} - u_{i-2} + \zeta_i(\alpha), \quad i = \overline{3, n}, \quad (17)$$

where the variance of the generating noise process  $\zeta_i(\alpha)$  is

$$\sigma_\zeta^2 = \sigma_g^2 (2 + \alpha^2).$$

Expression (17) shows that in the presence of a harmonic signal corresponding to the autoregressive model (13), the observed differences (18)

$$\zeta_i(\alpha) = (\mathcal{G}_i - \alpha \mathcal{G}_{i-1} + \mathcal{G}_{i-2}), \quad i = \overline{3, n} \quad (18)$$

have zero mean and variance

$$\sigma_\zeta^2 = \sigma_g^2 (2 + \alpha^2).$$

The covariance matrix of the sequence

$$\zeta_i(\alpha), \quad i = \overline{3, n}$$

has the form (19).

$$R = \sigma_g^2 \begin{pmatrix} (2 + \alpha^2) & -2\alpha & 1 & 0 & \dots & 0 \\ -2\alpha & (2 + \alpha^2) & -2\alpha & 1 & \dots & 0 \\ 1 & -2\alpha & (2 + \alpha^2) & -2\alpha & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & 1 & -2\alpha & (2 + \alpha^2) \end{pmatrix} \quad (19)$$

We use a quasi-optimal algorithm for estimating the frequency of HS [12], which is synthesized on the

maximum likelihood method using the AR-model of HS under conditions of additive uncorrelated Gaussian noise.

The steps sequence of the frequency estimation procedure is as follows. Based on the values of the input sample, the auxiliary coefficient is calculated (20).

$$B(\mathbf{X}) = \frac{\sum_{i=2}^{n-1} [(u_{i+1} + u_{i-1})^2 - 2u_i^2]}{\left[ 2 \sum_{i=1}^{n-2} (u_{i+1}u_i + u_i u_{i-1}) \right]} \quad (20)$$

There is one adequate root (parameter estimate)  $\hat{\alpha}$  of the quadratic equation (21).

$$\alpha^2 - 2B\alpha - 2 = 0 \quad (21)$$

Then the normalized frequency estimate is obtained (22).

$$\hat{\gamma} = \arccos(\hat{\alpha}/2) \quad (22)$$

### Synthesis of an adaptive algorithm for detecting an AR model of a harmonic signal

Based on the [22] we consider the residues between the actual values  $u_i$  and the predicted values  $u_n = \alpha u_{n-1} - u_{n-2}$  as follows (23).

$$z_i = u_i - \alpha u_{i-1} + u_{i-2} \quad (23)$$

The jointed distribution of residues has form (24):

$$L_{AR}^{(1)} = [2\pi]^{-\frac{n-2}{2}} |R|^{-\frac{1}{2}} \exp\left(-\frac{1}{2} \sum_{i=3}^n \sum_{k=3}^n R_{ik}^{-1} z_i z_k\right) \quad (24)$$

where  $R_{ik}^{-1}$  are elements of the inverse covariance matrix.

Signal detection algorithm is determined according to (12) as:

$$\mathcal{L}_M(Z|R(\alpha)) = \frac{\max_{\{\alpha\}, \{\sigma_g\}} L_{AR}^{(1)}(Z, R(\alpha) | H_1)}{\max_{\{\sigma_g\}} L^{(0)}(Z, \sigma_g | H_0)} > V_d, \quad (25)$$

where  $L^{(0)}(Z, \sigma | H_0) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi\sigma_g}} \exp(-z_i^2/2\sigma_g^2)$ .

Based on the assumptions of the residues independence (16) ( $R = \sigma_g^2(2 + \alpha^2)I$ ) and the type of

variance (16), the likelihood function of residues for the AR-model are defined as (26).

$$L_{AR}^{(1)} = \frac{1}{[2\pi\sigma_g^2(2 + \alpha^2)]^{-\frac{n-2}{2}}} \exp\left(-\frac{\sum_{i=3}^n (u_i - \alpha u_{i-1} + u_{i-2})^2}{2\sigma_g^2(2 + \alpha^2)}\right) \quad (26)$$

Its feature, in contrast to LF (3), is that it has  $n-2$  multipliers. Therefore, for the formation of LR it is advisable in LF (3) to discard two counts, we have (27).

$$\mathcal{L}_A = (2 + \hat{\alpha}^2)^{-\frac{n-2}{2}} \exp\left(-\frac{\sum_{i=1}^{n-2} u_i^2 - \sum_{i=3}^n (u_i - \hat{\alpha}u_{i-1} + u_{i-2})^2}{2\hat{\sigma}_g^2(2 + \hat{\alpha}^2)}\right) \quad (27)$$

If you neglect the first multiplier, then after taking the logarithm and transformations you can get the modified verified statistic (28).

$$\mathcal{T}_{Am, \hat{\gamma}, \hat{\sigma}} = \frac{1}{2\hat{\sigma}_g^2} \left[ \sum_{i=1}^{n-2} x_i^2 - \frac{\sum_{i=3}^n (u_i - \hat{\alpha}u_{i-1} + u_{i-2})^2}{(2 + \hat{\alpha}^2)} \right] > \tilde{V}_d, \quad (28)$$

where the parameters are replaced by their estimates.

Estimation of  $\hat{\alpha}$  can be obtained by some of numerical method. Due to complex multi extremal form of likelihood function (24) one can use random search genetic method.

The structure of joint detection algorithm and frequency measurement of radar HS is shown in Fig. 1.

### Analysis of HS detection-measurement algorithm

The purpose of the research is an analysis of the efficiency of the synthesized adaptive detection algorithms.

Statistical modelling by the Monte-Carlo method is carried out. We limit ourselves to the following typical research conditions: the FAR level is  $F=0.01$ , for which 10000 attempts (simulations) are enough.

Consider that a packet of size  $N = 16, 32, 64, 128$  contains different number of signal periods, that provides different phase shifts between adjacent signal samples; the signal to noise range (SNR) change is -20...15 dB, and the model noise variance is always 1; the number of detection attempts for each signal power is selected 1000.

The frequency estimating algorithm is to calculate the initial value of the autoregressive coefficient  $\alpha$  by formula (15) and then search for a more accurate value that provides the maximum of the likelihood function (19). The maximum can be searched by one of the numerical methods, for example, the method of random search in the vicinity of the initial value, and so on.

Detection characteristics and RMS of frequency estimations vs SNR for different sample volumes and different signal frequencies are shown in Fig. 2, 3.

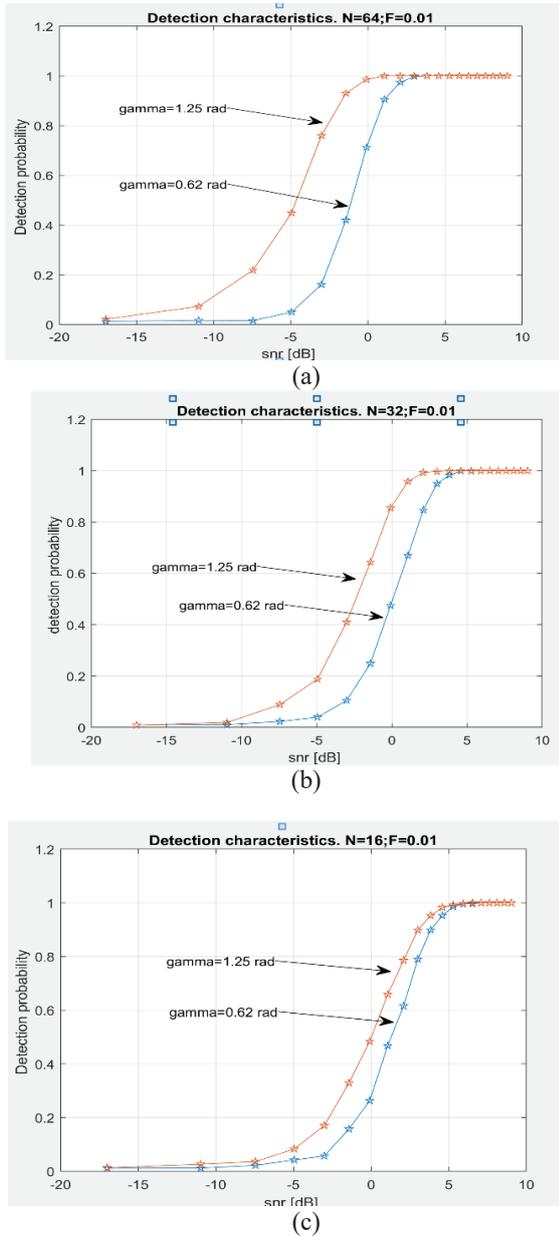


Fig.2. Detection characteristics of algorithm (20) by different signal frequencies  $\gamma=\{0.62;1.252\}$  [rad].

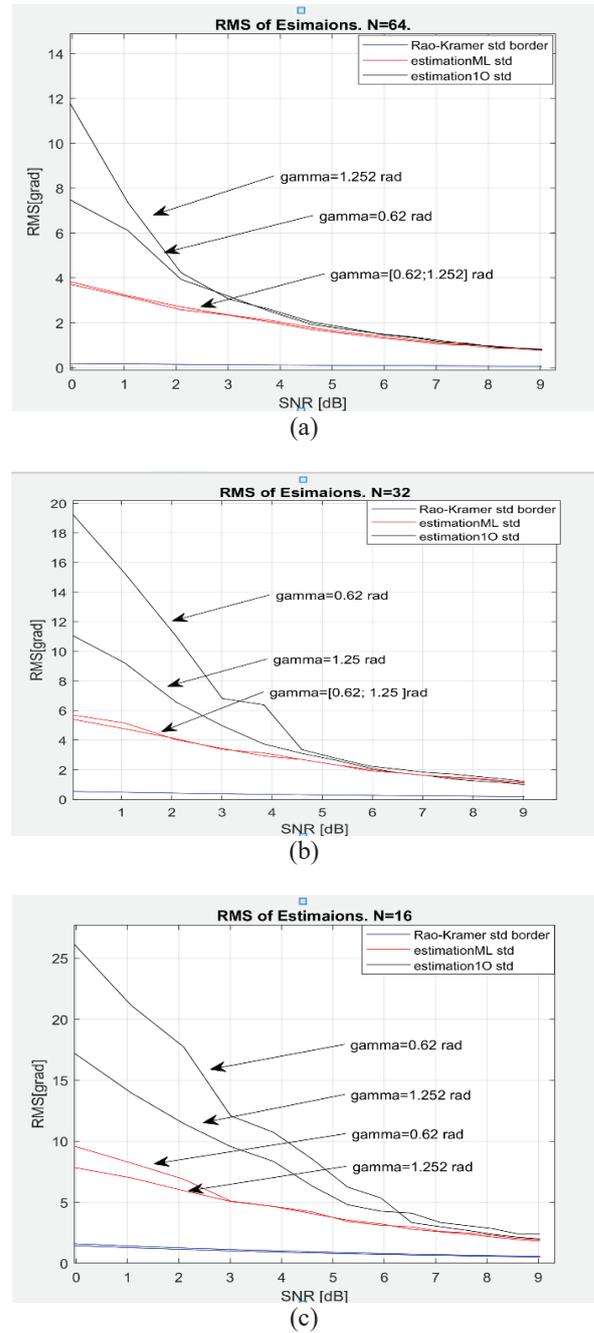
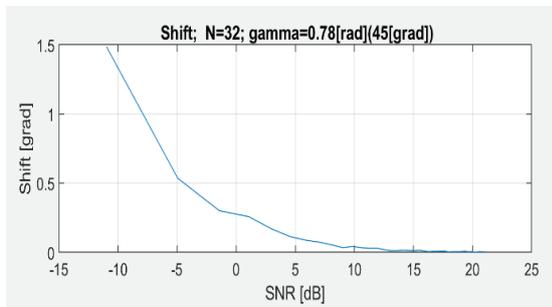


Fig.3. RMS of frequency estimations vs SNR. a)-N=64; b) - N=32; c)-N=16.

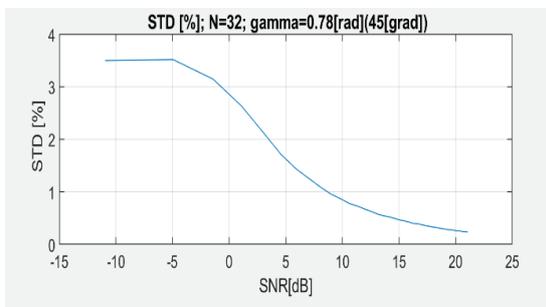
Fig. 2 shows the characteristics of the harmonic signal detection using the algorithm (20). When the SNR > 2.5 dB, the probability of detection reaches the level of 0.5 with the probability of the false alarm of 0.01. Thus, the synthesized adaptive algorithm provides a high quality joint detection of the HS and evaluation of its frequency.

Fig. 3 presents accuracy of the algorithms (20) using two HS frequency estimations: first – the initial estimation according to the root of equation (14), it's marked 'estimation1O', and second – the tuned estimation, according to Maximum Likelihood function (18) method, it's marked 'estimation ML'. Accuracy was calculated for different signal frequencies  $\gamma = \{0.62; 1.252\}$  [rad] according to  $\gamma$  marked on the figures 'gamma'. Blue bottom line is Rao-Kramer border of estimation standard deviation (RMS).

The results of the study of the accuracy of estimating the frequency of the harmonic signal by the method of maximum likelihood for the autoregressive model (13) are shown in Fig.4. The dependence of the evaluation offset on the signal-to-noise ratio (SNR) is shown in Fig 4 a). Fig. 4 b) shows the dependence of the root mean square error of the estimate on the signal-to-noise ratio in dB.

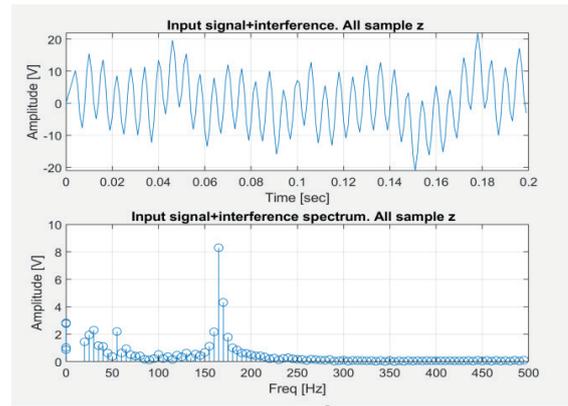


(a)

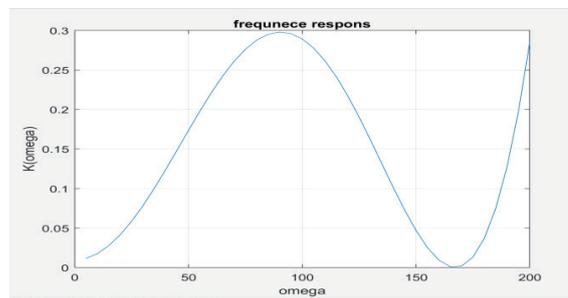


(b)

Fig.4. Shift (a) and STD (b) of frequency estimations.



(a)



(b)

Fig.5. Input sinal+interference mixture realization (a) and its FFT. SNR=-1dB.

Frequency respons of adaptive notch filter (b).

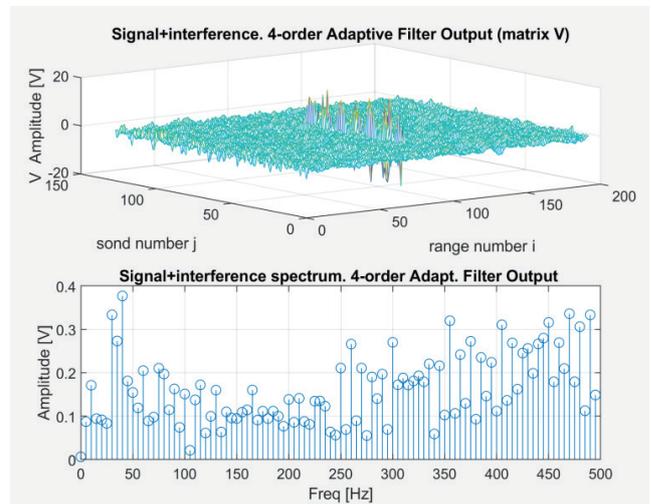
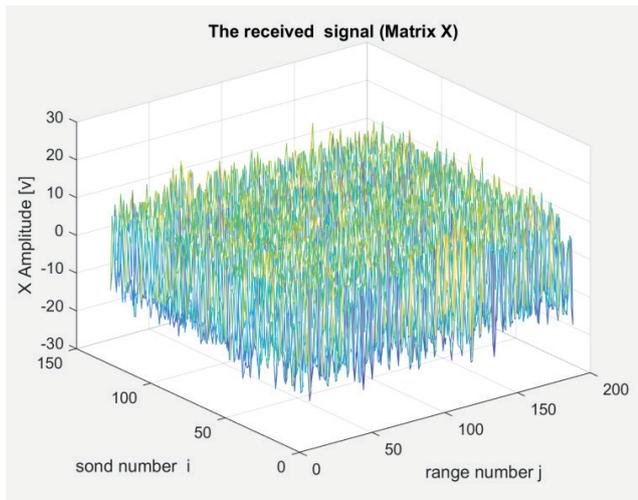
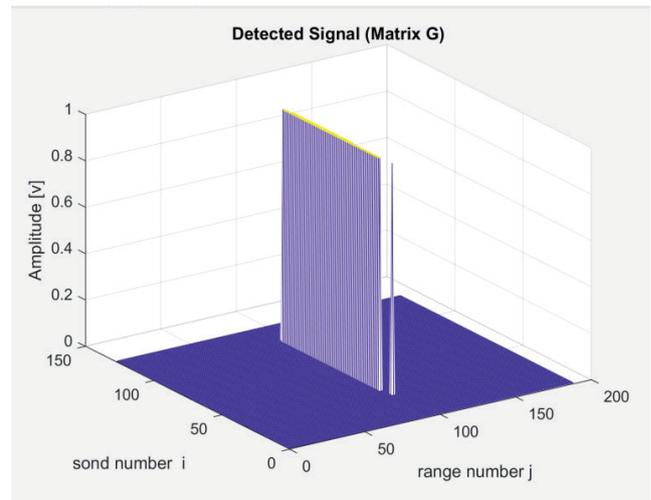


Fig.6. Shows the field of signals after processing by the 4th order notch filter and the Fourier transform module of the 100-thsond.

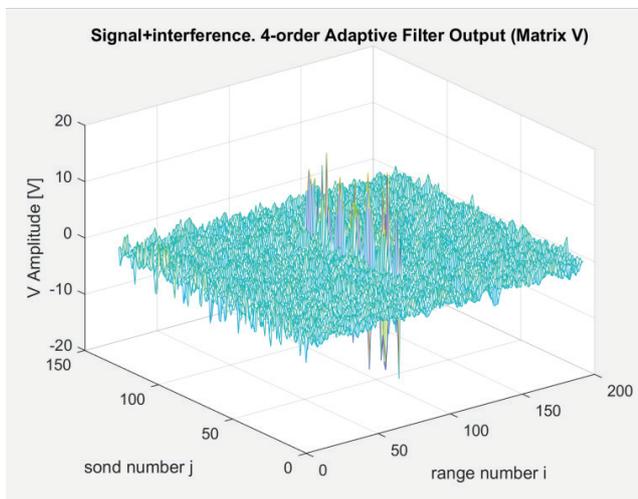
As a result of filtration, the active narrowband interference is almost completely suppressed and the low-frequency correlated interference is significantly suppressed.



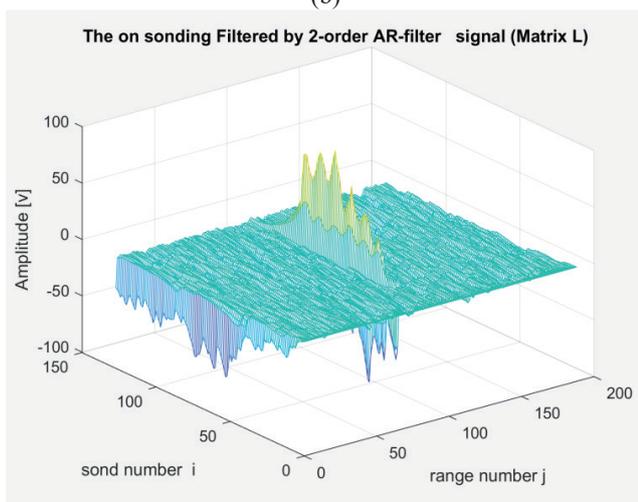
(a)



(d)



(b)



(c)

Fig.7. Simulated fields of radar prints at different stages of processing:

a) shows the field of the mixture of signal, correlated noise interference and narrowband interference. SNR = -1dB. The signal is not visible;

b) shows the field  $V$  after processing the field  $X$  with a 4-order notch filter. Correlated noise and narrowband interference is significantly suppressed. The hundredth element of the residuals shows a signal from the target, which occupies several probes - from the 30th to the 94th (packet size of 64 radar pulses);

c) shows the result of applying the algorithm for calculating test statistics (28) and exponential smoothing along the probes to the field  $L$ . The algorithm reduced background noise and increased the signal-to-noise ratio to 14 dB;

d) shows the result of applying the quantization operator  $C$  to the field  $L$ . The standard deviation of the Doppler frequency estimate was 3%. The shift was 0.2rad.

## Conclusion

It is proved that the empirical Bayesian approach is an effective working methodology in solving the joint problem of a HS detecting in the mixture with complex interference and estimation of signal frequency under different conditions of a priori uncertainty of harmonic signal and interference parameters.

New adaptive notch filters based on autoregressive random processes mathematical model provide effective reduction of spatial correlated interference.

New adaptive algorithms on the base of AR-model of harmonic signal are synthesized. These algorithms are invariant to the amplitude and phase of the signal and require only an estimate of its frequency. Reducing the number of estimated parameters and high effectivity of AR algorithms allows us to recommend their use to increase the efficiency of radio systems.

The synthesized adaptive algorithm provides a high quality joint detection of the harmonic signal and evaluation of its frequency in the complex interference condition.

The computer modelling shows that in sample 64, the algorithm estimates the parameters of the harmonic signal with an accuracy of several percent at fairly low ratio values of signal energy to interference energy ( $\text{SNR} > 0 \text{ dB}$ ).

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**Прокопенко И. Г., Омельчук И. П., Дмитрук А. Ю., Петрова Ю. В.**

**Виявлення гармонічного сигналу на тлі нестационарної гауссової завади зі складним спектром.**

**Проблематика.** Сучасні радіолокаційні станції різного призначення працюють в умовах перешкод, створюваних відбитками радіолокаційного сигналу від фонові поверхні, від метеорологічних утворень (опади, хмари тощо) та штучних джерел випромінювання. Забезпечення роботи РЛС у таких складних умовах потребує побудови адаптивних алгоритмів обробки сигналів, що володіють високими показниками ефективності та зберігають їх при зміні ситуацій сигнал/шум.

**Мета досліджень.** Створення адаптивного алгоритму виявлення гармонічного сигналу на тлі просторової корельованої завади та оцінки його параметрів.

**Методика реалізації.** Побудова двовимірної авторегресійної моделі суміші просторової корельованої завади та гармонічного сигналу та застосування емпіричного байєсівського підходу до синтезу адаптивного алгоритму виявлення та оцінки параметрів сигналу та завади.

**Результати досліджень.** Синтезовано двомірний адаптивний просторово-часовий алгоритм виявлення радіолокаційного сигналу, відбитого від цілі, що рухається, на тлі корельованої по дальності просторової завади. Проведено аналіз ефективності алгоритму методом Монте – Карло.

**Висновки.** Показано, що емпіричний байєсівський підхід є ефективною робочою методологією у вирішенні задачі виявлення гармонічного сигналу та оцінки його параметрів в умовах завад зі складним частотним спектром за різних умов апіорної невизначеності їх параметрів.

**Ключові слова:** гармонічний сигнал; виявлення-вимірювання; авторегресійні моделі; апіорна невизначеність.

*Прокопенко И. Г., Омельчук И. П., Дмитрук А. Ю., Петрова Ю. В.*

**Выявление гармонического сигнала на фоне нестационарной гауссовской помехи со сложным спектром.**

**Проблематика.** Современные радиолокационные станции разного назначения работают в условиях действия помех, создаваемых отражениями радиолокационного сигнала от фоновой поверхности, от метеорологических образований (осадки, облака и т.п.) и искусственными источниками излучения. Обеспечение работы РЛС в таких сложных условиях требует построения адаптивных алгоритмов обработки сигналов, обладающих высокими показателями эффективности и сохраняющими их при изменении сигнально-помеховых ситуаций.

**Цель исследований.** Создание адаптивного алгоритма обнаружения гармонического сигнала на фоне пространственно коррелированной помехи и оценки его параметров.

**Методика реализации.** Построение двумерной авторегрессионной модели смеси коррелированной пространственной помехи и гармонического сигнала и применения эмпирического байесовского подхода к синтезу адаптивного алгоритма обнаружения и оценки параметров сигнала и помех.

**Результаты исследований.** Синтезирован двухмерный адаптивный пространственно-временный алгоритм обнаружения радиолокационного сигнала, отраженного от движущейся цели, на фоне коррелированной по дальности-пространственной помехи. Проведен анализ эффективности алгоритма методом Монте – Карло.

**Выводы.** Показано, что эмпирический байесовский подход является эффективной рабочей методологией в решении задачи обнаружения гармонического сигнала и оценки его параметров в условиях помех со сложным частотным спектром при различных условиях апіорной неопределенности их параметров.

**Ключевые слова:** гармонический сигнал; обнаружение-измерение; авторегрессионные модели; апіорная неопределенность.