

LTE/MVNO NETWORKS STRUCTURE OPTIMIZATION BASED ON TENSOR DECOMPOSITION

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The usage of tensor methods on the decomposition basis is offered for the tasks solution of structure optimization for LTE/MVNO networks mobile communication. The choice problem of optimum topology of e-Node B base stations connections in the radio access of E-UTRAN/LTE network was solved. The assessment problem of QoS quality characteristics of complex LTE/MVNO network architecture was solved.

Introduction

The 3GPP concept connects the further technological development of mobile communication networks with the LTE technology introduction (*Long Term Evolution*) (Rel. 8-10), it will allow increasing considerably radio-interface capacity, to improve network architecture and to ensure the wide nomenclature provision of high-speed services. According to [1,2] the most appropriate for the LTE technology introduction, is the usage of the virtual mobile network – MVNO (*Mobile Virtual Network Operator*). The LTE/MVNO network architecture is based on the usage sharing principles of existing PLMN (*Public Land Mobile Network*) infrastructures of different operators through the creation of the general for several operators virtual MVNO network which will be "overlaid" on the existing infrastructure of operators and will be able to use the accessible frequency range. LTE/MVNO network implementation feature is a new network creation of E-UTRAN (*Evolved UMTS Terrestrial Radio Access Network*) radioaccess which will be shared by several operators (fig. 1) [1,2].

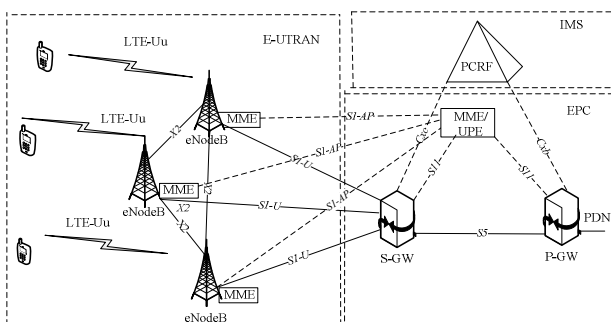


Fig. 1. LTE/MVNO network architecture

The research tasks complexity of the LTE/MVNO networks is that the solution requires considering not only the structural features of existing networks construction but also the technological features of their construction. The certain sequence of tasks solution is carried out at the network design stage: the reasonable choice of network architecture, the definition of quantity and functional characteristics of hardware and software network facilities and network interconnection methods. The tasks solution of structure optimization of mobile communication networks is based on the performance of the criterion of guaranteed parameters support in service quality *QoS* (*Quality of Service*). Let's define a number of design tasks which demand the solution at the stage of the LTE/MVNO networks implementation. First, the E-UTRAN network architecture excludes the usage of base stations controllers assuming the possibility of e-NodeB base stations connection for the purpose of the loading closing in the E-UTRAN radio access network. Therefore, it is an essential task solution of optimum topology choice of e-NodeB connections in the E-UTRAN network taking into account the traffic transmission directions and providing guaranteed parameters of quality of service *QoS*. Secondly it is required to solve the task of quality characteristics assessment of complex architecture and large LTE/MVNO network dimension as a whole as well as its separate objects, for example, S-GW/P-GW network gateway. Such task is rather intricate and difficult, and finding solutions in an analytical form that will consider complexity of topology and functional properties of network objects and is extremely difficult.

Therefore the authors of their works [3-6] offered the use of tensor methods for the LTE/MVNO network research of big dimension and difficult architecture.

They allow carrying out an assessment of structural characteristics and functional properties of the considered network at the same time, to analyze the network state of the certain period of time, taking into account topology and features of network equipment technological implementation, and also consider features of used technologies and protocols. Besides, it is reasonable to apply the tensor decomposition method [3-6] which allows carrying out the *LTE/MVNO* network architecture division into final amount of objects, when each of them possesses system properties with the subsequent independent research of these objects and transferring the results for the system as a whole.

The purpose of this article is the tasks solution of structure optimization of *LTE/MVNO* mobile communication network on the tensor decomposition basis.

1. The task solution of the optimum topology choice of e-NodeB connection in the E-UTRAN/LTE radio access network on the tensor decomposition basis

Let's consider the task solution of the optimum topology choice of *e-NodeB* connection in the *E-UTRAN/LTE* radio access network taking into account the traffic transmission directions and loading locking in the radio access network providing the guaranteed parameters of *QoS* service quality [5]. Considering that the *E-UTRAN* network is shared by several *LTE/MVNO* operators, it is possible to state that it has complex topology and construction architecture. Therefore for the task solution we will use tensor decomposition. For this purpose, according to [5], the *E-UTRAN* network decomposition is carried out by its division into the separate subnets providing the use of a boundary tensor research method will allow to receive results of quality characteristics for separate objects of each subnet and to use them for the network as a whole. We will investigate each subnet separately in case of *E-UTRAN* network decomposition on separate subnets taking into account the introduction for consideration the "boundary" branches concepts which appeared on subnets crossing in case of network division into subnets and boundary contours which include boundary branches.

To define the principles of *e-NodeB* connections for the purpose of traffic closing locking in *E-UTRAN/LTE* providing parameters of quality *QoS*, we will find quality characteristics values of all functional objects of the network such as: duration of packages delays in the definite contours of a network T_r , capacities of routes L_v and contours of a network L_r , the length of package queues in contours H_r and network routes H_v . The values of the length of initial package queue for each H_{vk}^+

network route and the durations of packages delay in the routes of T_{vk}^+ network are known.

Let's consider architecture of the *E-UTRAN/LTE* network fragment (fig. 2). It is presented in the form of the directed graph $G(N, V)$ where $N = \{N_j, j=1, m\}$ the set of its tops are the base *e-NodeB* stations, $m=10$, and $V = \{v_i, i=1, n\}$ is a set of the directed edges presented by the networks routes, $n=20$, which connect network objects and provide a package traffic transmission.

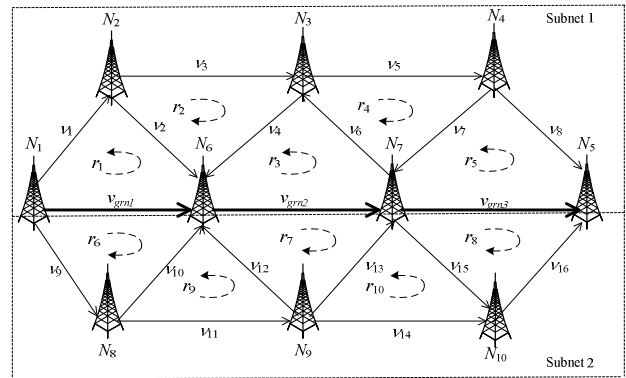


Fig.2. *E-UTRAN/LTE* network fragment

Let's randomly set the closed contours $r_l, l=1, 10$ and the directions of a package traffic transmission for the network directed graph. Let's divide architecture of the *E-UTRAN/LTE* network fragment into two subnets, then boundary contours will be contours $r_1, r_3, r_5, r_6, r_7, r_8$, and $v_{gm1}, v_{gm2}, v_{gm3}$ will be boundary branches.

According to [5-8], let's use Little's formula as the invariant functional equation. For the systems of coordinates (SC) of branches and contours it has following form:

$$H_v = T_v L_v, \quad H_r = T_r L_r, \quad (1)$$

where H_v, H_r – are covariant tensors of package queue lengths in the SC of branches and independent contours of a network, thus T_v, T_r – are contra variant tensors of packages transfer delays in the SC of branches and independent contours, thus L_v, L_r – are capacity covariant tensors in the SC of branches and independent networks contours.

Transformation formulae between the SC have a form [5-7]:

$$H_r = B_v^t H_v^+, \quad T_r = B_v^t T_v B_v, \quad L_v = L_r B_v, \quad (2)$$

where B_v – is a basic matrix of transformation between the SC, H_v^+ – is the initial length of package queue.

To solve this problem, we write basic transformation matrix B_v between coordinate systems (CS) for each subnet according to (Fig.2).

$$B_{v1} = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}, \quad B_{v2} = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & -1 \\ 0 & 0 & -1 & 0 & 0 \end{pmatrix}. \quad (3)$$

For calculation of each subnet let's use a topological matrix $B_s^{(k)}$, which defines a condition of each subnet after decomposition, where $s=1,2,3$ – are states characterizing the relations between network objects, k – is a subnet number. Each s states defines: 1 – relations between contours and boundary contours; 2 – relations between boundary contours and boundary branches; 3 – relations between boundary branches of a subnet and all boundary branches.

We write the topological matrix for each subnet:

$$B_1^{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad B_1^{(2)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad B_2^{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$B_2^{(2)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad B_3^{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad B_3^{(2)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (4)$$

We will find the $B_s^{(k)}$ matrix, defining the state of each k -th subnet according to the obtained topological matrices that determine the state of each subnet after decomposition (4):

$$B^{(1)} = B_1^{(1)} \cdot B_2^{(1)} \cdot B_3^{(1)}, \quad (5)$$

$$B^{(2)} = B_1^{(2)} \cdot B_2^{(2)} \cdot B_3^{(2)}, \quad (6)$$

where the matrices $B^{(1)}$ and $B^{(2)}$ characterize the relation between the contours and the boundary contours of each subnet, respectively, and all network boundary branches.

According to (5) and (6) we obtain the matrix B , which has the form [5]:

$$B = \begin{pmatrix} B^{(1)} \\ B^{(2)} \end{pmatrix}. \quad (7)$$

According to expression (2), we define the tensor of package delay T_{rk} in the contours of k -th subnets [5]:

$$T_{rk} = B_{vk}^t T_{vk}^+ B_{vk}, \quad (8)$$

where T_{rk} – is twice a contra variant tensor of packages delays in the SC of independent contours for each k -th subnet, B_{vk} – is a basic transformation matrix of

each k -th subnet of type (3), T_{vk}^+ – is twice a contra variant tensor of packages delays in the SC of network branches of the k -th subnet, given in the initial data.

According to the expression (2), basic transformation matrices B_v (3), and given values of durations of package queue of H_{vk}^+ sub-networks, let we find the value of tensor package queue in contours H_{rk} of each k -th sub-networks (5):

Let's find a tensor value of package queue in contours H_{rk} of each k -th subnet:

$$H_{rk} = B_{vk}^t H_{vk}^+, \quad (9)$$

where H_{rk} – is a length tensor of package queue in the k -th subnet contours, H_{vk}^+ – is a length tensor of initial package queue in routes of each k -th subnet.

Let's define a tensor of traffic capacities L_{rk} in contours of each k -th subnet according to (1) [5]:

$$L_{rk} = [T_{rk}]^{-1} H_{rk}. \quad (10)$$

where $[T_r]^{-1}$ – inverse matrix to the matrix package delay in the SC independent network contours.

Let's define a delays tensor T_{vgrmk} for boundary branches of k -th subnet taking into account temporary delays T_{vgrm} , which introduce boundary branches [5]:

$$T_{vgrmk} = B_s^{(k)t} T_{rk} B_s^{(k)} + T_{vgrm}, \quad (11)$$

where $B_s^{(k)}$ – is a topological matrix for each k -th subnet which characterizes the relation between contours of a subnet and boundary branches of (5) and (6) types, T_{vgrm} – is a temporary delays tensor of boundary branches for the k -th subnet, $T_{vgrm} = 0$.

Then the tensor of packages delays in the SC of branches T_v can be defined with a formula [5] taking into account delays in boundary branches, is define by the sum of package delay in all subnets:

$$T_v = \sum_{i=1}^k T_{vi}, \quad (12)$$

where, $i = \overline{1, k}$, k – is a quantity of subnets.

Let's find capacities L_{vk} for boundary branches of each k -th subnet according to the expression (2), using as the transformation matrix the received topological matrixes under the decomposition of $B_s^{(k)}$ network of (5) and (6) types, according to [5]:

$$L_{vk} = B_s^{(k)t} L_{rk}. \quad (13)$$

Then the capacity tensor L_v of initial network routes is defined by the sum of capacity routs of each k -th subnet [5]:

$$L_v = \sum_{i=1}^k L_{vi}, \quad (14)$$

where, $i = \overline{1, k}$, k – is a quantity of subnets.

Using the expression (1) and received tensor values of package delays in SC of T_v initial network (12) and capacity tensor L_v of initial network routes (14), we find the value of package queue lengths of the H_v initial networks.

Then, the length of package queue in contours of initial network H_r is defined according to the expression (2) by topological matrix B of (7) type, received under the decomposition of the network [5]:

$$H_r = B H_v. \quad (15)$$

Let's define a capacity gain ΔL_{rk} in the network contours by subnets integration in an initial network by the formula (1) [5]:

$$\Delta L_{rk} = \Delta [T_{rk}]^{-1} \Delta H_{rk}, \quad (16)$$

where ΔL_{rk} – is a define a capacity gain in network contours, ΔT_{rk} – is a define of temporary delays in network contours, ΔH_{rk} – is a define length of package queue in network contours due to subnet integration in an initial networks, k – is a quantity of subnets.

Then it is possible to determine contours capacities L_k of the initial network [5]:

$$L_k = L_{rk} + \Delta L_{rk}. \quad (17)$$

Let we consider the solution example of above given problem. The values of the initial package queue length of each route (thousands of packages), connecting network objects for given subnets and package delay durations in the routs of each subnet (sec), the providing the package traffic transmission between network objects are given (tab.1).

Table 1. Initial data for calculations

Package delay in branches, τ_v		Package queue in branches, h_v^+		Package delay in branches, τ_v		Package queue in branches, h_v^+	
Subnet 1				Subnet 2			
τ_{v1}	0,011	h_{v1}^+	1000	τ_{v9}	0,011	h_{v9}^+	1000
τ_{v2}	0,017	h_{v2}^+	6000	τ_{v10}	0,014	h_{v10}^+	3000
τ_{v3}	0,013	h_{v3}^+	7500	τ_{v11}	0,015	h_{v11}^+	5000
τ_{v4}	0,014	h_{v4}^+	2000	τ_{v12}	0,014	h_{v12}^+	4000
τ_{v5}	0,015	h_{v5}^+	5000	τ_{v13}	0,013	h_{v13}^+	6000
τ_{v6}	0,015	h_{v6}^+	8000	τ_{v14}	0,015	h_{v14}^+	2000
τ_{v7}	0,012	h_{v7}^+	6000	τ_{v15}	0,012	h_{v15}^+	9000
τ_{v8}	0,016	h_{v8}^+	3000	τ_{v16}	0,017	h_{v16}^+	1000

The results of quality characteristics calculation of E -UTRAN/LTE subnet are shown in fig.3.

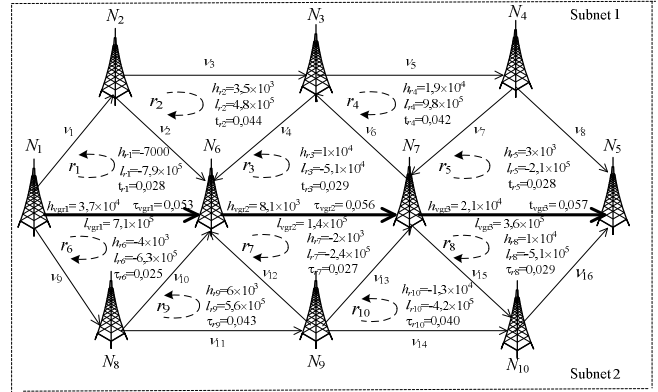


Fig.3. The results of quality characteristics of E -UTRAN/LTE network

Thus, for the network research of complex topology and architecture of $LTE/MVNO$ the use of a boundary tensor method is offered on the basis of decomposition, which allows receiving values of quality characteristics for the E -UTRAN network as a whole and its subnets, using the concept of boundary branches which are in the intersection of divided subnets.

2. The assessment task solution of QoS characteristics of $LTE/MVNO$ network architecture and functional objects of the S -GW/ P -GW network core on the basis of tensor decomposition

The assessment task solution of QoS characteristics of E -UTRAN network architecture, CN -host operator networks and objects of the $LTE/MVNO$ network kernel of S -GW/ P -GW gateways at the stage of designing supposes the QoS characteristics definition aimed at hardware and software network facilities and the principles of $LTE/MVNO$ network objects connection reasonable chose. Thus, it is necessary to consider that the $LTE/MVNO$ network is formed on the basis of core networks of several $LTE/MVNO$ operators. Therefore it is possible to state that it has a large dimension and a complex topological structure of the construction. So, let's use tensor decomposition on the basis of a nodal tensor method for the task solution [3,4]. In this case the $LTE/MVNO$ network is considered as a whole, as geographically distributed structure where the core network of $LTE/MVNO$ core is defined on the S -GW/ P -GW gateways and a subnet basis on the basis of E -UTRAN radio access network and on CN -host operators networks. Then, according to [3,4], it is possible to use tensor decomposition to define necessary characteristics of QoS service quality for each of E -UTRAN and CN -host operator subnets, the core GW/P -GW network and also interaction routes.

Let's consider a fragment of the $LTE/MVNO$ network architecture (fig. 3) and let's perform its consideration in the form of geographically distributed structure.

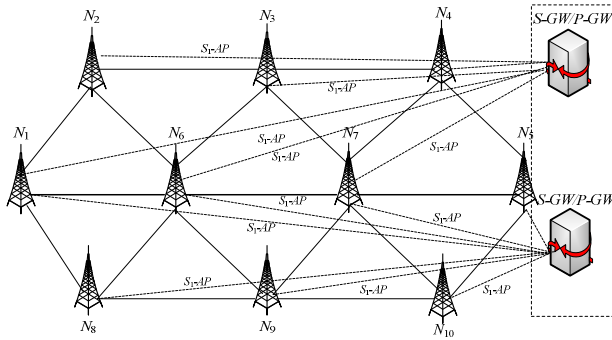


Fig.3. E-UTRAN/LTE network fragment

Let's sort *LTE/MVNO* network by the branches removal connecting the core network with k geographically distributed subnets. Thus, we receive independent subnets of the *E-UTRAN* radio access network, *CN-host* operator's subnets and a core network of *S-GW/P-GW* core. Let's enter the following definition and set *LTE/MVNO* network objects as $A-i/j-p$, where A – is a type of the network object (*e-NodeB*, *S-GW/P-GW*, *CN-host* operator), i – is a serial number of the network object, j – is a number of a subnet, p – is a number of the network object in this subnet. Similar enumeration is also applied for interaction routes.

Let's define the main *QoS* quality characteristics of the *LTE/MVNO* network, the *E-UTRAN* radioaccess network, *CN-host* operators networks and objects of *S-GW/P-GW* gateways core: values of the minimum time of packages delay for network objects $T_{\eta_LTE/MVNO}^{j_LTE/MVNO}$ and connecting them routes $T_{v_LTE/MVNO}^{j_LTE/MVNO}$, the length of package queue $H_{\eta_LTE/MVNO}$ and connections routes $H_{v_LTE/MVNO}$.

By analogy with tensor method, suggested by G.Kron and develop in the works [3-7], *LTE/MVNO* networks structure is represented by one-dimensional network, consisted of m nodes. Thereby the network branches v_i , $i = \overline{1, n}$ modulate *LTE/MVNO* network routes and *LTE/MVNO* network nodes N_j , $j = \overline{1, m}$ – are basic network stations. In the introduced m - dimensional space we perform tensor description in the frames of node networks. As informative system of coordinates (SC) we introduce into consideration two systems of coordinates. The first is the SC of branches, and the second is the SC of network node pairs. Using the Little's formula is invariant functional equation (1), let we write it in vector matrix form in the branches and node pairs SC for the considered *LTE/MVNO* network:

$$H_{v_LTE/MVNO} = L_{v_LTE/MVNO} \cdot T_{v_LTE/MVNO},$$

$$H_{\eta_LTE/MVNO} = L_{\eta_LTE/MVNO} \cdot T_{\eta_LTE/MVNO} \quad (18)$$

where $H_{v_LTE/MVNO}$ и $H_{\eta_LTE/MVNO}$ – covariant tensors of package queue lengths in SC of branches and network node pairs, respectively; $L_{v_LTE/MVNO}$ and $L_{\eta_LTE/MVNO}$ – twice contravariant tensors of traffic capacities the in SC of branches and network node pairs, respectively; $T_{v_LTE/MVNO}$ and $T_{\eta_LTE/MVNO}$ – covariant tensors package transmission delay time in SC of branches and network node pairs, respectively.

Formulas of transformation system of coordinate are define according to node tensor methods, suggested in works [3-6]:

$$H_{\eta_LTE/MVNO} = B_{\eta_LTE/MVNO} \cdot H_{v_LTE/MVNO}^+,$$

$$L_{\eta_LTE/MVNO} = B_{\eta_LTE/MVNO} \cdot L_{v_LTE/MVNO} \cdot B_{\eta_LTE/MVNO}^t, \quad (19)$$

$$T_{v_LTE/MVNO} = B_{\eta_LTE/MVNO}^t \cdot T_{\eta_LTE/MVNO},$$

where $H_{v_LTE/MVNO}^+$ – the length of initial package queue, t – sign of matrix transposition.

According to [3,4] the considered fragment of the *LTE/MVNO* network which includes the core *S-GW/P-GW* network and *E-UTRAN*, *CN-host* operator subnets and the set transmission directions let's form a basic matrix of network reserves:

$$B_{\eta_LTE/MVNO} = \begin{bmatrix} B_{\eta_E-UTRAN}^{1,j_1} & 0 & \dots & 0 & 0 \\ 0 & B_{\eta_CN}^{2,j_2} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & B_{\eta_CN}^{k,j_k} & 0 \\ 0 & 0 & \dots & 0 & B_{\eta_S-GW/P-GW}^{j_S-GW/P-GW} \end{bmatrix}, \quad (20)$$

where $B_{\eta_E-UTRAN}^{i,j_i}$ – is a basic reserves matrix of k -th *E-UTRAN* subnets, $B_{\eta_CN}^{i,j_i}$ – is a basic reserves matrix of k -th *CN-host* operator subnets, $i = \overline{1, k}$, j_i – is a number of network objects i -th subnets, $j_1 + j_2 + \dots + j_k + j_{S-GW/P-GW} = n$, $k \leq n-1$, $B_{\eta_S-GW/P-GW}^{j_S-GW/P-GW}$ – is a basic reserves matrix of a basic core of the *S-GW/P-GW* network, k – is a quantity of subnets.

To determine the length of package queue $H_{\eta_LTE/MVNO}$ in the *E-UTRAN* and *CN-host* operator subnets and the basic *S-GW/P-GW* network let's use an expression using the expression (19) [3,4]:

$$\begin{bmatrix} H_{\eta_E-UTRAN}^{1,j_1} \\ H_{\eta_CN}^{2,j_2} \\ \vdots \\ H_{\eta_CN}^{k,j_k} \\ H_{\eta_S-GW/P-GW}^{j_S-GW/P-GW} \end{bmatrix} = \begin{bmatrix} B_{\eta_E-UTRAN}^{1,j_1} & 0 & \dots & 0 & 0 \\ 0 & B_{\eta_CN}^{2,j_2} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & B_{\eta_CN}^{k,j_k} & 0 \\ 0 & 0 & \dots & 0 & B_{\eta_S-GW/P-GW}^{j_S-GW/P-GW} \end{bmatrix} \begin{bmatrix} H_{v_E-UTRAN}^{+1} \\ H_{v_CN}^{+2} \\ \vdots \\ H_{v_CN}^{+k} \\ H_{v_S-GW/P-GW}^{+} \end{bmatrix}, \quad (21)$$

where $H_{\eta_E-UTRAN}^{i,j_i}$, $H_{\eta_CN}^{i,j_i}$ and $H_{\eta_S-GW/P-GW}^{j_S-GW/P-GW}$ – are tensors of package queues lengths in the k -th *E-UTRAN*, *CN-host* operator subnets and the basic *S-GW/P-GW*

network, $H_{v,E-UTRAN}^{+,i}$, $H_{v,CN}^{+,i}$ and $H_{v,S-GW/P-GW}^{+,i}$ – are tensors of source package queue of k -th $E-UTRAN$, CN -host operator subnets and the basic $S-GW/P-GW$ network, thus $i=\overline{1,k}$, $k \leq n-1$, k – is a quantity of subnets.

Let's find delay time of packages transmission in the nodes of $E-UTRAN$, CN -host operator subnets and the core $S-GW/P-GW$ network according to using the expression (18), which gives that $T_{\eta_LTE/MVNO}$ [3,4]:

$$\begin{bmatrix} T_{\eta,E-UTRAN}^{1,j_1} \\ T_{\eta,CN}^{2,j_2} \\ \vdots \\ T_{\eta,CN}^{k,j_k} \\ T_{\eta,S-GW/P-GW}^{j_s,GW/P-GW} \end{bmatrix} = \begin{bmatrix} L_{\eta,E-UTRAN}^{1,j_1} & 0 & \cdots & 0 & 0 \\ 0 & L_{\eta,CN}^{2,j_2} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & L_{\eta,CN}^{k,j_k} & 0 \\ 0 & 0 & \cdots & 0 & L_{\eta,S-GW/P-GW}^{j_s,GW/P-GW} \end{bmatrix}^{-1} \begin{bmatrix} H_{\eta,E-UTRAN}^{1,j_1} \\ H_{\eta,CN}^{2,j_2} \\ \vdots \\ H_{\eta,CN}^{k,j_k} \\ H_{\eta,S-GW/P-GW}^{j_s,GW/P-GW} \end{bmatrix}, \quad (22)$$

where $L_{\eta,E-UTRAN}^{i,j_i}$, $L_{\eta,CN}^{i,j_i}$ and $L_{\eta,S-GW/P-GW}^{j_s,GW/P-GW}$ – are tensors of traffic capacities in the network nodes of k -th $E-UTRAN$, CN -host operator subnets and the core $S-GW/P-GW$ subnet, $T_{\eta,CN}^{i,j_i}$, $T_{\eta_LTE/MVNO}^{j_s,GW/P-GW}$ and $T_{\eta,E-UTRAN}^{i,j_i} = (\tau_{\eta,E-UTRAN}^1 \quad \tau_{\eta,E-UTRAN}^2 \quad \cdots \quad \tau_{\eta,E-UTRAN}^j)^i$ – are tensors of packages transmission delays in the nodes of $E-UTRAN$, CN -host operator subnets and the basic $S-GW/P-GW$ network, thus $i=\overline{1,k}$, k – is a quantity of subnets.

To calculate the tensor length of package queue $H_{v,E-UTRAN}$ in the routes of the $LTE/MVNO$ network let's use the expression using the expression (18) [3,4]:

$$\begin{bmatrix} H_{v,E-UTRAN}^{1,j_1} \\ H_{v,CN}^{2,j_2} \\ \vdots \\ H_{v,CN}^{k,j_k} \\ H_{v,S-GW/P-GW}^{j_s,GW/P-GW} \end{bmatrix} = \begin{bmatrix} L_{v,E-UTRAN}^{1,j_1} & 0 & \cdots & 0 & 0 \\ 0 & L_{v,CN}^{2,j_2} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & L_{v,CN}^{k,j_k} & 0 \\ 0 & 0 & \cdots & 0 & L_{v,S-GW/P-GW}^{j_s,GW/P-GW} \end{bmatrix} \begin{bmatrix} T_{v,E-UTRAN}^{1,j_1} \\ T_{v,CN}^{2,j_2} \\ \vdots \\ T_{v,CN}^{k,j_k} \\ T_{v,S-GW/P-GW}^{j_s,GW/P-GW} \end{bmatrix}, \quad (23)$$

where $H_{v,E-UTRAN}^{i,j_i}$, $H_{v,CN}^{i,j_i}$ and $H_{v,S-GW/P-GW}^{j_s,GW/P-GW}$ – are tensors of packages queue lengths in the routes of k -th $E-UTRAN$, CN -host operator subnets and the core $S-GW/P-GW$ network, thus $L_{v,E-UTRAN}^{i,j_i}$, $L_{v,CN}^{i,j_i}$ and $L_{v,S-GW/P-GW}^{j_s,GW/P-GW}$ – are tensors of traffic capacities in the routes of $E-UTRAN$, CN -host operator subnets and the core $S-GW/P-GW$ network, thus $T_{v,E-UTRAN}^{i,j_i}$, $T_{v,CN}^{i,j_i}$ and $T_{v,S-GW/P-GW}^{j_s,GW/P-GW}$ – are tensors of packages transfer delays in the routes of $E-UTRAN$, CN -host operator subnets and the basic $S-GW/P-GW$ network, thus $i=\overline{1,k}$, k – is a quantity of subnets.

Having defined the necessary values of the minimum delay time of packages and the package queue length in the core $S-GW/P-GW$ network and in each $E-UTRAN$, CN -host operator subnet, it is necessary to define connections characteristics with the core $S-GW/P-$

GW network. For their calculation let's consider a network model where all subnets are presented as the network nodes connected to the core $S-GW/P-GW$ network, it is also presented as a node, and connections are branches of a network model. Then according to fig. 4 the basic matrix B_{η_Link} has the following form where basic matrixes of all $E-UTRAN$, CN -host operator subnets are its objects [3, 4]:

$$B_{\eta_Link} = \begin{bmatrix} B_{\eta_Link}^{1,j_1} & 0 & 0 & 0 \\ 0 & B_{\eta_Link}^{2,j_2} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & B_{\eta_Link}^{k+1,j_{k+1}} \end{bmatrix}, \quad (24)$$

where $B_{\eta_Link}^{i,j_i}$ – is a basic reserves matrix of k -th subnet, $B_{\eta_Link}^{k+1,j_{k+1}}$ – is a basic selections matrix of $LTE/MVNO$ network, $i=\overline{1,k}$, k – is a quantity of subnets.

Values of packages delays in the network nodes are defined as the sum of packages transmission delays of each k -th subnet. Using values $\tau_{\eta_LTE/MVNO}^{i,j}$ let's define packages delays in the subnets $\tau_{\eta_Link}^i$:

$$\tau_{\eta_Link}^i = \sum_{j=1}^p \tau_{\eta_LTE/MVNO}^{i,j}, \quad (25)$$

where $\tau_{\eta_Link}^i$ – is a delay of packages transmission in the i -th network, $\tau_{\eta_LTE/MVNO}^{i,j}$ – is a delay of packages transmission in the j -th object, $j=\overline{1,p}$ of the i -th network $i=\overline{1,k+1}$, k – is a quantity of subnets.

Let's define delays of packages transmission T_{v_link} in the $Links$ as [3,4]:

$$\begin{bmatrix} \tau_{v_Link}^1 \\ \tau_{v_Link}^2 \\ \vdots \\ \tau_{v_Link}^{k+1} \end{bmatrix} = \begin{bmatrix} B_{\eta_Link}^{1,j_1} & 0 & \cdots & 0 \\ 0 & B_{\eta_Link}^{2,j_2} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & B_{\eta_Link}^{k+1,j_{k+1}} \end{bmatrix}^{-1} \begin{bmatrix} \tau_{\eta_Link}^1 \\ \tau_{\eta_Link}^2 \\ \vdots \\ \tau_{\eta_Link}^{k+1} \end{bmatrix} \quad (26)$$

where $\tau_{\eta_Link}^i$ – is a delay of packages in the i -th subnet, $\tau_{\eta_Link}^{k+1}$ – is a delay of packages in the network links $LTE/MVNO$, k – is a quantity of subnets.

Then the package queue length H_{v_link} in the $LTE/MVNO$ network connections is defined as (18):

$$\begin{bmatrix} H_{v_Link}^1 \\ H_{v_Link}^2 \\ \vdots \\ H_{v_Link}^{k+1} \end{bmatrix} = \begin{bmatrix} L_{v_Link}^1 & 0 & \cdots & 0 \\ 0 & L_{v_Link}^2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & L_{v_Link}^{k+1} \end{bmatrix} \begin{bmatrix} \tau_{v_Link}^1 \\ \tau_{v_Link}^2 \\ \vdots \\ \tau_{v_Link}^{k+1} \end{bmatrix}, \quad (27)$$

where $l_{v_Link}^i$ – is a traffic capacity in the *Link* routes of the *i*-th subnet, $l_{v_Link}^{k+1}$ – is a traffic capacity in the *Link* routes *LTE/MVNO* network, $h_{v_Link}^i$ – is the package queue length in the route on the *Link* segment of the *i*-th subnet, $h_{v_Link}^{k+1}$ – is the package queue length in the route on the *Link* segment of the *LTE/MVNO* network, $i = \overline{1, k}$, k – is a quantity of subnets.

Let us consider the above given method of problem solving on specific example. Thereby, let us consider *LTE/MVNO* network, given on the fig.3, and initial data for calculating are given in the tab. 2.

Table 2. Values of traffic capacities in network branches

Number of branch	1	2	3	4	5	6	7
Package capacity	200	800	0	500	300	100	400
Number of branch	8	9	10	11	12	13	14
Package capacity	500	200	100	0	600	0	900

The calculation results of quality characteristics of *E-UTRAN/LTE* network are shown on fig.4.

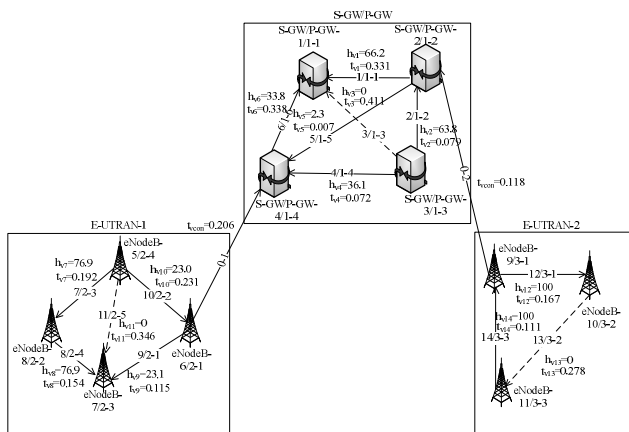


Fig.4. The calculation results of *LTE/MVNO* network

Thus, characteristics of *QoS* service quality for each considered *E-UTRAN* and *CN-host* operator subnets and the core *GW/P-GW* network of geographically distributed *LTE/MVNO* network, and also interaction routes are defined by the tensor decomposition.

Conclusions

1. To research a network with complex topology and *LTE/MVNO* architecture the use of tensor decomposition is offered and it allows analyzing an initial network, dividing it into subnets and receiving necessary characteristics of *QoS* quality as well as for each subnet separately, and for all network as a whole.

2. The application of the decomposition approach for research of systems with complex topological struc-

tures allowed investigating the simpler objects – subnets with the subsequent usage of results in the initial structure. Thus, the finding process of quality characteristics of the complex structure is considerably simplified and the number of computing operations is decreased.

3. The task solution of an optimum topology choice of *e-NodeB* connections in the *E-UTRAN/LTE* network is proposed taking into account a choice of the traffic transmission directions and loading closing in the radio access network providing the guaranteed parameters of *QoS* service quality using the tensor decomposition on the basis of a boundary tensor method.

4. The assessment task of *QoS* quality characteristics of *LTE/MVNO* network architecture is solved on the tensor decomposition basis. The necessary characteristics of service quality are received for the *LTE/MVNO* network and *E-UTRAN*, *CN-host* operator subnets and a *S-GW/P-GW* core, they will allow to make a reasonable choice of hardware and software network facilities and the connection principles of network objects at the designing stage and the *LTE/MVNO* networks implementations.

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