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HOMOMORPHIC SUPPRESSION OF NOISES IN TRANSIENTS BASED ON WAVELET DECOMPOSITION

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The new procedure of sequential noise suppression in transient signals has been developed. The additive, convolutional and multiplicative noises are reduced to the additive forms by appropriate homomorphic transformation. In the decomposition of the signal by wavelets, the additive noise component is identified with the detailizing parts of this expansion. It is found that the Daubechies wavelets would be used as the basis functions. The specific features of the thresholds choice for noise suppression are discussed. According to the results of numerical simulations, it is ascertained that in certain cases the main role in the forming of transients belongs to convolutional noise.

Introduction

In passive remote sensing of artifical or natural sources of electromagnetic (EM) radiation [1, 2], an important task is to estimate the parameters of a current pulse $I(z,t)$ in the origin point $z=0$ of this source by analyzing the emitted electromagnetic transient. At a point of observation, the experimental data are limited often to registration of the only one of transient components of the electromagnetic field. Mostly, it is the vertical component of electric field $E_z(t)$ which in the linear model frames can be described as

$$E_z(t) = I(0,t) * h(t), \qquad (1)$$

where the symbol * denotes the operator of convolution and $h(t)$ is an impulse response (IR) of some fictitious four-terminal network, which converts a current at the radiator base ($z=0$) into electric field observed at the receiving point. Considering a radiator as a set or some array of elementary electric dipoles, we obtain

$$h(t) = h_{el}(t) * h_{arr}(t) * h_{trace}(t); \qquad (2)$$

$$h_{el}(t) = \mathrm{F}^{-1}\{\dot{H}_{el}(i\omega)\} = \mathrm{F}^{-1}[\frac{1}{r^2} + i(-\frac{1}{kr^3} + \frac{k}{r})] \qquad (3)$$

where $h_{el}(t)$ is the IR described by the inverse Fourier transform F^{-1} from the frequency transfer function $\dot{H}_{el}(i\omega)$ for one of these dipoles. This function is described by expressions in square brackets, where $k=\omega/c$; $\omega=2\pi f$; f denotes the current frequency; c defines the light velocity in free space; r is the distance between the radiation source and observation point.

The function $h_{arr}(t)$ describes the of an antenna array formed by the set of dipoles. For some radiators, these dipoles can be identified as several structural inhomogeneities combined into a certain discrete array.

The function $h_{trace}(t)$ in (2) represents the IR of signal propagation path. For example, let us define the function $h_{trace}(t)$ with reference to the very low frequency antennas or natural lightning channels.

High frequencies in received transient spectra of EM pulses radiated by these antennas are located right up to 200–500 kHz. Let the distance r does not exceed a few tens of kilometers and the ground is assumed to be perfectly conducting. The analysis shows that the function $h_{trace}(t)$ for the given case can be considered as a constant value close to 2.

Thus, the form of pulse (1) is mainly determined by the result of convolution of the current $I(0,t)$ with radiation characteristic $h_{arr}(t)$ of EM source that displays main information about geometry and electrophysical properties of the radiator structure. In real terms, an analyzed signal (1) comprises some additional noise components. Their presence complicates the solution of the inverse problem, which consists in evaluation of temporal form and parameters of the current form in (1) and/or convolution members in (2) by the analysis of recorded transient pulse. Appearance of these noise components can be stimulated by different physical causes. We consider additive, convolutional, and multiplicative noises.

For additive noise $n_{add}(t)$, one can assume that the registration of transient pulses (1) in frequency range below 1 MHz are contaminated mainly by a certain global background of atmospheric interferences. Namely, the total flow of pulses with relatively low amplitudes exists always at any point of registration due to electromagnetic radiation of distant storms [3].

A convolutional noise $n_{conv}(t)$ is generated by small inhomogeneities randomly spaced along the radiator.

Under the current wave influence, it reduces to a set of multiple reflections of small levels. This phenomenon can be interpreted as reproduction of waves or co-current flow. This process may be described as an input of a filter with infinite impulse response [4]. Naturally, some inhomogeneities, obstacles or reflective surfaces along the propagation trace can provide the additional influence on reproduction of waves.

A multiplicative noise $n_{mult}(t)$ can be caused by a variety of reasons, for example, by the modulation of antenna current parameters owing to wind or ground vibrations. In particular, the running conductivity of the lightning channel may change due to random variation of electron plasma density along the channel. It is usually supposed that the process $n_{mult}(t)$ is non-negative and slow with respect to $E_z(t)$.

Thus, taking into account the influence of factors discussed above, the model (1) of transient signal can be represented by the expression

$$\hat{E}_z(t) = [E_z(t) \cdot n_{mult}(t)] * n_{conv}(t) + n_{add}(t) . \quad (4)$$

The purpose of this paper is to develop an approach for suppressing the noises indicated above by digital signal processing methods. The proposed algorithm is based on homomorphic mapping of the investigated transient into such domain, where a wanted kind of noise will be additive with informative part of the signal. In such a way, this kind of noise can be eliminated by linear filtering methods. The wavelet transform (WT) [5, 6] will be considered below as one of them. It can be applied also in association with other up-to-date methods, such as empirical mode decomposition [7, 8].

Highly complicated structures are demonstrated by atmospherics or unique EM pulses radiated by lightning strokes [3]. That is the reason why one of them had been selected as an example of processing below.

To achieve the delivered purpose, the calculation of the separate components from the pulse in conformity with (3) is performed, the additive noise suppression by using the WT is investigated, the demonstration of homomorphic methods capabilities to multiplicative and convolutional noises reduction is carried out.

Evaluation of transient separate components

As follows from (3), the most simple expression (directly proportional) connects a current at the radiator with induction component $\hat{E}_z(t)$ of the generated field. If the distance r from the point of observation to the EM source is known, this component can be easily distinguished from the received transient by using inverse filtering. As a result, we obtain

$$\hat{E}_{ind}(t) = \mathrm{F}^{-1}\{\frac{\mathrm{F}[\hat{E}_z(t)]}{\dot{H}_{el}(i\omega)} \frac{1}{r^2}\} , \quad (5)$$

where $\hat{E}_z(t)$, $\hat{E}_{ind}(t)$ denote the transient and its induction component estimations, F and F^{-1} are operators of the direct and inverse Fourier transforms, respectively.

In a similar way, we can estimate the radiation constituent component of the transient EM field

$$\hat{E}_{rad}(t) = \mathrm{F}^{-1}\{\frac{\mathrm{F}\{\hat{E}_z(t)\}}{\dot{H}_{el}(i\omega)} i\frac{k}{r}\} . \quad (6)$$

This component is determined by derivative of current $I(z,t)$ in time.

Elimination of the additive noise

One of the promising methods to reduce the level of noise in the signal is based on its expansion in some elementary waves named as wavelets. A wavelet is a test signal with finite energy given by the compact supported function, which average value is equal to zero [6]. It should be noted that the term "wavelet" has been used in literature on signal processing not long ago. The experience in geophysical papers is more long standing and reflecting the physical nature of the "elementary wave" phenomenon as a structural unit of a more complex process.

It is possible to construct a complete orthonormal system of functions for most of the commonly used wavelets using an appropriate scaling and shifting. The signal decomposition on the elementary waves called a wavelet transform is reduced to the convolution of the signal with these waves.

The continuous wavelet transform (CWT) can be expressed as follows

$$\mathrm{CWT}(\tau, s) = \frac{1}{\sqrt{|s|}} \int_0^t E(t)\, \psi(\frac{t-\tau}{s}) \mathrm{d}t , \quad (7)$$

where ψ is some basic wavelet. Thus, (7) is the function of two variables: the scale factor s, which value determines appropriate stretching or contraction of wavelet, and wavelet shift τ along the time axis t. When the variables s and τ are determined by rules of a geometric progression with ratio of 2, the discrete wavelet transform (DWT) can be used.

By using these procedures, a waveform $E(t)$ can be displayed on the plane "time — scale". Because the scale s is varied in a certain range of values, the WT can detect differences of signal characteristics with various scales and assess their discrepancies. In the region of low values of s, it is possible to observe some fast (or small-scale) variations of the function (7), carrying the information about structural details of the signal $E(t)$ and about noises. Conversely, for large values of s, the region of slow, or large-scale, variations in (7) reflects the main features of this signal. Finally, by varying the shift τ, it is possible to move the wavelet along all the definition domain of the function $E(t)$. It allows analysis of the local properties of the signal in different temporal regions.

To realize this transformation, the serious problem of a wavelet $\psi(t)$ choice must be solved. Its formalization is difficult since the characteristics of the analyzed signal must be taken into account. Some criterion of this choice is the variance of noised signal part which foregoes to the primary transient. Its minimum value has been selected as a measure for wavelet choice. Preliminary computational experiments with real data of transient EM field registrations including some antennas and lightning stroke radiation signals allowed coming the conclusion that it is advisable to use Daubechies wavelets as basis functions described in [6]. The wavelet of order 4 is used below everywhere.

A certain atmospherics (Fig. 1*a*) generated by return lightning stroke and registered in the frequency band 0.5—250 kHz on the distance $r = 30$ km from the source was selected for further studies.

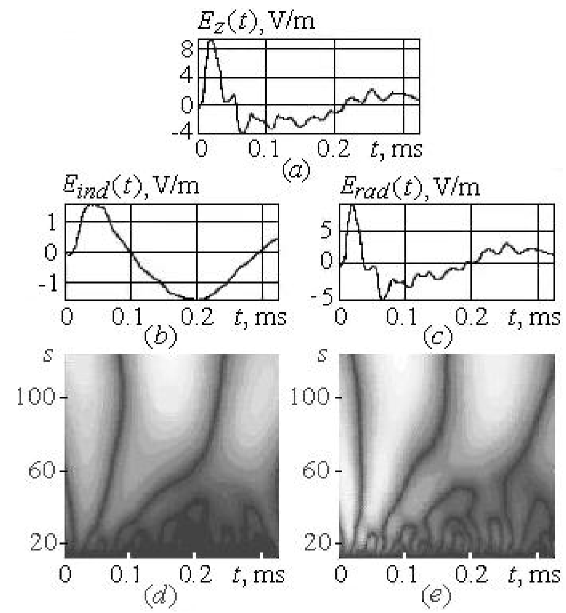

Fig. 1. The transient (*a*); its induction (*b*) and radiation (*c*) components; (*d*), (*e*) CWT from *b* and *c*, respectively.

Its induction (5) and radiation (6) components are illustrated in Fig. 1*b* and in Fig. 1*c*, respectively. Calculated results of CWT moduli according to (7) are demonstrated in Fig. 1*d* and Fig. 1*e*. They were obtained by the Matlab program code cwt.m [7], which allows carrying out the necessary calculations and visualizing the results. Increasing these moduli is reflected by increment of brightness. Thus, the dark lines can be interpreted as the locus of the points with values of CWT, which are close to zero.

Fig. 1*d* and Fig. 1*e* show, how nature of information containing in the signal changes with scale increase. At small scales ($s < 40$), rapid variations with small amplitudes are detected. They contain the information about the fine details of the signal. These variations can be identified as mapping the additive noise in the model (4) on the plane "time — scale". CWT for the signal (6) is more noised as it has been expected.

Weakening the additive noise is based on the appropriate filtering the function (7) and applying the inverse WT. It allows restoring some analogue of the signal by using the new values of its CWT. For example, after processing the signal (7) with limiting $s \in [s_1, s_2]$ and considering that $\mathrm{CWT}(s,\tau) = 0$ beyond the bounds of this interval, we obtain

$$E(t)\Big|_{s\in[s_1,\,s_2]} = \frac{1}{C_\psi}\int_0^{\tau}\int_{s_1}^{s_2} \mathrm{CWT}(s,\tau)\,\psi(t)\frac{d s\, d\tau}{s^2}, \quad (8)$$

where the normalizing factor is defined as

$$C_\psi = \int_{-\infty}^{+\infty} |\,\psi(x)|^2 dx\big/|x|\,. \quad (9)$$

To increase the computation rate in (7)—(9) and their accuracy, it is advisable to use the discrete wavelet transform (DWT) instead of CWT as the basis of the additive noise suppression algorithm. The DWT finds the coefficient vector

$$\mathbf{w} = \mathbf{W}\hat{\mathbf{E}}, \quad (10)$$

where $\mathbf{W}$ denotes $N\times N$ orthonormal matrix, and N is the signal length. The value of N is chosen more often so that $N = 2^j$ for integer j. Structure of matrix and values of its elements are determined by the type of selected wavelet.

Procedure (10) allows the signal decomposition based on assumption that the vector $\mathbf{w}$ contains the coefficients of two kinds. First of them are approximating ones that describe the signal roughly, and the second are detailing coefficients, which reflect subtle features of signal structure. Thus, an additive noise is transmitted in detailing components, and its suppression is, in essence, a threshold filtering of the vector (10) to weaken influence of the coefficients with low values.

The algorithm (8) described above is based on signal weighing by using some window. It is a kind of cut-off filters with so-called hard threshold. For the treatment of atmospherics, one of the many options for soft-threshold filtering [7] was used, where *n*th sample of detailing component in (10) is set as

$$\hat{\mathbf{w}}_n = \begin{cases} \operatorname{sign}(\mathbf{w}_n)(|\mathbf{w}_n| - \delta), & |\mathbf{w}_n| > \delta; \\ 0, & |\mathbf{w}_n| \le \delta, \end{cases} \quad (11)$$

and if the noise is assumed as Gaussian with variance equal to σ^2, the threshold δ in (11) can be presented as

$$\delta = \sqrt{2\sigma^2 \log(N)}\,. \quad (12)$$

By applying the median med[$d(n)$] for a set $d(n)$ contained detailing coefficients, one can modify the threshold (12) to the form [7]

$$\delta_c = \sigma \operatorname{med}[d(n)]/0.6745\,. \quad (13)$$

Equations (10)–(13) allow restoring the modified signal

$$\tilde{\mathbf{E}} = \mathbf{W}^{\mathrm{T}} \hat{\mathbf{w}}. \quad (14)$$

This signal has to demonstrate a lower noise level compared with the original transient.

Successive application of the DWT (10) allows performing the multilevel decomposition by using approximate components after every stage of the described procedure as an input data for subsequent one. Curve *1* in Fig. 2 shows the effect of DWT levels number of the signal (5) on the Euclidian norm of recovered signal (14).

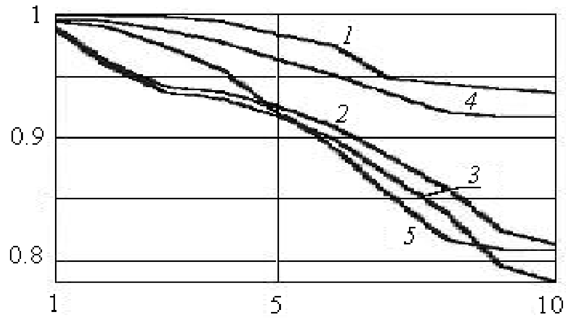

Fig. 2. Influence of the number of levels on the norm of the recovered signal.

The reduction of this norm can be considered as a measure of noise suppression.

Suppression of convolution and multiplicative noises using homomorphic transformations

Homomorphic transformation introduces a sequence of operations which should be imposed to the signal in order to map the latter onto such domain, where its components are additive and can be separated by means of linear filtering [9]. Results of separation can be returned to the original domain by using the inverse transform. A certain procedures of homomorphic filtering may be proposed to solve the problem of noise reduction for such signals that contain convolutional and/or multiplicative noises.

As follows from (4), the homomorphic transformation with reference to the processing of convolutional noise is gradual transition from the original signal to the complex spectrum by applying the direct Fourier transform, and then to calculating the logarithmic spectrum

$$\begin{aligned} \dot{S}_L(i\omega) = \operatorname{Ln}[\mathrm{F}\{\hat{E}(t)\}] = \left|\dot{S}_L(i\omega)\right| + n_L(\omega) + \\ + i[\arg \dot{S}_L(i\omega) + n_\varphi(\omega)], \end{aligned} \quad (15)$$

where symbol Ln denotes the operation of complex logarithm calculation. According to the procedure of finding the complex spectrum [9], calculation of the imaginary part in (15) has to include unwrapping of phase curve $\arg S_L$. It is necessary in order to eliminate the discontinuities in phase spectrum and to remove the linear component from the latter.

As follows from (15), the above transformation allows filtering noises $n_L(\omega)$ and $n_\varphi(\omega)$ separately with respect to the real and imaginary parts of the logarithmic spectrum, respectively.

Fig. 3*a* and Fig. 3*c* show real and imaginary parts of the logarithmic spectrum. Frequency *f* measures in kHz.

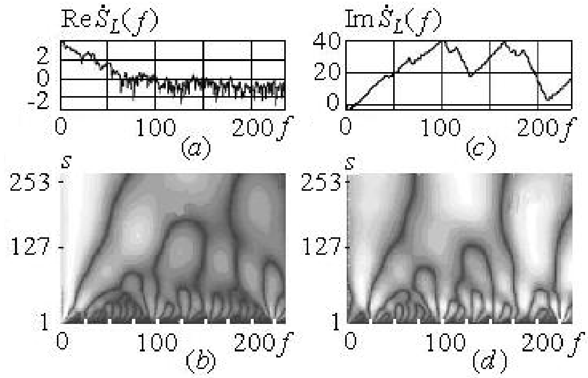

Fig. 3. The real (*a*) and imaginary (*c*) parts of the logarithmic spectrum of the transient (5); (*b*), (*d*) their CWT.

Figures 3*b*, 3*d* display results of application to these parts of the CWT (7). It should be noted that the structures of amplitude noise $n_L(\omega)$ (Fig. 3*b*) and phase noise $n_\varphi(\omega)$ (Fig. 3*d*) are not only likely close, but also show some features of fractal processes. Additive properties of the corresponding components in logarithmic spectrum may be used to perform the filtering operations (10)–(14) with the amplitude and phase noises separately. Suppression grades of $n_L(\omega)$ and $n_\varphi(\omega)$ noises as proper Euclidian norms are illustrated by curves *2* and *3* in Fig. 2. Sequence of actions to adequate convert the multiplicative noise modeled by (4) into additive one is suggested below. This conversion ought to be homomorphic with respect to multiplication. Corresponding algorithm can be constructed by the transition to the analytical signal [10, 11] with subsequent calculation of its complex logarithm:

$$\begin{aligned} \dot{\hat{A}}_L(t) &= \mathrm{Ln}[\hat{E}(t) + i\mathrm{Hb}\{\hat{E}(t)\}] = \\ &= \mathrm{Ln}[n_{mult}(t)A(t)\exp\{i\varphi(t)\}] = \\ &= \left|\dot{A}_L(t)\right| + n_{mA}(t) + i[\arg \dot{A}_L(t) + n_{m\varphi}(t)] \end{aligned} \quad (16)$$

where Hb denotes a symbol of Hilbert transform; $A(t)$, $\varphi(t)$ are an envelope and an argument of the analytical signal, respectively; the term $n_{mA}(t)$ presents an amplitude component of multiplicative noise mapped into the complex logarithm domain; $n_{m\varphi}(t)$ is a phase noise. It should be taken into account in (16) that it is permissible to carry out the slow multiplier from the Hilbert transform.

As in the previous analysis, finding the imaginary part in (16) has to include unwrapping of the curve that describes the instantaneous phase and removing the linear component from it. Suppressions of amplitude $n_{mA}(t)$ and phase $n_{m\varphi}(t)$ noises are illustrated as proper Euclidian norms by curves *4* and *5* in Fig. 2.

As an instance, Fig. 4 represents the effect caused by successive noise suppression on shape of the radiation component (6) of the atmospherics displayed in Fig. 1*c* and repeated in Fig. 4*a*. Figures 4*b* — 4*d* show the results of applying the procedures (10)—(16) to suppress additive, multiplicative and convolutional noises, respectively. The output signal of the previous proceeding stage is used as the input signal in the next stage.

By applying the described approach to the induction component (5) of transient signal, solutions concerned the evaluation [2] of current pulse form $I(0, t)$ at the radiator input can be noticeably simplified.

Conclusion

The method of suppression of noises with different physical nature has been proposed. It is based on appropriate homomorphic mapping of noises described by the model (4) into additive ones. By displayed examples of some signal proceeding, it was shown that convolutional and multiplicative noise components may play in certain cases more prominent role than additive noise. In turn, convolutional noise may manifest itself more than multiplicative one.

In comparison with amplitude and phase noises, the latter may show the more influence on the inherent structure of certain transients, in particular, atmospherics. The performed analysis decisively shows that the reduction of noises has to produce substantial signal compression.

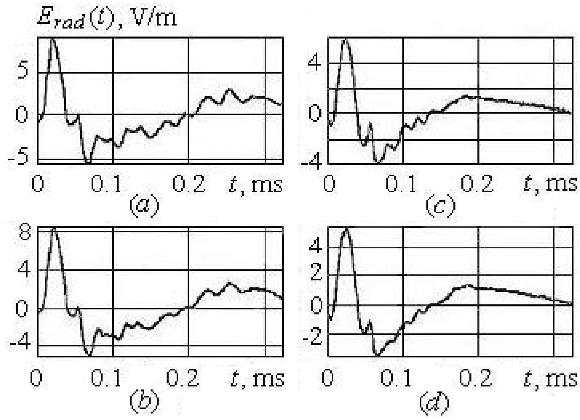

Fig. 4. Influence of noise suppression on the shape of the transient (*a*) after successive elimination: additive (*b*), multiplicative (*c*) and convolutional (*d*) noise components.

Obtained results can be applied to increase the signal to noise ratio in telecommunication channels.

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