UDC 621.372

MUTUAL COUPLING COEFFICIENTS OF ROTATING RECTANGULAR DIELECTRIC RESONATORS IN CUT-OFF RECTANGULAR WAVEGUIDE

Alexander A. Trubin

Institute of Telecommunication Systems
Igor Sikorsky Kyiv Polytechnic Institute, Kyiv, Ukraine

Background. A further increase in the speed of information transfer is determined by more stringent requirements for the elements of communication devices. One of the most important components of such devices is various filters, which are often made on the basis of dielectric resonators. Calculation of the parameters of multi-section filters is impossible without further development of the theory of their design. The development of filter theory is based on electrodynamic modelling, which involves calculating the coupling coefficients of dielectric resonators in various transmission lines.

Objective. The aim of the research is to calculate and study the coupling coefficients of rectangular dielectric resonators with a rectangular metal waveguide when their axes rotate. Investigation of new effects to improve the performance of filters and other devices based on them.

Methods. Methods of technical electrodynamics are used to calculate and analyse the coupling coefficients. The end result is to obtain new analytical formulas for new structures with rectangular dielectric resonators, which make it possible to analyse and calculate their coupling coefficients.

Results. New analytical expressions are found for the coupling coefficients of dielectric resonators with the rotation of their axes in a rectangular waveguide.

Conclusions. The theory of designing filters based on new structures of dielectric resonators with rotation of their axes in metal waveguides has been expanded. New analytical relationships and new patterns of change in the coupling coefficients are found.

Keywords: dielectric filter; rectangular dielectric resonator; rotation; coupling coefficients.

Introduction

Multi-section band-pass and band-stop filters based on dielectric resonators (DR) of different shapes are used in various devices of telecommunication systems [1-8]. Further improvement of filter characteristics can be achieved by applying less traditional structures, such as dielectric resonators with rotation of their axes relative to each other and the transmission line [1, 2]. Theoretical analysis of the characteristics of such filters, it is required to calculate the mutual coupling coefficients of the DR located at an arbitrary angle with respect to the waveguides.

Statement of the problem

The purpose of this article is to calculation and study mutual coupling coefficients of the rectangular DRs in a rectangular metal cut-off waveguide.

Calculation of fields during rotation of dielectric resonators in a waveguide

The internal field $(\vec{e},\vec{h})$ of a rectangular dielectric resonator with magnetic type oscillations H_{nml} in the local coordinate system (x',y',z') (Fig.1) with good accuracy can be represented as:

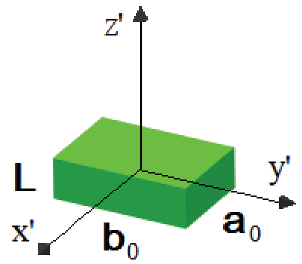

Fig. 1. Rectangular dielectric resonator in the local coordinate system (x',y',z').

$$e_{x'} = \frac{-h_1 i\omega\mu_0}{k_1^2-\beta_z^2}\beta_y \begin{pmatrix} \sin\beta_x x' \\ \cos\beta_x x' \end{pmatrix} \begin{bmatrix} \cos\beta_y y' \\ -\sin\beta_y y' \end{bmatrix} \begin{Bmatrix} \sin\beta_z z' \\ \cos\beta_z z' \end{Bmatrix};$$

$$e_{y'} = \frac{h_1 i\omega\mu_0}{k_1^2-\beta_z^2}\beta_x \begin{pmatrix} \cos\beta_x x' \\ -\sin\beta_x x' \end{pmatrix} \begin{bmatrix} \sin\beta_y y' \\ \cos\beta_y y' \end{bmatrix} \begin{Bmatrix} \sin\beta_z z' \\ \cos\beta_z z' \end{Bmatrix};$$

$$e_{z'} = 0; \quad (1)$$

$$h_{x'} = \frac{h_1}{k_1^2 - \beta_z^2}\beta_x\beta_z \begin{pmatrix} \cos\beta_x x' \\ -\sin\beta_x x' \end{pmatrix} \begin{bmatrix} \sin\beta_y y' \\ \cos\beta_y y' \end{bmatrix} \begin{Bmatrix} \cos\beta_z z' \\ -\sin\beta_z z' \end{Bmatrix};$$

$$h_{y'} = \frac{h_1}{k_1^2 - \beta_z^2}\beta_y\beta_z \begin{pmatrix} \sin\beta_x x' \\ \cos\beta_x x' \end{pmatrix} \begin{bmatrix} \cos\beta_y y' \\ -\sin\beta_y y' \end{bmatrix} \begin{Bmatrix} \cos\beta_z z' \\ -\sin\beta_z z' \end{Bmatrix};$$

$$h_{z'} = h_1 \begin{pmatrix} \sin\beta_x x' \\ \cos\beta_x x' \end{pmatrix} \begin{bmatrix} \sin\beta_y y' \\ \cos\beta_y y' \end{bmatrix} \begin{Bmatrix} \sin\beta_z z' \\ \cos\beta_z z' \end{Bmatrix}.$$

Where $(\beta_x, \beta_y, \beta_z)$ - are the wave numbers; $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$, $k_1 = \omega\sqrt{\mu_0\varepsilon_1}$; h_1 - is the amplitude; ω - is the resonant frequency; μ_0 - is the magnetic permeability; ε_0; ε_1 - is the dielectric permittivity of the external space and resonator, respectively.

An analytical expressions for the coupling coefficients obtained from [10] for cut-off waveguide

$$k_{12} = \frac{-1}{\omega W}\sum_{t \ge t_M} (c_t^{1\pm})_0 (c_t^{2\mp})_0^* e^{-\Gamma|z_2 - z_1|}. \quad (2)$$

Here W - energy stored in the dielectric of the resonator; Γ - is the guided wavelength; t - is the multi-index, defining non-propagating line wave type; z_n - longitudinal coordinate of the n - th DR centre ($n = 1, 2$); $*$ - is the complex conjugate symbol.

We need to calculate the expansion coefficients $(c_t^{n\pm})_0$ of the field of natural oscillations of the DR over the waveguide field in the resonator centre coordinate system.

For a rectangular resonator, it is more convenient to use representation for expansion coefficients:

$$c_t^{n\pm} = i / 2\omega(\varepsilon_1 - \varepsilon_0)\int_V (\vec{e}^n, (\vec{E}_t^{\pm})^*)dv. \quad (3)$$

Here $\vec{E}_t^{\pm}$ - electric field of the t-th eigen wave of a waveguide; $\vec{e}^n$ - is the field (1); V - is the resonator volume; $n = 1, 2$.

For simplicity, we will rewrite (2), (3) in the waveguide coordinate system (x, y, z) (Fig. 2 - a) in a more convenient form:

$$k_{12} = k_{12}^0 \sum_{t \ge t_M} f_t^1(\mp i\Gamma)(f_t^2(\pm i\Gamma))^* e^{-\Gamma|z_2 - z_1|} \quad (4)$$

where for rectangular DR

$$k_{12}^0 = \frac{16}{w_0}(\varepsilon_{1r} - 1)^2 \frac{p_x p_y p_z q_x q_y q_z}{\upsilon H};$$

$$\upsilon H = \varepsilon_{1r} k_0^2 [\beta_y^2 \pi_x^{\mp}\pi_y^{\pm}\pi_z^{\mp} + \beta_x^2 \pi_x^{\pm}\pi_y^{\mp}\pi_z^{\mp}] +$$
$$+ \beta_x^2\beta_z^2 \cdot \pi_x^{\pm}\pi_y^{\mp}\pi_z^{\pm} + \beta_y^2\beta_z^2 \cdot \pi_x^{\mp}\pi_y^{\pm}\pi_z^{\pm} + (\beta_x^2 + \beta_y^2)^2 \cdot \pi_x^{\mp}\pi_y^{\mp}\pi_z^{\mp}$$

$\pi_v^{\pm} = p_v \pm \sin p_v \cos p_v$; $v = (x, y, z)$. The upper and lower signs of $\pi_v^{\pm}$ correspond to the distributions of the DR field shown in brackets (1). Here $\varepsilon_{1r} = \varepsilon_1 / \varepsilon_0$; $w_0 = (\mu_0 / \varepsilon_0)^{1/2}$; $p_x = \beta_x a_0 / 2$; $p_y = \beta_y b_0 / 2$; $p_z = \beta_z L / 2$ as well as $q_x = k_0 a_0 / 2$; $q_y = k_0 b_0 / 2$; $q_z = k_0 L / 2$ - is the characteristic parameters of the resonator.

Rotation of the dielectric resonator relatively x-axis of the waveguide

In the case of rotation of the resonator relatively to the x-axis in the waveguide coordinate system (Fig. 2, a), the functions $f_t^n(\mp i\Gamma)$ of (4) take the form for $n = 1, 2$:

$$f_t^n(\mp i\Gamma) = \begin{pmatrix} i\sin\chi_{sx}x_n \\ \cos\chi_{sx}x_n \end{pmatrix} \cdot \{-E_{x0}^* \beta_y \omega_x(\xi_{sx}) \cdot$$
$$\cdot\Big[e^{-i\chi_{uy}y_n} \varpi_y(-(\eta_{uy}\cos\beta_n \mp i\gamma\sin\beta_n))\omega_z(-(\eta_{uy}\sin\beta_n \pm i\gamma\cos\beta_n)) -$$
$$-e^{i\chi_{uy}y_n} \varpi_y(\eta_{uy}\cos\beta_n \pm i\gamma\sin\beta_n)\omega_z(\eta_{uy}\sin\beta_n \mp i\gamma\cos\beta_n)\Big] -$$
$$-E_{y0}^*\beta_y\cos\beta_n \varpi_x(\xi_{sx}) \cdot$$
$$\cdot\Big[e^{-i\chi_{uy}y_n} \omega_y(-(\eta_{uy}\cos\beta_n \mp i\gamma\sin\beta_n))\omega_z(-(\eta_{uy}\sin\beta_n \pm i\gamma\cos\beta_n)) +$$
$$+e^{i\chi_{uy}y_n} \omega_y(\eta_{uy}\cos\beta_n \pm i\gamma\sin\beta_n)\omega_z(\eta_{uy}\sin\beta_n \mp i\gamma\cos\beta_n)\Big] -$$
$$-iE_{z0}^*\beta_x \sin\beta_n \varpi_x(\xi_{sx}) \cdot \quad (5)$$
$$\cdot\Big[e^{-i\chi_{uy}y_n} \omega_y(-(\eta_{uy}\cos\beta_n \mp i\gamma\sin\beta_n))\omega_z(-(\eta_{uy}\sin\beta_n \pm i\gamma\cos\beta_n)) -$$
$$-e^{i\chi_{uy}y_n} \omega_y(\eta_{uy}\cos\beta_n \pm i\gamma\sin\beta_n)\omega_z(\eta_{uy}\sin\beta_n \mp i\gamma\cos\beta_n)\Big]\}$$

Here and below $\chi_{sx} = s\pi / a$; $\chi_{uy} = u\pi / b$; $\xi_{sx} = \chi_{sx} / k_0$; $\eta_{uy} = \chi_{uy} / k_0$; $\Gamma = \sqrt{(\chi_{sx})^2 + (\chi_{uy})^2 - k_0^2}$; $\gamma = \Gamma / k_0$ - is the wave numbers of a rectangular cut-off waveguide with a cross section $a \times b$ (Fig. 2, a);

$E_{x0} = i\chi_{uy}\left|w_{su}^{H}\right|h_{su}^{0}$; $E_{y0} = -i\chi_{sx}\left|w_{su}^{H}\right|h_{su}^{0}$ for H-waves;

$E_{x0} = \mp\chi_{sx}e_{su}^{0}$; $E_{y0} = \mp\chi_{uy}e_{su}^{0}$; $E_{z0} = \chi^2/|\Gamma|e_{su}^{0}$ for E-waves [10];

$$h_{su}^{0} = \frac{2}{\chi}\left[\frac{|\Gamma|}{w_0 k_0 ab}\right]^{1/2}\frac{1}{(1+\delta_{s0}+\delta_{u0})^{1/2}};$$

$$e_{su}^{0} = \frac{2}{\chi}\left[\frac{|\Gamma| w_0}{k_0 ab}\right]^{1/2}(1+\delta_{s0}+\delta_{u0})^{1/2};$$

$w_{su}^{H} = \omega\mu_0/\Gamma$; $\chi = [\chi_{sx}^2 + \chi_{uy}^2]^{1/2}$; (x_n, y_n, z_n) - is the coordinates of the centre of the resonator in the waveguide; β_n - is the angle between the z'- axes of the local coordinate system of the resonator and the z- axes waveguide (Fig. 2, a); $a_0 \times b_0 \times L$- dimensions of the Rectangular DR (Fig. 1);

$$\omega_v(\xi) = \frac{1}{p_v^2 - (q_v\xi)^2}\cdot \begin{pmatrix} -i[p_v\cos p_v \sin(q_v\xi) - q_v\xi\sin p_v\cos(q_v\xi)] \\ p_v \sin p_v\cos(q_v\xi) - q_v\xi\cos p_v\sin(q_v\xi)\end{pmatrix};$$

$$\varpi_v(\xi) = \frac{1}{p_v^2 - (q_v\xi)^2}\cdot \begin{pmatrix} p_v \sin p_v\cos(q_v\xi) - q_v\xi\cos p_v\sin(q_v\xi) \\ i[p_v\cos p_v \sin(q_v\xi) - q_v\xi\sin p_v\cos(q_v\xi)]\end{pmatrix};$$

for $v = (x, y, z)$.

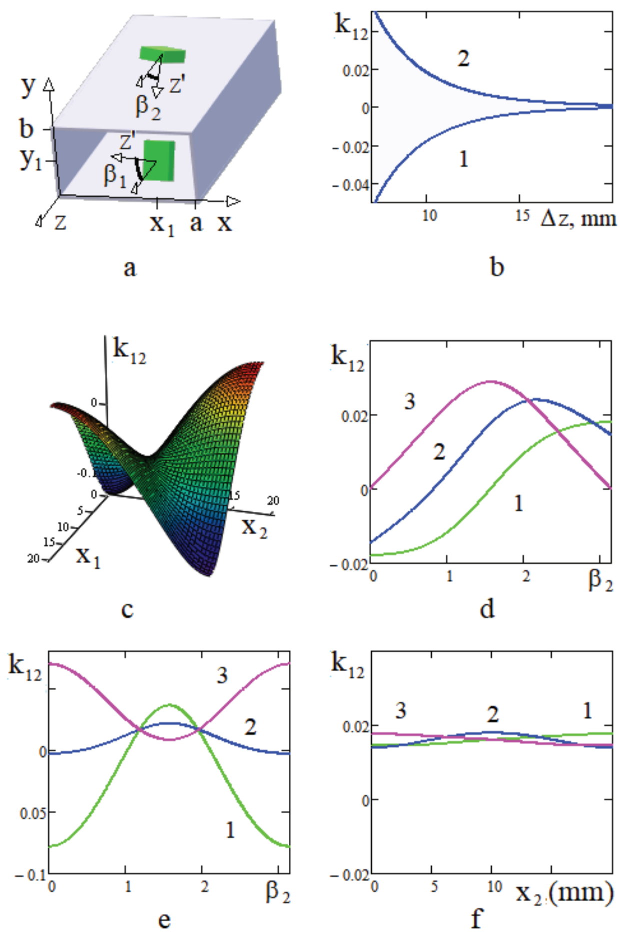

Fig. 2. Two rectangular dielectric resonators rotating relatively the x axis (a) in the rectangular waveguide. Variation of the mutual coupling coefficients at different longitudinal distances Δz between the centres dielectric resonators in the waveguide $a \times b = 20 \times 15 mm^2$ for $\varepsilon_{1r} = 36$; $a_0/b_0 = 1$; $L/a_0 = 0{,}4$; $x_1 = x_2 = a/2$; $y_1 = y_2 = b/2$; 1 - $\beta_1 = \beta_2 = 0$; 2 - $\beta_1 = 0$; $\beta_2 = \pi$ (b). Mutual coupling coefficient on the coordinates (x_1, x_2) for $\beta_1 = \beta_2 = 0$; $y_1 = y_2 = b/2$; $\Delta z = |z_2 - z_1| = 10mm$ (c). Dependence of the mutual coupling coefficient on the angle of rotation of the second resonator (d): 1 - $\beta_1 = 0$; 2 - $\beta_1 = \pi/4$; 3 - $\beta_1 = \pi/2$. Dependence of the coupling coefficient on the angle of rotation of the resonators for $\beta_1 = \beta_2$ (e): 1 - $x_1 = x_2 = a/4$; 2 - $x_1 = a/4$; $x_2 = a/2$; 3 - $x_1 = a/4$; $x_2 = 3a/4$. The coupling coefficient on the coordinate x_2 for $y_1 = y_2 = b/2$; $\Delta z = 10mm$ (f) and $\beta_1 = \beta_2 = 1{,}175$; 1 - $x_1 = a/4$; 2 - $x_1 = a/2$; 3 - $x_1 = 3a/4$.

Rotation of the dielectric resonator relatively y-axis of the waveguide

The coupling coefficient of the dielectric resonators rotated relatively the y-axis determines by the functions:

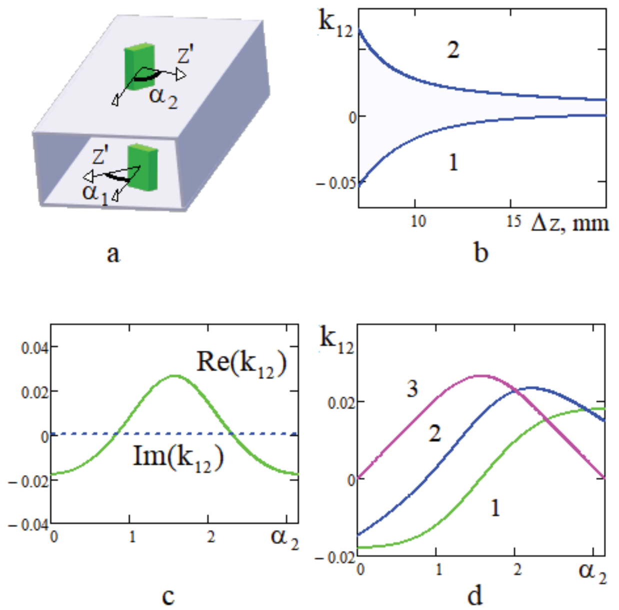

Fig. 3. Two rectangular dielectric resonators rotating relatively the y - axes (a). Variation of the mutual coupling coefficients at different longitudinal distances between the resonator centres for $a \times b = 20 \times 15\,\mathrm{mm}^2$; $\varepsilon_{1r} = 36$; $a_0 / b_0 = 1$; $L / a_0 = 0,4$; $x_1 = x_2 = a/2$; $y_1 = y_2 = b/2$; 1 - $\alpha_1 = \alpha_2 = 0$; 2 - $\alpha_1 = \alpha_2 = \pi/2$ (b). Dependence of coupling coefficient on the angle of rotations of the second resonator α_2 for $\Delta z = 10\mathrm{mm}$ (c, d); 1 - $\alpha_1 = 0$; 2 - $\alpha_1 = \pi/4$; 3 - $\alpha_1 = \pi/2$ (d).

$$f_t^n(\mp i\Gamma) = \begin{pmatrix} i\sin\chi_{uy}y_n \\ \cos\chi_{uy}y_n \end{pmatrix} \cdot \Big\{ E_{y0}^* \beta_x \omega_y(\eta_{uy}) \cdot$$

$$\cdot \Big[e^{-i\chi_{uy}x_n} \varpi_x(-(\xi_{sx}\cos\alpha_n \mp i\gamma\sin\alpha_n))\omega_z(-(\xi_{sx}\sin\alpha_n \pm i\gamma\cos\alpha_n)) -$$

$$- e^{i\chi_{sx}x_n} \varpi_x(\xi_{sx}\cos\alpha_n \pm i\gamma\sin\alpha_n)\omega_z(\xi_{sx}\sin\alpha_n \mp i\gamma\cos\alpha_n) \Big] +$$

$$+ E_{x0}^* \beta_y \cos\alpha_n \varpi_y(\eta_{uy}) \cdot$$

$$\cdot \Big[e^{-i\chi_{sx}x_n} \omega_x(-(\xi_{sx}\cos\alpha_n \mp i\gamma\sin\alpha_n))\omega_z(-(\xi_{sx}\sin\alpha_n \pm i\gamma\cos\alpha_n)) +$$

$$+ e^{i\chi_{sx}x_n} \omega_x(\xi_{sx}\cos\alpha_n \pm i\gamma\sin\alpha_n)\omega_z(\xi_{sx}\sin\alpha_n \mp i\gamma\cos\alpha_n) \Big] +$$

$$+ iE_{z0}^* \beta_y \sin\alpha_n \varpi_y(\eta_{uy}) \cdot \qquad (6)$$

$$\cdot \Big[e^{-i\chi_{sx}x_n} \omega_x(-(\xi_{sx}\cos\alpha_n \mp i\gamma\sin\alpha_n))\omega_z(-(\xi_{sx}\sin\alpha_n \pm i\gamma\cos\alpha_n)) -$$

$$- e^{i\chi_{sx}x_n} \omega_x(\xi_{sx}\cos\alpha_n \pm i\gamma\sin\alpha_n)\omega_z(\xi_{sx}\sin\alpha_n \mp i\gamma\cos\alpha_n) \Big] \Big\}$$

Here α_n - is the angle between z'- axes of the local coordinate system of the resonator (Fig. 1) and z- axes of the waveguide (Fig. 3, a) ($n = 1,2$).

Rotations of the dielectric resonator relatively z-axis of the waveguide

In the case of rotation of the dielectric resonator relatively z-axis at the initial position of the resonator axis z' parallel to the waveguide axis (Fig. 4, a), the coupling coefficient defined by functions:

$$f_t^n(\mp i\Gamma) = \omega_z(\mp i\gamma) \cdot \Big\{ E_{x0}^* \beta_y \cos\alpha_n \cdot$$

$$\Big[\omega_x(\xi_{sx}\cos\alpha_n - \eta_{uy}\sin\alpha_n)\varpi_y(\xi_{sx}\sin\alpha_n + \eta_{uy}\cos\alpha_n)\cos(\chi_{sx}x_n + \chi_{uy}y_n) -$$

$$- \omega_x(\xi_{sx}\cos\alpha_n + \eta_{uy}\sin\alpha_n)\varpi_y(\xi_{sx}\sin\alpha_n - \eta_{uy}\cos\alpha_n)\cos(\chi_{sx}x_n - \chi_{uy}y_n) \Big] +$$

$$+ E_{y0}^* \beta_y \sin\alpha_n \cdot$$

$$\Big[\omega_x(\xi_{sx}\cos\alpha_n - \eta_{uy}\sin\alpha_n)\varpi_y(\xi_{sx}\sin\alpha_n + \eta_{uy}\cos\alpha_n)\cos(\chi_{sx}x_n + \chi_{uy}y_n) +$$

$$+ \omega_x(\xi_{sx}\cos\alpha_n + \eta_{uy}\sin\alpha_n)\varpi_y(\xi_{sx}\sin\alpha_n - \eta_{uy}\cos\alpha_n)\cos(\chi_{sx}x_n - \chi_{uy}y_n) \Big] -$$

$$- E_{x0}^* \beta_x \sin\alpha_n \cdot$$

$$\Big[\varpi_x(\xi_{sx}\cos\alpha_n - \eta_{uy}\sin\alpha_n)\omega_y(\xi_{sx}\sin\alpha_n + \eta_{uy}\cos\alpha_n)\cos(\chi_{sx}x_n + \chi_{uy}y_n) -$$

$$- \varpi_x(\xi_{sx}\cos\alpha_n + \eta_{uy}\sin\alpha_n)\omega_y(\xi_{sx}\sin\alpha_n - \eta_{uy}\cos\alpha_n)\cos(\chi_{sx}x_n - \chi_{uy}y_n) \Big] +$$

$$+ E_{y0}^* \beta_x \cos\alpha_n \cdot \qquad (7)$$

$$\Big[\varpi_x(\xi_{sx}\cos\alpha_n - \eta_{uy}\sin\alpha_n)\omega_y(\xi_{sx}\sin\alpha_n + \eta_{uy}\cos\alpha_n)\cos(\chi_{sx}x_n + \chi_{uy}y_n) +$$

$$+ \varpi_x(\xi_{sx}\cos\alpha_n + \eta_{uy}\sin\alpha_n)\omega_y(\xi_{sx}\sin\alpha_n - \eta_{uy}\cos\alpha_n)\cos(\chi_{sx}x_n - \chi_{uy}y_n) \Big] \Big\}$$

Here α_n can be defined as the angle between the y - axis of the waveguide (Fig. 4, a) and the y' - axis (Fig. 1) of the n -th resonator.

$$\left[\omega_z(\xi_{sx}\cos\alpha_n-\eta_{uy}\sin\alpha_n)\varpi_y(\xi_{sx}\sin\alpha_n+\eta_{uy}\cos\alpha_n)\sin(\chi_{sx}x_n+\chi_{uy}y_n)-\right.$$
$$\left.-\omega_z(\xi_{sx}\cos\alpha_n+\eta_{uy}\sin\alpha_n)\varpi_y(\xi_{sx}\sin\alpha_n-\eta_{uy}\cos\alpha_n)\sin(\chi_{sx}x_n-\chi_{uy}y_n)\right]\Big\}$$

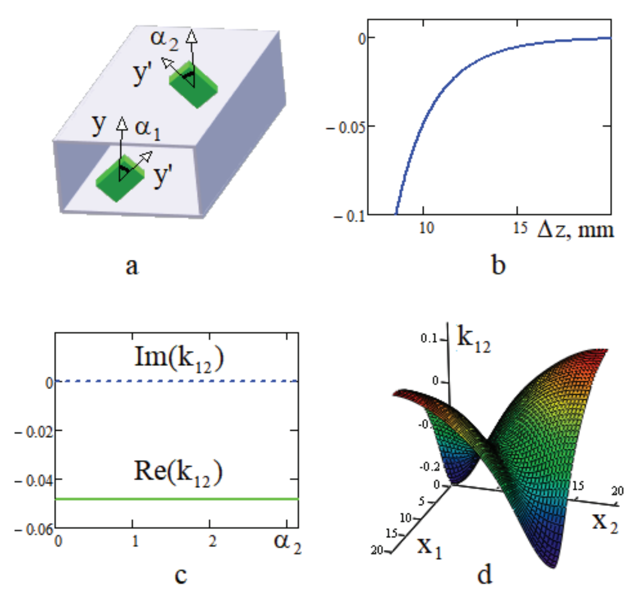

Fig. 4. Two rectangular dielectric resonators rotating relatively z - axis (a) of the waveguide. Variation of the mutual coupling coefficients at different longitudinal distances between the centres dielectric resonators for $a\times b=20\times15\text{mm}^2$; $\varepsilon_{1r}=36$; $a_0/b_0=1$; $L/a_0=0,4$; $x_1=x_2=a/2$; $y_1=y_2=b/2$; $\alpha_1=\alpha_2=0$ and $\alpha_1=0$, $\alpha_2=\pi/2$ (b). Dependence of the mutual coupling coefficient on the angle of rotation of the second resonator for $\alpha_1=0$; $\Delta z=10\text{mm}$ (c, d). Mutual coupling coefficient on the coordinates (x_1,x_2); $\alpha_1=0$; $\alpha_2=\pi/4$; $y_1=y_2=b/2$ (d).

In the case of rotation of the dielectric resonator relative z-axis at the initial position of the resonator y' - axis parallel to the waveguide y -axis (Fig. 5, a):

$$f_t^n(\mp i\Gamma)=i\cdot\Big\{-E^*_{x0}\,\beta_x\sin\alpha_n\varpi_x(\mp i\gamma)\cdot$$
$$\left[\omega_z(\xi_{sx}\cos\alpha_n-\eta_{uy}\sin\alpha_n)\omega_y(\xi_{sx}\sin\alpha_n+\eta_{uy}\cos\alpha_n)\sin(\chi_{sx}x_n+\chi_{uy}y_n)-\right.$$
$$\left.-\omega_z(\xi_{sx}\cos\alpha_n+\eta_{uy}\sin\alpha_n)\omega_y(\xi_{sx}\sin\alpha_n-\eta_{uy}\cos\alpha_n)\sin(\chi_{sx}x_n-\chi_{uy}y_n)\right]-$$
$$-E^*_{y0}\beta_x\cos\alpha_n\varpi_x(\mp i\gamma)\cdot$$
$$\left[\omega_z(\xi_{sx}\cos\alpha_n-\eta_{uy}\sin\alpha_n)\omega_y(\xi_{sx}\sin\alpha_n+\eta_{uy}\cos\alpha_n)\sin(\chi_{sx}x_n+\chi_{uy}y_n)+\right.$$
$$\left.+\omega_z(\xi_{sx}\cos\alpha_n+\eta_{uy}\sin\alpha_n)\omega_y(\xi_{sx}\sin\alpha_n-\eta_{uy}\cos\alpha_n)\sin(\chi_{sx}x_n-\chi_{uy}y_n)\right]-$$
$$+iE^*_{z0}\beta_y\omega_x(\mp i\gamma)\cdot \qquad (8)$$

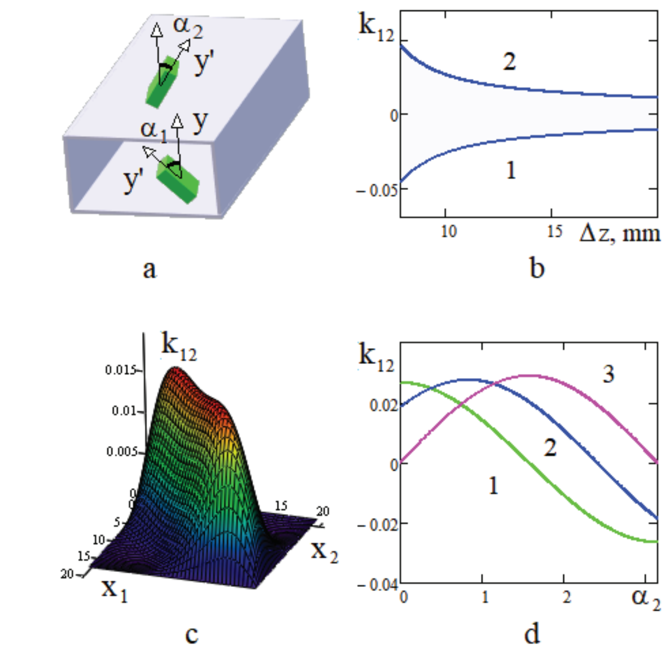

Fig. 5. Two rectangular dielectric resonators rotating relatively the z - axis (a). Variation of the mutual coupling coefficients at different longitudinal distances between dielectric resonators for $a\times b=20\times15\text{mm}^2$ for $\varepsilon_{1r}=36$; $a_0/b_0=1$; $L/a_0=0,4$; $x_1=x_2=a/2$; $y_1=y_2=b/2$; 1 - $\alpha_1=\alpha_2=0$; 2 - $\alpha_1=0$, $\alpha_2=\pi$ (b). Mutual coupling coefficient on the coordinates (x_1,x_2): $\alpha_1=0$; $\alpha_2=\pi/4$; $y_1=y_2=b/2$ (c). Dependence of the mutual coupling coefficient on the angle of rotation of the second resonator for: 1 - $\alpha_1=0$; 2 - $\alpha_1=\pi/4$; 3 - $\alpha_1=\pi/2$ (d).

Calculation and analysis of mutual coupling coefficients

Relations (4–8) were used for calculations mutual coupling coefficient dependences.

In Fig. 2-5, b depending on the coupling coefficients on the longitudinal distance between resonator centres with $\varepsilon_{1r}=36$; $a_0=b_0$; $L/a_0=0,4$. The cross section of the waveguide was $a\times b=20\times15$ mm; frequency of the fundamental magnetic oscillation of the resonator

H_{111} (for $h_z = h_1 \cos\beta_x x' \cos\beta_y y' \cos\beta_z z'$ in (1)) and $f = 7\,\text{GHz}$.

As follows from the indicated dependencies, the coupling coefficients can take both positive and negative values. This can be due to physical and mathematical reasons. If co-directional z'-axes of the resonators with H_{111} oscillations lie on the z-axis of the waveguide and $\Delta z > L$, the mutual coupling coefficient is negative. If the co-directed z'- axes are parallel and the resonators are coupled along the side wall, the coupling coefficients are usually positive for H_{111} oscillations. If there is continuous rotation between these two positions of the DR, it leads to a change in the sign of the coupling (see fig. 2, 3, b - e). However, the rotation of one of the resonators at an angle π can also lead to a change in the sign of the coupling. This purely mathematically compensates for the change in the direction of the natural oscillation field of one of the resonators (see, for example, fig. 2, b or fig. 5, b).

As can be seen from the results of calculations, in spite of the fact that the coupling functions are complex (see (4) - (8)), all coupling coefficients of the resonators in the rectangular cut-off waveguide are purely real (see fig. 3, 4, c).

For the dipole type of natural oscillations H_{111}, the field distribution is close to azimuthally uniform, therefore, the rotation of the resonators about the z' (fig. 1) axis does not change the value of the coupling, or does not change significantly (see fig. 4, c).

A more nontrivial case takes place at the rotation of both resonators relatively the x- axis (fig. 2, e), there is an angle at which the coupling coefficient weakly depends on the transverse coordinates of their centres. For the considered cross-section of the cut-off waveguide with tilt of the resonators with $\beta_1 = \beta_2 = 1{,}175$, the indicated phenomenon is demonstrated by the curves in Fig. 2, f.

As expected, when the transverse coordinates are varied, the maximum coupling is achieved at the walls of the waveguide with the longitudinal arrangement of the resonator axes (Fig. 2, 4, d), and with the transverse arrangement of the axes, this maximum lies near the waveguide axis (Fig. 5, c).

Conclusions

In the paper, new analytical expressions are obtained for the mutual coupling coefficients for rectangular dielectric resonators in the rectangular waveguide, when the resonators are rotated relative to one of the waveguide axis.

The dependences of the coupling coefficients on the angles of rotation of the axes of dielectric resonators are studied.

It is shown that the change in the sign of the coupling can be due to both the physical nature and the spatial change in the direction of the field of natural oscillations of the resonators.

In the case of rotation about the x axis, the existence of an angle is shown at which the coupling coefficients weakly depend on the second resonator coordinate in the waveguide symmetry plane $y = b/2$.

With the longitudinal arrangement of the axes of resonators with magnetic types of oscillations H_{111} relative to the axis of the waveguide, a weak dependence of the coupling on the angles of rotation has been established.

The obtained analytical relations can be used to develop mathematical models for a wide class of elements of telecommunication systems, such as filters, channel dividers, multiplexers, to name a few.

References

1. Fesenko V.I., Kupriianov A.S., Sherbinin A., Trubin A.A., Tuz V.R.. All-dielectric Vogel metasurface antennas with bidirectional radiation pattern, Journal of Optics, UK, 22, 035104, 2020, pp. 1- 8.
2. Trubin A.A. Adaptive planar antennas on lattices of rotating Dielectric Resonators. 14 Int. Scientific Conf. Modern Challenges in Telecommunications. Kiev, 2020, pp. 89-91.
3. Shi T., Wang Y., Deng Z.-L., Ye X., Dai Z., Cao Y., Guan B.-O., Xiao S., Li X. All-Dielectric Kissing-Dimer Metagratings for Asymmetric High Diffraction, Adv. Optical Mater. WILEY-VCH Verlag, 2019, pp. 1901389 - 1901389.
4. Wang X., Nie Z., Liang Y., Wang J., Li T., Jia B. Recent advances on optical vortex generation, Review article. Nanophotonics 2018; 7(9): pp. 1533–1556
5. Sallehuddin N.F., Jamaluddin M.H., Kamarudin M.R., Dahri M.H., Anuar S.U.T. Dielectric Resonator Reflectarray Antenna Unit Cells for 5G Applications // International Journal of Electrical and Computer Engineering (IJECE) 2018, Vol. 8, No. 1, February pp. 101 - 109.
6. Pidgurska T.V. Trubin A.A. Dual-bandpass filter built on rectangular dielectric resonators, Radioelectronics and Communications Systems, 2015. Vol. 58. pp.174–180.

7. Iveland T.D. Dielectric Resonator Filters for Application in Microwave Integrated Circuites // IEEE Trans. on MTT. 1971, MTT-19, No 7, pp. 643 - 652.
8. Wang K., Titchener J. G., Kruk S.S., Xu L., Chung H.-P., Parry M., Kravchenko I. I., Chen Y.-H., Solntsev A. S., Kivshar Y. S., Neshev D. N., Sukhorukov A. A. Quantum metasurface for multi-photon interference and state reconstruction / Science. 2018. vol. **361** (6407), pp. 1104-1108. http://science.sciencemag.org/content/361/6407/1104
9. Trubin A.A. , Kupriianov A.S., Fesenko V.I., Tuz V.R. Coupling coefficients for dielectric cuboids located in free space // Applied Optics. OSA, 2020 / Vol. 59, No 23/10. pp. 6918 - 6924.
10. Ilchenko M.E., Trubin A.A. Electrodynamics of Dielectric Resonators. Kiev. Naukova Dumka. 2004, 265 p.

Трубін О.О.

Коеффіцієнти зв'язку обертових прямокутних діелектричних резонаторів з позамежним прямокутнім хвилеводом при обертанні їх осей

Проблематика. Підвищення швидкості передачі інформації визначається жорсткими вимогами, що пред'являються до елементної бази приймально-передавальних пристроїв. Одної із важливих складових частин таких пристроїв є смугові фільтри, яки часто виконуються на діелектричних резонаторах. Розрахунок параметрів багатоланкових смугових фільтрів неможливо без подальшого розвитку теорії їх проектування. Розвиток теорії фільтрів базується на електродинамічному моделюванні яке ґрунтується на попередніх розрахунках коєфіцієнтів взаємного зв'язку діелектричних резонаторів в різноманітних лініях передачі.

Мета дослідження. Метою досліджень є розрахунок та дослідження коєфіцієнтів взаємного зв'язку прямокутних діелектричних резонаторів які розташовані у прямокутному металевому хвилеводі при застосуванні нових структур з обертанням їх осей. Дослідження нових ефектів, що дозволяють покращувати характеристики розсіювання смугових фільтрів та інших пристрів на їх основі.

Методика реалізації. Для розрахунку та аналізу коєфіцієнтів взаємного зв'язку використовуються методи технічної електродинаміки. Кінцевим результатом є отримання нових аналітичних формул для нових структур з прямокутними діелектричними резонаторами, що дозволяють аналізувати і розраховувати їх коефіцієнти взаємного зв'язку.

Результати дослідження. Знайдено нові аналітичні вирази для коефіцієнтів взаємного зв'язку діелектричних резонаторів з обертанням їх осей в прямокутному позамежному хвилеводі.

Висновки. Розширено теорія конструювання фільтрів на нових структурах діелектричних резонаторів з обертанням їх осей в металевих хвилеводах. Знайдено нові аналітичні співвідношення та досліджені нові закономірності зміни коефіцієнтів зв'язку.

Ключові слова: діелектричний фільтр; прямокутний діелектричний резонатор; обертання; коефіцієнти взаємного зв'язку.

Трубин А.А.

Коэффициенты связи прямоугольных диэлектрических резонаторов с прямоугольным волноводом при вращении их осей

Проблематика. Дальнейшее повышение скорости передачи информации определяется более жесткими требованиями, предъявляемыми к элементам приемо-передающих устройств. Одной из важнейших составных частей таких устройств являются различные фильтры, которые часто выполняются на диэлектрических резонаторах. Расчет параметров многозвенных фильтров невозможен без дальнейшего развития теории их проектирования. Развитие теории фильтров базируется на электродинамическом моделировании, которое предполагает расчет коэффициентов связи диэлектрических резонаторов в различных линиях передачи.

Цель исследования. Целью исследований является расчет и исследование коэффициентов связи прямоугольных диэлектрических резонаторов с прямоугольным металлическим волноводом при вращении их осей. Исследование новых эффектов, позволяющих улучшать характеристики фильтров и других устройств на их основе.

Методика реализации. Для проведения расчета и анализа коэффициентов связи используются методы технической электродинамики. Конечным результатом является получение новых аналитических формул для новых структур с прямоугольными диэлектрическими резонаторами, которые позволяют анализировать и рассчитывать их коэффициенты связи.

Результаты исследования. Найдены новые аналитические выражения для коэффициентов связи диэлектрических резонаторов с вращением их осей в прямоугольном волноводе.

Выводы. Расширена теория конструирования фильтров на новых структурах диэлектрических резонаторов с вращением их осей в металлических волноводах. Найдены новые аналитические соотношения и новые закономерности изменения коэффициентов связи.

Ключевые слова: диэлектрический фильтр; прямоугольный диэлектрический резонатор; вращение; коэффициенты связи.