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COUPLING COEFFICIENTS OF DIFFERENT SPHERICAL DIELECTRIC MICRORESONATORS

A. A. Trubin

The Institute of Telecommunication Systems
Igor Sikorsky Кyiv Politechnic Institute, Кyiv, Ukraine

Background. Nowadays, further increase of processing and transmission speed of information is associated with the development of hybrid integrated circuits, combining electrical and optical components. One of the important constituent parts of future optical integrated circuits are filters that can be conveniently implemented using so-called disc microresonators with whispering gallery oscillations. Technically, the problem of manufacturing such filters in the infrared and even in the visible wavelength range has been solved, but calculation of parameters and tuning of multilink filters is impossible without further development of the theory of their building. The development of such a theory is based on the electrodynamic modeling of processes that occur in complex systems of coupled microcavities, coupled with transmission line. At present, the study of filters that built on different microresonators hasn't been carried out.

Objective. The aim of the research is to construct the theory of electromagnetic wave scattering of the integral optical transmission lines on systems of coupled diversiform optical microresonators with whispering gallery modes, as well as development of mathematical models of filters constructed using various disk microresonators, and investigation of new structures of coupled microresonators with acceptable scattering characteristics.

Methods. To construct a mathematical model of filters, an approximate solution of the Maxwell equations based on perturbation theory is used. The application of perturbation theory made it possible to find a solution to the problem of calculating the S-matrix of the filter in an analytical form.

Results. An electrodynamic model for the scattering of optical transmission line waves by a system of coupled diversiform microresonators made of different dielectrics is developed. New structures of microresonators, realizing bandpass and bandstop filters, are investigated and their scattering characteristics are calculated.

Conclusions. The theory of scattering of electromagnetic waves by systems of coupled diversiform optical microresonators is expanded. A new definition of the coupling coefficients of different microresonators is given. New filter models are constructed.

Keywords: infrared range; integrated optics; optical filter; bandstop filter; bandpass filter; microresonator.

Introduction

Spherical shape is the most convenient for manufacturing of resonators in the optical wavelength range. In addition, the quality factor of such resonators for whispering gallery modes is one of the highest. All this makes spherical microresonators the most attractive elements of optical devices. Today the spherical microresonators are being actively studied with a view to their subsequent application [1 - 15].

The possibility of using microresonators of various forms in the filters and other optical devices significantly expands the capabilities of developers. To calculate parameters of the microresonators in various structures knowledge of its mutual coupling coefficients is assumed. Mutual coupling coefficients of different spherical dielectric microresonators yet to be fully studied.

Statement of the problem

The aim of the paper is the calculation, analysis and study of the coupling coefficients of different spherical microresonators with whispering gallery modes in the open space, as well as investigation of new structures of microresonators for the construction of filters and other devices of infrared and optical wavelengths, and development of mathematical models of infrared wavelength filters on different spherical microresonators with improved scattering parameters.

Classification of the modes of spherical microresonators

To calculate the coupling coefficients information on the fields of the isolated microresonators is needed. The eigenmode field of the spherical microresonator is well known [16].

In local spherical coordinate system, associated with microresonator center (Fig. 1, a), the field of

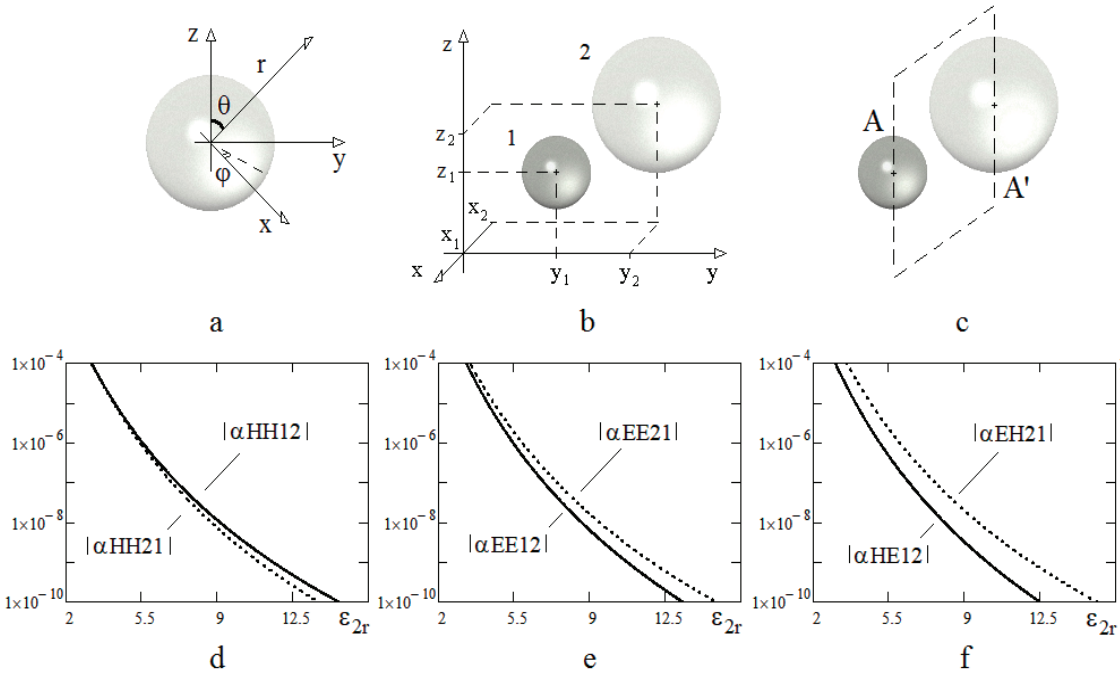

Fig. 1. The spherical microresonator in the open space (a). Two spherical microresonators in the rectangular coordinate system (b). Plane of symmetry AA' of two different spherical microresonators (c). Dependence of module of coupling coefficient multipliers $|\alpha|$ as a function of the second microresonator dielectric permittivity (d-f).

the H_{nml} (or E_{nml}) mode is described by radial magnetic-field (or electric-field) component in the local spherical coordinate system (r,θ,φ) (Fig. 1, a):

$$h_r(e_r)=\frac{n(n+1)}{r}f_n^{(s)}(k_s r)P_n^m(\cos\theta)\begin{bmatrix}\sin\\ \cos\end{bmatrix} m\varphi, \quad (1)$$

Here $k_s=\omega\sqrt{\varepsilon_s\mu_0}$ is the wave number in s-th dielectric ($s=0,1$); $P_n^m(\cos\theta)$ - is the associated Legendre polynomial; $f_n^{(s)}(z)$ - one of the spherical Bessel functions: $j_n(z)=(\pi/2z)^{1/2}J_{n+1/2}(z)$; $y_n(z)=(\pi/2z)^{1/2}Y_{n+1/2}(z)$; $h_n^{(2)}(z)=(\pi/2z)^{1/2}H_{n+1/2}^{(2)}(z)$ [17], or their linear combinations.

The indexes $n>0$; $0\le m\le n$ are integers; they determine a number of the field's variations in the meridional plane: $\varphi=\text{const}$ and on the surface: $\theta=\text{const}$, respectively (Fig. 1, a). In the dielectric volume ($r\le r_0$): $f_n^{(1)}(k_1 r)=a_1 j_n(k_1 r)$; in the open space: ($r\ge r_0$): $f_n^{(0)}(k_0 r)=a_0 h_n^{(2)}(k_0 r)$ (r_0 - is the radius of the microresonator). Satisfaction to the boundary conditions leads to determination of the unknown constants ($a_0; a_1$). At that, the characteristic parameters $p=k_1 r_0$, $q=k_0 r_0$ would satisfy to the equations [16].

Coupling coefficient calculating

The coupling coefficients of the spherical microresonators can be obtained from already known formulae. We used analytical formulae for the coupling coefficients of the different spherical microresonators, situated in the metal rectangular waveguide [18]. When the waveguide walls tend to infinity, the sums on the waveguide wave numbers transform to the integrals on non-dimensional parameters. As a result of integration next relationships for different spherical microresonators have been obtained

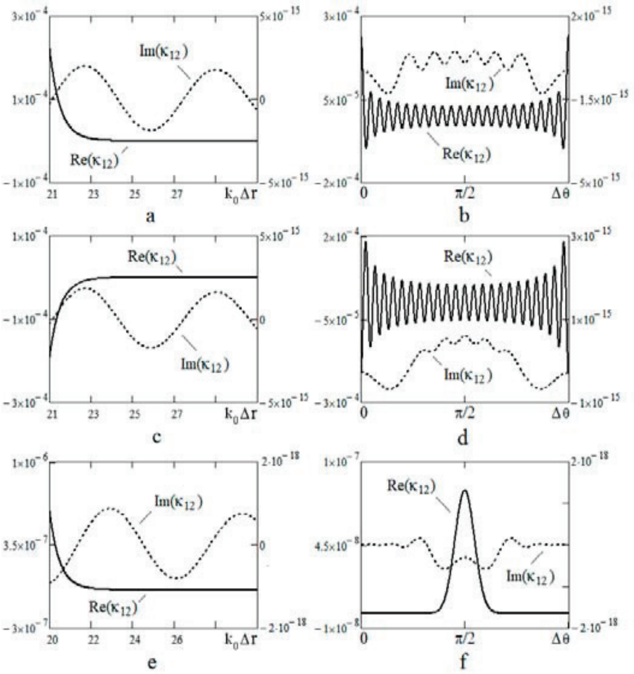

Fig. 2. The coupling coefficients as functions of the distance between microresonator centers for the same parity whispering gallery modes $H_{n_1m_1l_1}$, $H_{n_2m_2l_2}$ ($n_1 = 22$; $n_2 = 24$; $\varepsilon_{1r} = 4$; $\varepsilon_{2r} = 16$). A, b: $m_1 = m_2 = 0$; c, d: $m_1 = m_2 = 1$; e, f: $m_1 = n_1$; $m_2 = n_2$. A, c: $\Delta\theta = 0,01$; e: $\Delta\theta = \pi/2$; a, c, e: $k_0\Delta r = 21$. The ordinate axes on the left correspond to the real part, and the y-axis to the right of the imaginary part of the coupling coefficients.

1) with magnetic mode $H_{n_1m_1l_1}$ in the first microresonator, the magnetic-type modes in the second microresonator $H_{n_2m_2l_2}$ and also for the same parity with respect to m_1, m_2:

$$\kappa_{12} = \kappa_0 \alpha_{n_1n_2}^{HH}(p_1,q_1;p_2,q_2)\cdot \qquad (2)$$

$$\cdot\int_0^\infty e^{-i\gamma k_0\Delta z}\{[(1-\gamma^2)^2 \frac{dP_{n_1}^{m_1}(\gamma)}{d\gamma}\cdot\frac{dP_{n_2}^{m_2}(\gamma)}{d\gamma} + m_1m_2P_{n_1}^{m_1}(\gamma)P_{n_2}^{m_2}(\gamma)]J_{m_1-m_2}(\eta k_0\Delta\rho)\mp$$

$$\mp(-1)^{m_2}[(1-\gamma^2)^2 \frac{dP_{n_1}^{m_1}(\gamma)}{d\gamma}\cdot\frac{dP_{n_2}^{m_2}(\gamma)}{d\gamma} - m_1m_2P_{n_1}^{m_1}(\gamma)P_{n_2}^{m_2}(\gamma)]J_{m_1+m_2}(\eta k_0\Delta\rho)\}\frac{d\eta}{\gamma\eta}$$

In the case of electric oscillations $E_{n_1m_1l_1}$, $E_{n_2m_2l_2}$, the function $\alpha_{n_1n_2}^{HH}$ should be replaced by $\alpha_{n_1n_2}^{EE}$.

2) with modes $H_{n_1m_1l_1}$, $E_{n_2m_2l_2}$ and for the same parity with respect to m_1, m_2:

$$\kappa_{12} = \kappa_0 w_0 \alpha_{n_1n_2}^{HE}(p_1,q_1;p_2,q_2)\cdot \qquad (3)$$

$$\cdot\int_0^\infty e^{-i\gamma k_0\Delta z}\{\mp[\frac{dP_{n_1}^{m_1}(\gamma)}{d\gamma}m_2P_{n_2}^{m_2}(\gamma) + m_1P_{n_1}^{m_1}(\gamma)\frac{dP_{n_2}^{m_2}(\gamma)}{d\gamma}]J_{m_1-m_2}(\eta k_0\Delta\rho)+$$

$$+(-1)^{m_2}[\frac{dP_{n_1}^{m_1}(\gamma)}{d\gamma}m_2P_{n_2}^{m_2}(\gamma) - m_1P_{n_1}^{m_1}(\gamma)\frac{dP_{n_2}^{m_2}(\gamma)}{d\gamma}]J_{m_1+m_2}(\eta k_0\Delta\rho)\}\frac{\eta d\eta}{\gamma}$$

3) with modes $E_{n_1m_1l_1}$, $H_{n_2m_2l_2}$:

$$\kappa_{12} = \frac{\kappa_0}{w_0}\alpha_{n_1n_2}^{EH}(p_1,q_1;p_2,q_2)\cdot \qquad (4)$$

$$\cdot\int_0^\infty e^{-i\gamma k_0\Delta z}\{\mp[\frac{dP_{n_1}^{m_1}(\gamma)}{d\gamma}m_2P_{n_2}^{m_2}(\gamma) + m_1P_{n_1}^{m_1}(\gamma)\frac{dP_{n_2}^{m_2}(\gamma)}{d\gamma}]J_{m_1-m_2}(\eta k_0\Delta\rho)+$$

$$+(-1)^{m_2}[\frac{dP_{n_1}^{m_1}(\gamma)}{d\gamma}m_2P_{n_2}^{m_2}(\gamma) - m_1P_{n_1}^{m_1}(\gamma)\frac{dP_{n_2}^{m_2}(\gamma)}{d\gamma}]J_{m_1+m_2}(\eta k_0\Delta\rho)\}\frac{\eta d\eta}{\gamma}$$

For different parity with respect to m_1, m_2, the sign (4) changes to the opposite.

Here $w_0 = \sqrt{\mu_0/\varepsilon_0} = 120\pi$ - is the wave impedance of the open space;

$$\kappa_0 = 2\frac{i^{1+n_1-n_2-(m_1-m_2)}}{(1+\delta_{m_20})}\cdot\frac{(2n_2+1)}{n_2(n_2+1)}\cdot\frac{(n_2-m_2)!}{(n_2+m_2)!};$$

$$\alpha^{HH}_{n_1n_2}(p_1,q_1;p_2,q_2)=\frac{j_{n_1}(p_1)}{y_{n_1}(q_1)}\cdot\frac{1}{q_2}\frac{j_{n_2}(p_2)}{y_{n_2}(q_2)}/$$

$$/\left\{\left[p_2^2-n_2(n_2+1)\right]j_{n_2}^2(p_2)+\left[n_2j_{n_2}(p_2)-p_2j_{n_2-1}(p_2)\right]^2\right\};$$

$$\alpha^{EE}_{n_1n_2}(p_1,q_1;p_2,q_2)=\varepsilon_{1r}\frac{j_{n_1}(p_1)}{y_{n_1}(q_1)}\cdot\frac{1}{q_2}\frac{j_{n_2}(p_2)}{y_{n_2}(q_2)}/ \quad (5)$$

$$/\left\{\left[p_2^2-n_2(n_2+1)\right]j_{n_2}^2(p_2)+\left[n_2j_{n_2}(p_2)-p_2j_{n_2-1}(p_2)\right]^2\right\};$$

$$\alpha^{HE}_{n_1n_2}(p_1,q_1;p_2,q_2)=\frac{j_{n_1}(p_1)}{y_{n_1}(q_1)}\cdot\frac{1}{q_2}\frac{j_{n_2}(p_2)}{y_{n_2}(q_2)}/$$

$$/\left\{\left[p_2^2-n_2(n_2+1)\right]j_{n_2}^2(p_2)+\left[n_2j_{n_2}(p_2)-p_2j_{n_2-1}(p_2)\right]^2\right\};$$

- is the functions, determining the dependence of coupling coefficients on the parameters of oscillations and material of microresonators; $\gamma=\sqrt{1-\eta^2}$; $\Delta\rho=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$; $\Delta z=|z_2-z_1|$ (Fig. 1, b). Where also $p_s=k_sr_s$ and $q_s=k_0r_s$ - is the characteristic parameters of the spherical microresonator modes [16]; $k_s=\omega\sqrt{\varepsilon_s\mu_0}$ ($s=0,1,2$); ω - is the circular frequency.

We examined the case, when the microsphere centers situated in the plane of symmetry AA', parallel to the plane xz: $y_1=y_2$ (Fig. 1, c). The upper and lower signs in (2-4) correspond to the parity of the field shown in (1), with respect to the plane $\varphi=0$.

In the special cases of the identical microresonators, the obtained formula (2, 3) coincides with previously known [19].

The integrals (4 - 6) can be calculated, if used [19]:

$$\int_0^\infty e^{-ib\sqrt{1-t^2}}J_m(ct)P_n^m\left(\sqrt{1-t^2}\right)t\,dt/\sqrt{1-t^2}=$$

$$=i^{(m-n)}P_n^m\left(b/\sqrt{b^2+c^2}\right)h_n^{(2)}\left(\sqrt{b^2+c^2}\right),\quad (6)$$

(b, c - are the real constants), as well as expansions:

$$[(1-\gamma^2)\frac{dP^{m_1}_{n_1}(\gamma)}{d\gamma}\cdot\frac{dP^{m_2}_{n_2}(\gamma)}{d\gamma}+m_1m_2\frac{P^{m_1}_{n_1}(\gamma)P^{m_2}_{n_2}(\gamma)}{(1-\gamma^2)}]=$$

$$=\sum_{s=|n_1-n_2|}^{n_1+n_2}a^{m_1m_2}_{n_1n_2s}P_s^{|m_1-m_2|}(\gamma);$$

$$[(1-\gamma^2)\frac{dP^{m_1}_{n_1}(\gamma)}{d\gamma}\cdot\frac{dP^{m_2}_{n_2}(\gamma)}{d\gamma}-m_1m_2\frac{P^{m_1}_{n_1}(\gamma)P^{m_2}_{n_2}(\gamma)}{(1-\gamma^2)}]=$$

$$=\sum_{s=|n_1-n_2|}^{n_1+n_2}b^{m_1m_2}_{n_1n_2s}P_s^{m_1+m_2}(\gamma);$$

$$[\frac{dP^{m_1}_{n_1}(\gamma)}{d\gamma}m_2P^{m_2}_{n_2}(\gamma)+m_1P^{m_1}_{n_1}(\gamma)\frac{dP^{m_2}_{n_2}(\gamma)}{d\gamma}]=$$

$$=\sum_{s=|n_1-n_2|}^{n_1+n_2}c^{m_1m_2}_{n_1n_2s}P_s^{|m_1-m_2|}(\gamma);$$

$$[m_1P^{m_1}_{n_1}(\gamma)\frac{dP^{m_2}_{n_2}(\gamma)}{d\gamma}-\frac{dP^{m_1}_{n_1}(\gamma)}{d\gamma}m_2P^{m_2}_{n_2}(\gamma)]=$$

$$=\sum_{s=|n_1-n_2|}^{n_1+n_2}d^{m_1m_2}_{n_1n_2s}P_s^{m_1+m_2}(\gamma);\quad (7)$$

The first two expansions (7) are easy to determine if we use the well-known decompositions [20, 21]:

$$P^{m_1}_{n_1}(z)P^{m_2}_{n_2}(z)=(-1)^{m_1}\sqrt{\frac{(n_1+m_1)!(n_2+m_2)!}{(n_1-m_1)!(n_2-m_2)!}}\cdot$$

$$\cdot\sum_{s=|n_1-n_2|}^{n_1+n_2}G_s^-\sqrt{\frac{(s-|m_1-m_2|)!}{(s+|m_1-m_2|)!}}P_s^{|m_1-m_2|}(z);$$

$$P^{m_1}_{n_1}(z)P^{m_2}_{n_2}(z)=\sqrt{\frac{(n_1+m_1)!(n_2+m_2)!}{(n_1-m_1)!(n_2-m_2)!}}\cdot$$

$$\cdot\sum_{s=|n_1-n_2|}^{n_1+n_2}G_s^+\sqrt{\frac{(s-m_1-m_2)!}{(s+m_1+m_2)!}}P_s^{m_1+m_2}(z),$$

where

$$G_s^{\pm}=(-1)^{m_1\pm m_2}(2s+1)\begin{pmatrix}n_1 & n_2 & s\\ 0 & 0 & 0\end{pmatrix}\cdot$$

$$\cdot\begin{pmatrix}n_1 & n_2 & s\\ \pm m_1 & m_2 & -|m_1\pm m_2|\end{pmatrix},$$

and $\begin{pmatrix} a & b & c \\ \alpha & \beta & \gamma \end{pmatrix}$ - is the 3-j Wigner symbol [17]. Using the recurrence relations for the Legendre polynomials and (6, 7), we obtain:

$$a^{m_1m_2}_{n_1n_2s} = (-1)^{m_2}\sqrt{\frac{(n_1+m_1)!(n_2+m_2)!}{(n_1-m_1)!(n_2-m_2)!}}\sqrt{\frac{(s-|m_1-m_2|)!}{(s+|m_1-m_2|)!}}\cdot$$
$$\cdot\{m_1m_2G^-_s(m_1,m_2)-$$
$$-\frac{1}{2}\sqrt{(n_1-m_1)(n_1+m_1+1)(n_2-m_2)(n_2+m_2+1)}\cdot$$
$$\cdot G^-_s(m_1+1,m_2+1)-$$
$$-\frac{1}{2}\sqrt{(n_1+m_1)(n_1-m_1+1)(n_2+m_2)(n_2-m_2+1)}\cdot$$
$$\cdot G^-_s(m_1-1,m_2-1)\};$$
$$b^{m_1m_2}_{n_1n_2s} = -\sqrt{\frac{(n_1+m_1)!(n_2+m_2)!}{(n_1-m_1)!(n_2-m_2)!}}\sqrt{\frac{(s-m_1-m_2)!}{(s+m_1+m_2)!}}\cdot$$
$$\cdot\{m_1m_2G^+_s(m_1,m_2)+$$
$$+\frac{1}{2}\sqrt{(n_1-m_1)(n_1+m_1+1)(n_2+m_2)(n_2-m_2+1)}\cdot$$
$$\cdot G^+_s(m_1+1,m_2-1)+$$
$$+\frac{1}{2}\sqrt{(n_1+m_1)(n_1-m_1+1)(n_2-m_2)(n_2+m_2+1)}\cdot$$
$$\cdot G^+_s(m_1-1,m_2+1)\} \qquad (8)$$

The coefficients of the last two expansions (7) are calculated by using:

$$c^{m_1m_2}_{n_1n_2s} = \frac{(2s+1)(s-|m_1-m_2|)!}{2(s+|m_1-m_2|)!}\cdot$$
$$\cdot\int_{-1}^{1}[(1-x^2)\frac{dP^{m_1}_{n_1}(x)}{dx}\cdot\frac{dP^{m_2}_{n_2}(x)}{dx}+m_1m_2\frac{P^{m_1}_{n_1}(x)P^{m_2}_{n_2}(x)}{(1-x^2)}]P^{|m_1-m_2|}_s(x)dx$$
$$d^{m_1m_2}_{n_1n_2s} = \frac{(2s+1)(s-m_1-m_2)!}{2(s+m_1+m_2)!}\cdot \qquad (9)$$
$$\cdot\int_{-1}^{1}[(1-x^2)\frac{dP^{m_1}_{n_1}(x)}{dx}\cdot\frac{dP^{m_2}_{n_2}(x)}{dx}-m_1m_2\frac{P^{m_1}_{n_1}(x)P^{m_2}_{n_2}(x)}{(1-x^2)}]P^{m_1+m_2}_s(x)dx$$

Substituting (7) in the (2-4) and using (6), obtains finally:

1) for $H_{n_1m_1l_1}$, $H_{n_2m_2l_2}$ and for the same parity with respect to m_1, m_2:

$$\kappa_{12} = \kappa_0\cdot\alpha^{HH}_{n_1n_2}(p_1,q_1;p_2,q_2)\cdot \qquad (10)$$
$$\cdot\sum_{s=|n_1-n_2|}^{n_1+n_2} i^{m_1-m_2-s}[a^{m_1m_2}_{n_1n_2s}P^{|m_1-m_2|}_s(\cos\Delta\theta)\mp$$
$$\mp b^{m_1m_2}_{n_1n_2s}P^{m_1+m_2}_s(\cos\Delta\theta)]h^{(2)}_s(k_0\Delta r);$$

2) for $H_{n_1m_1l_1}$, $E_{n_2m_2l_2}$ and for the same parity with respect to m_1, m_2:

$$\kappa_{12} = \kappa_0 w_0\cdot\alpha^{HE}_{n_1n_2}(p_1,q_1;p_2,q_2)\cdot \qquad (11)$$
$$\cdot\sum_{s=|n_1-n_2|}^{n_1+n_2} i^{m_1-m_2-s}[\mp c^{m_1m_2}_{n_1n_2s}P^{|m_1-m_2|}_s(\cos\Delta\theta)-$$
$$-d^{m_1m_2}_{n_1n_2s}P^{m_1+m_2}_s(\cos\Delta\theta)]h^{(2)}_s(k_0\Delta r);$$

3) for $E_{n_1m_1l_1}$, $H_{n_2m_2l_2}$:

$$\kappa_{12} = \frac{\kappa_0}{w_0}\cdot\alpha^{EH}_{n_1n_2}(p_1,q_1;p_2,q_2)\cdot \qquad (12)$$
$$\cdot\sum_{s=|n_1-n_2|}^{n_1+n_2} i^{m_1-m_2-s}[\mp c^{m_1m_2}_{n_1n_2s}P^{|m_1-m_2|}_s(\cos\Delta\theta)-$$
$$-d^{m_1m_2}_{n_1n_2s}P^{m_1+m_2}_s(\cos\Delta\theta)]h^{(2)}_s(k_0\Delta r).$$

Here the $\Delta r = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$ is the relative distance between the centers of the microresonators (Fig. 1, b); $\cos\Delta\theta = (z_1-z_2)/\Delta r$.

Coupling coefficient analysis

The relationships (10)-(12) was used for the calculating of the coupling coefficients for whispering gallery modes of different spherical microresonators. Fig. 1, d-f shows the dependence of the module of the coupling coefficient multipliers (5) on the dielectric constant of the second microresonator in the case of two magnetic type oscillations $H_{n_1m_1l_1}$, $H_{n_2m_2l_2}$ (d); in the case of two oscillations of the electric type $E_{n_1m_1l_1}$, $E_{n_2m_2l_2}$ (e) and for oscillations of magnetic type $H_{n_1m_1l_1}$ in the first microresonator and the electric type $E_{n_2m_2l_2}$ in the second (f). The calculations show "nonreciprocity" of the coupling coefficients for microresonators of different types.

The real part of the mutual coupling coefficient is determined by the overlap of the stored field of the microresonators, and the imaginary part is the magnitude of the radiations of the structure. If such overlap is mainly determined by the electrical components of the field, the real part of the coupling coefficient becomes negative, and if the overlap is mainly determined by the magnetic components of the field, it is positive. Thus, the dependence of the real part of the mutual coupling coefficient is determined by the distribution of the stored fields of microcavities in the near spatial region, while the imaginary part of the coupling coefficient is determined by the radiation fields of the coupled oscillations. As can be seen from the graphs shown in Fig. 2, 3, this statement is in good agreement with the results of the calculations.

And as follows from the results, presented in Fig. 2, 3, the coupling coefficients of the whispering gallery modes obtains sufficiently large values in the near-field region. At that, the relative motion in the tangent directions leads to a complex interference of their mutual influence, determining by significant eigenmode field variation nearby their surfaces (Fig. 2, b, d, f; Fig. 3, b, d). In the majority cases the imaginary part values of the coupling coefficients at least one tenth as many as it real parts.

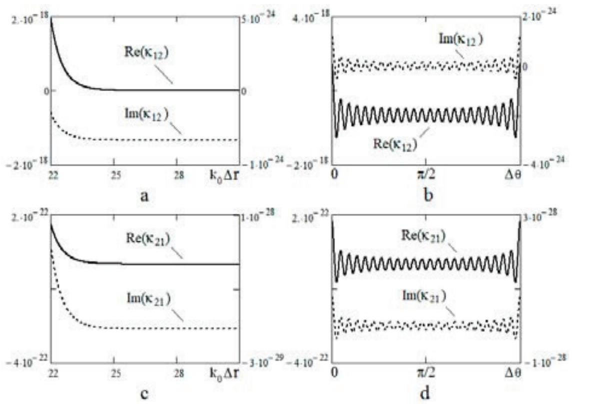

Fig.3. Coupling coefficients as functions of the microresonator relative coordinates for the $H_{n_1m_1l_1}$, $E_{n_2m_2l_2}$ oscillations (a, b); $E_{n_2m_2l_2}$, $H_{n_1m_1l_1}$ oscillations (c, d); ($n_1 = 22$; $n_2 = 24$; $m_1 = 1$; $m_2 = 1$; $\varepsilon_{1r} = 4$; $\varepsilon_{2r} = 16$). A, c: $\Delta\theta = 0{,}01$; b, d: $k_0\Delta r = 22$. The ordinate axes on the left and on the right correspond to the real and imaginary part of the coupling coefficients.

Filter parameters calculation

Obtained results allow us to design electrodynamic models of various filters in the infrared and optical wavelengths ranges. Of greatest interest is the search for new structures of microresonators that most simply realize acceptable scattering characteristics [22]. In this case it is desirable that the systems of microresonators are excited only on the given type of oscillations. In the case of band-stop filters, an additional condition is added, which is necessary to realize partially frequency-symmetric frequency response.

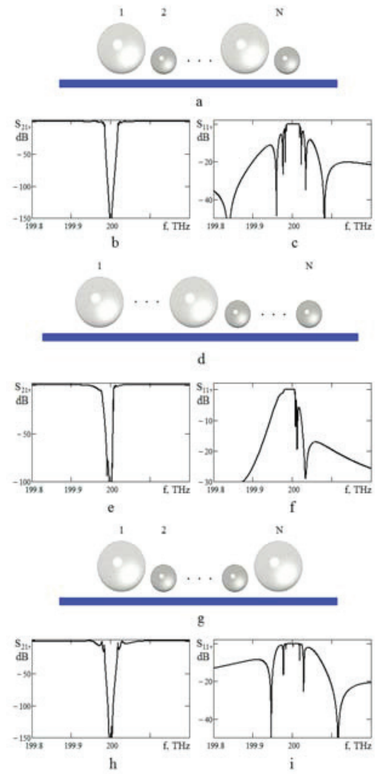

Fig. 4. Sketch of bandstop filters on coupled spherical microresonators and dielectric waveguide (a, d, g). S-matrix responses of one band of the 10-section (b, c); 8-section (e, f); 9-section (h, i) filters on spherical microresonator H_{nml} modes ($\varepsilon_{1r} = 9$; $\varepsilon_{2r} = 16$; $Q_1^D = 1\cdot 10^6$; $Q_2^D = 2\cdot 10^6$; $l_1 = l_2 = 1$; $m_1 = m_2 = n_1 = n_2 = 26$). S-parameters of the bandstop filters (b, c; e, f; h, i).

Example of calculation on the S-parameters of the bandstop filter on whispering gallery modes H_{mml} ($m = 26$) is shown in Fig. 4. The coupling coefficients between microresonators were calculated on (10). The coupling coefficients between microresonators and dielectric waveguide are equal: $k_L = 1{,}7 \cdot 10^{-4}$ (a); $k_L = 1 \cdot 10^{-4}$ (d); $k_L = 1{,}5 \cdot 10^{-4}$ (g). Here and below $S_{21} = S_{21}(f) = 20\lg|T(f)|$; $S_{11} = S_{11}(f) = 20\lg|R(f)|$, are the scattering matrix elements, where T(f) - is the transmission coefficient and R(f) - is the reflection coefficient response of the microresonator system in the optical transmission line.

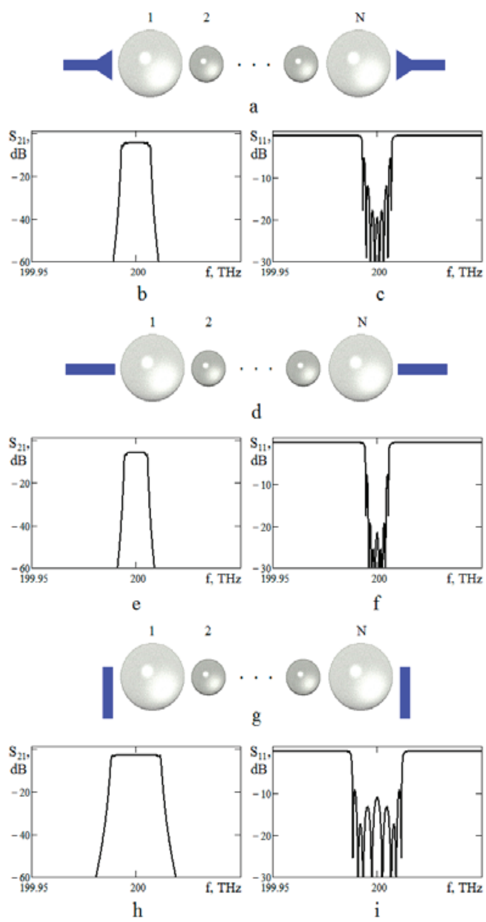

Fig. 5. Sketch of bandpass filters on different spherical microresonators (a, d, g). S-matrix responses of 8-section ($m_1 = m_2 = 0$) (b, c); ($m_1 = m_2 = 1$) (e, f) ($n_1 = 24$; $n_2 = 22$); ($m_1 = m_2 = n_1 = n_2 = 24$) (h, i) filter ($\varepsilon_{1r} = 9$; $\varepsilon_{2r} = 16$; ; $l_1 = l_2 = 1$).

The Fig. 5 shows theoretical scattering parameters of several bandpass filters on the spherical microresonators with different H_{nml} whispering gallery modes. In this case practically zero coupling between not adjacent microresonators supports a good symmetry of the S-matrix parameters relatively central frequency. The very high quality of microcavity radiation causes a minimum loss in the pass band of filters without additional screening.

Conclusions

Analytical relationships for the coupling coefficients of different spherical microresonators in the open space has been obtained and investigated.

It's shown that the whispering gallery mode coupling coefficients are described by more complicated spatial dependencies.

The real and imaginary parts of the coupling coefficients of the whispering gallery modes can be differed more than one degree.

Structures of various microcavities, most suitable for use in bandpass and bandstop filters are determined.

Mathematical models of filters are created, their scattering characteristics are adjusted and analyzed.

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Трубін О.О.

Коефіцієнти зв'язку різних сферичних діелектричних мікрорезонаторів

Проблематика. Подальше підвищення швидкості передавання інформації визначається розробкою гібридних схем, які одночасно об'єднують в собі електричні та оптичні компоненти. Одної із важливих складових частин оптичних інтегральних схем є різноманітні фільтри, яки зручно виконувати із так званих сферичних мікрорезонаторів з коливаннями шепочучої галереї. Розрахунок параметрів багатоланкових фільтрів неможлива без подальшого розвитку теорії їх проектування. Розвиток цей теорії базується на електродинамічному моделюванні процесів, які відбуваються у складних системах зв'язаних мікрорезонаторів, зв'язаних також і з лінією передачі. В наступний час дослідження фільтрів, побудованих на різних мікрорезонаторах не проводилось.

Мета досліджень. Метою досліджень є розробка теорії розсіювання електромагнітних хвиль інтегральних оптичних ліній на системах зв'язаних між собою різних оптичних мікрорезонаторів з коливаннями шепочучої галереї. Розробка математичних моделей фільтрів, виконаних із застосуванням різних сферичних мікрорезонаторів з різними видами коливань. Дослідження нових структур зв'язаних мікрорезонаторів з прийнятними характеристиками розсіювання.

Методика реалізації. Для побудови математичної моделі фільтрів застосовано приблизне вирішення системи рівнянь Максвела, засноване на застосуванні теорії збурень. Застосуванні теорії збурень дозволило знайти вирішення задачі розрахунку S - матриці фільтра у аналітичному вигляді.

Результати досліджень. Розроблена електродинамічна модель розсіювання хвиль оптичної лінії на системі зв'язаних сферичних мікрорезонаторів різних розмірів і виконаних із різного діелектрика. Досліджені нові структури мікрорезонаторів, які реалізують смугові та режекторні фільтрі та розраховані їх характеристики розсіювання.

Висновки. Розширена теорія розсіювання електромагнітних хвиль на системах різних зв'язаних оптичних мікрорезонаторів. Розраховані коефіцієнти зв'язку сферичних мікрорезонаторів у відкритому просторі. Побудовані нові моделі фільтрів.

Ключові слова: інфрачервоний діапазон; інтегральна оптика; оптичний фільтр; смуговий фільтр; режекторний фільтр; мікрорезонатор.

Трубин А.А.

Коэффициенты связи различных сферических диэлектрических микрорезонаторов

Проблематика. В настоящее время дальнейшее повышение скорости обработки и передачи информации связывается с расработкой гибридных интегральных схем, объединяющих в себе электрические и оптические компоненты. Одной из важных составляющих частей будущих оптических интегральных схем являются фильтры, которые удобно выполнять, используя так называемые дисковые микрорезонаторы с колебаниями шепчущей галлереи. Технически задача изготовления подобных фильтров в инфракрасном и даже в видимом диапазоне длин волн решена, однако расчет параметров и настройка многозвенных фильтров невозможна без дальнейшего развития теории их проектирования. Развитие такой теории основывается на электродинамическом моделировании процессов, которые происходят в сложных системах связанных микрорезонаторов, связанных также и с линией передачи. В настоящее время исследование фильтров, построенных на различных микрорезонаторах не проводилось.

Цель исследований. Целью исследований является построение теории рассеяния электромагнитных волн интегральных оптических линий передачи на системах связанных между собой различных оптических микрорезонаторов с колебаниями шепчущей галлереи. Разработка математических моделей фильтров, выполненных с применением различных дисковых микрорезонаторов. Исследование новых структур связанных микрорезонаторов с приемлемыми характеристиками рассеяния.

Методика реализации. Для построения математической модели фильтров использовано приближенное решение уравнений Максвелла, основанное на применении теории возмущений. Применение теории возмущений позволило найти решение задачи расчета S - матрицы фильтра в аналитическом виде.

Результаты исследований. Разработана электродинамическая модель рассеяния волн оптической линии на системе связанных микрорезонаторов разной формы и выполненных из разного диэлектрика. Исследованы новые структуры микрорезонаторов, реализующих полосовые и режекторные фильтры и рассчитаны их характеристики рассеяния.

Выводы. Расширена теория рассеяния электромагнитных волн на системах разных связанных оптических микрорезонаторов. Дано новое определение коэффициентов связи разных микрорезонаторов. Построены новые модели фильтров.

Ключевые слова: инфракрасный диапазон; интегральная оптика; оптический фильтр; полосовой фильтр; режекторный фильтр; микрорезонатор.