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MATHEMATICAL MODEL OF E-LEARNING TRANSACTION PROCESS

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Background. In the case of hyper-converged management, granular updating of the system is impossible. In addition, the processing time of transactions compared to distributed systems increases. Therefore, the task of ensuring the timely execution of transactions is relevant.

Objective. The aim of the paper is to develop a mathematical model of e-learning transactions in a hyper-converged environment supporting electronic educational resources. In this model, timely execution of transaction requests minimizes the costs of the computing resource.

Methods. A two-stage method for constructing a mathematical model is proposed. At the first stage, a set of valid transaction options for resources is defined. At the second stage, there is an optimal time allocation of resources between transactions.

Results. A mathematical model of the transaction process is developed. The adequacy of the model is verified with the help of the simulation model.

Conclusions. A mathematical model of e-learning transactions in a hyper-converged environment supporting electronic educational resources is proposed. This model allows minimizing the cost of a computational resource with timely execution of transaction requests.

Keywords: E-learning; electronic educational resources; hyper-converged platform.

I. Introduction

Application of structured, purposeful use of computer technology to support the learning process (e-learning) is increasingly used in the teaching process of higher education institutions of various countries. So, according to the Babson Survey Research Group in higher education institutions in 2015 in the USA, in varying degrees more than 8 million students were enrolled in online education [1].

The purpose of using e-learning systems is to manage the knowledge of each learner based on a reliable assessment of his knowledge (in the process of training, advanced training, etc.), as well as on the basis of an individual, adaptive learning plan, taking into account the available opportunities and features. E-learning is characterized by the independence of the student's territorial position, flexible individual schedule of the curriculum, the lack of subjectivity of the examiner in assessing knowledge, etc. The consequence of using e-learning is costs reduction, ensuring the quality of the educational process and time saving for students.

The basis of e-learning is electronic educational resources (EER). EER is generally understood as a combination of software, information, technical and organizational support, electronic publications placed on electronic media. EER can be classified according to [2]:

- the purpose of creation - educational, sociocultural, etc.;
- the user category - teacher, student, etc.;
- the form of organization of the educational process - classroom activities, independent educational activities;
- special needs - without restrictions, with limited health possibilities;
- the nature of basic information - textual, elementary audio-visual, multimedia;
- the technology of distribution - local, network, combined distribution;
- the function in the educational process - information, practical, control and other types of EER, etc.

Because of the excessive enthusiasm for the concept of straight and direct use of information and telecommunication Internet capabilities, in contrast to the content aspects of training, many higher education institutions paid little attention to the development and implementation of EER in their educational activities. However, as shown by numerous studies in the field of e-learning [3 - 5], it is impossible to achieve quality education without own EER.

Creation and development of university EER require the investments which are fully unbearable for most universities in Ukraine. Therefore, when creating EER, attention is paid to platforms, which at least partially reduce costs.

Currently, in the IT market, distributed cloud platforms are gradually being replaced by convergent and hyper-converged platforms [6]. The infrastructure created on the converged platform involves the integration of memory, computing and network resources into a pool preconfigured to work in the data center [7], and with a hyper-converged infrastructure, computing power, storage, servers, networks are integrated into a single whole with the help of software tools, and they are managed through a common administration console [8].

With a hyper-converged structure, one system administrator is often sufficient to manage the EER. This significantly reduces the cost of maintaining the system. Therefore, this platform is preferable for university e-learning.

II. Problem formulation

Hyper-converged structures have several advantages over other platforms. One hyper-converged node combines computing resources and data storage resources, which, therefore, leads to the reduction in the number of individual devices and, as a consequence, to the reduction in the number of objects that need to be bought, installed and maintained. In addition, the easier deployment and maintenance of hyper-converged devices are facilitated by the fact that they are based on standard server components. Finally, the availability of integrated management tools in many solutions simplifies administration tasks. However, accordingly, there are a number of shortcomings. So, granular updating or operative reconfiguration of system are impossible. You may experience problems configuring storage disks for a particular application. In addition, despite the higher performance of hyper-converged devices, the processing time of transactions compared to distributed systems is increasing (by e-learning transaction we mean a group of logically combined sequential operations on data processing, processed or canceled entirely). Therefore, the task of ensuring the timely execution of transactions is relevant.

The aim of the article is to propose a formal statement of the task of planning the implementation of e-learning transactions in a hyper-converged environment supporting electronic educational resources, in which timely execution of transaction requests minimizes the costs of the computing resource.

III. Mathematical model of e-learning transactions planning

The initial data for the mathematical model are:

T_z - a predetermined planning time interval,

Δt - is the value of the sampling step for the interval T_z, equal to the minimum continuous time interval available to the e-learning transaction when requesting an EER;

Z - set of transactions of the e-learning system users, serviced during the interval.

In this case, the time interval can be divided into subintervals by points from the set

$$T = \left\{ t_{z_1}, t_{z_2}, \ldots, t_{z_i}, \ldots, t_{z_{h_t}} \right\},$$

Where t_{z_i} - is the beginning of the i-th sub-interval, $0 \le t_{z_i} < T, 1 \le i \le h_t$; h_t - Is the number of timeslots of the time interval T_z. Each transaction $z_b \in Z$, $1 \le b \le h_z$, $card\ Z = h_z$, is characterized by parameters ϕ_{z_b}, $M_\phi^{(\gamma)}$, where ϕ_{z_b} - the size of the computing resource required to obtain the required EER by z_b transaction; $T_{z_b} = \left\{ t_{z_{b_1}}, t_{z_{b_2}} \right\}$ - the time interval during which the required EER should be provided; $t_{z_{b_1}}$ - initial time subinterval of the interval T_{z_b}; $t_{z_{b_2}}$ - a finite time subinterval of the interval T_{z_b}.

Each possible allocation of transactions γ is given by a matrix $M_\varphi^{(\gamma)} = \left(m_{\varphi_{b,i}} \right)$ with the size of $h_z \times h_t$ in which each transaction $z_b \in Z$ is co-supplied with a row vector $\mathrm{m}_{\phi_b} = \left(m_{\phi_{b,1}}, \ldots, m_{\phi_{b,h_t}} \right)$, representing a schedule for allocating computational resources (CR) for transaction z_b processing, where the $m_{\phi_{b,i}}$ component determines the allocated computing resource for the transaction z_b in the i-th time sub-interval.

To determine the quality of fixed distribution γ, we introduce the penalty function for the allocation of the transaction $z_b \in Z$ of the subintervals. A penalty is imposed if the resource is not provided for the time interval during which the required EER should be provided. If the EER request for a transaction z_b, characterized by the processing time interval $T_{z_b} = \{ t_{z_{b_1}}, t_{z_{b_2}} \}$, is performed in the i-th sub-interval, then the penalty corresponding to this element of the matrix $M_\phi^{(\gamma)}$, is determined thus:

$$s_{t_{b,i}} = \begin{cases} 0, & \text{если} \quad t_{z_{b_1}} \le t_{z_i} \le t_{z_{b_2}}; \\ \left(t_{z_{b_1}} - t_{z_i} \right) \Big/ \phi_{z_b}, & \text{если} \quad t_{z_i} < t_{z_{b_1}}; \\ \left(t_{z_i} - t_{z_{b_2}} \right) \Big/ \phi_{z_b}, & \text{если} \quad t_{z_i} > t_{z_{b_2}}. \end{cases} \qquad (2)$$

So, for each transaction $z_b \in Z$, we have a penalty vector $\mathbf{s}_{t_b} = \left(\mathbf{s}_{t_{b_1}}, \dots, \mathbf{s}_{t_{b_{h_t}}} \right)$, in which the component $\mathbf{s}_{t_{b_i}}$, $1 \le i \le h_t$, determines the amount of penalty when allocating a transaction z_b of a unit of ENP into the i-th time quantum. The resulting set of penalty vectors determines the quality of planning in the distribution γ.

The value of the penalty characterizing the obtained distribution γ, allocated by the EER for processing the set of transactions Z determines the quality function

$$F^{(\gamma)} = \sum_{b=1}^{h_z} \sum_{i=1}^{h_t} m_{\phi_{b,i}} \cdot s_{t_{b,i}}. \quad (3)$$

When constructing the distribution of transactions γ EOR by quanta in a given time interval T_z, it is necessary to minimize the value $F^{(\gamma)}$. In this case, the distribution of γ must satisfy the following conditions:

$$\forall z_b \in Z,\ \forall t_{z_i} \in T_{z_b},\ m_{\phi_{b,i}} \ge 0,\ s_{t_{b,i}} \ge 0; \quad (4)$$

$$\forall z_b \in Z, \quad \sum_{i=1}^{h_t} m_{\phi_{b,i}} \le \phi_{z_b}; \quad (5)$$

$$\forall t_{z_i} \in T_z \quad \sum_{b=1}^{h_z} m_{\phi_{b,i}} \le \phi_{t_i}, \quad (6)$$

where ϕ_{t_i} - is the total available computing resource at the i-th sub-interval of the specified time interval T_z.

If we introduce a scale with step Δt in the time interval T_z, then the set of points of the partition (1) can be approximated by a continuous subset of the series of natural numbers: $T \to T_{h_t} = \{1,...,h_t\}$.

We introduce the following vectors:

$\overline{\phi}_z = \left(\phi_{z1}, \phi_{z2}, ..., \phi_{zn_b} \right)$ - vector that determines the required CR for each transaction $z_b \in Z$ to process it;

$\overline{t_{z,1}} = \left(t_{z_{1,1}}, t_{z_{2,1}}, ..., t_{zn_{b,1}} \right)$;

$\overline{t_{z,2}} = \left(t_{z_{1,2}}, t_{z_{2,2}}, ..., t_{zn_{b,2}} \right)$ - vectors indicating for each transaction $z_b \in Z$ the time interval for its processing;

$\overline{\phi}_t = \left(\phi_{t_1}, \phi_{t_2}, ..., \phi_{t_{n_t}} \right)$ - a vector that determines for each sub-interval $t_{z_i} \in T_z$ the total accessible computing resource.

Then the distribution $\Gamma(\gamma)$ obtained under the condition that satisfied constraints (4) - (6) is described by means of a tuple

$$\Gamma(\gamma) = \left\langle Z, \phi_z, T_z, \phi_t, M_\phi^{(\gamma)}, F^{(\gamma)} \right\rangle. \quad (7)$$

On the set of all possible distributions $\{\Gamma\}$ we construct a subset $\Im$ in the following way:

$$\Im = \left\{ \Gamma(\gamma) \middle| F^{(\gamma)} = 0 \right\}; \quad (8)$$

$$card\ \Im = \varnothing \Rightarrow$$

$$\Im = \left\{ \Gamma(\gamma^*) \middle| F^{(\gamma^*)} = \min_\gamma F^{(\gamma)} \right\}. \quad (9)$$

As a result, the mathematical model of the e-learning transaction planning is determined by the formulae (1) - (9).

IV. SETTING THE TASK OF SELECTION OF E-LEARNING TRANSACTION PLAN

The transaction execution plan is based on the developed mathematical model.

If $card\ \Im = 1$, then the distribution, which is its only element, is the exact plan.

In case $card\ \Im > 1$, then it is necessary to make a choice from its elements, each of which meets the requirements to transaction requests e-learning.

Define the value of the maximum total allocated CR, which is one sub-interval of the given time interval T_z in the distribution $\gamma \in \Im$ for all transactions of the set Z:

$$m_{\phi_{\max}}^{(\gamma)} = \max_{i=1,\dots,h_t} \sum_{b=1}^{h_z} m_{\phi_{b,i}}$$

In this case, the average value of the minimum total required CR, per one subinterval of the interval T_z, can be defined as

$$\phi_{z_{cp}} = \frac{1}{h_t} \sum_{b=1}^{h_z} \phi_{z_b}$$

To select the required transaction plan, we will provide further detail of the hyper-converged infrastructure under consideration (HCI).

Consider a subset of HCI nodes involved in processing transactions from the set Z –

$$Y = \left\{ y_1, \dots, y_a, \dots y_{h_y} \right\} \ card\ Y = h_t$$

Then, within the fixed distribution γ , one can define the transaction intensity matrix of the set Z with the nodes of the set Y: $U_z = \left\| u_{z_{b,i}} \right\|$, in which each element $u_{z_{b,i}}$ - is the transaction traffic intensity $z_b \in Z$ with the node y_i, calculated as $u_{z_{b,j}} = \sum_{i=1}^{h_t} \left(m_{\varphi_{b,i}} \cdot I_{b,j,i} \right)$ where $I_{b,j,i}$ - is a boolean function equal to 0 if and only if the processing of the transaction $z_b \in Z$ request by the node $y_i \in Y$ does not fall into the i-th time interval.

For each column vector $m_{\phi_i} = \left(m_{\phi_{1,i}}, \ldots, m_{\phi_{h_z,i}} \right)$ of the matrix $M_{\phi}^{(\gamma)}$ defining the allocated BP for processing the transactions of the set Z in the i-th subinterval of the given time interval, it is necessary to find such partitioning of the set of transactions Z into subsets and their distribution over nodes of the hyper-converged infrastructure (HCI), so that the total execution time of the queries transactions took on a minimal value. Then the objective function of the task of searching for the rational partitioning of the set of transactions Z, whose requests are processed by elements of the hyper-converged infrastructure Y, into subsets and their distributions over the nodes $y_a \in Y$, is defined by the formula:

$$F^{(\gamma)} = \frac{1}{u_{z_{\max}}} \cdot \sum_{b=1}^{h_z} \sum_{a=1}^{h_y} m_{z_{b,a}} \cdot s_{y_{b,a}} \quad (10)$$

Where $u_{z_{\max}}$ is a value independent of the distribution $\Gamma(\gamma)$, which determines the maximum aggregate intensity of the transaction exchange with the nodes Y in accordance with the formula $u_{z_{\max}} = \sum_{b=1}^{h_z} \sum_{i=1}^{h_y} u_{z_{b,i}}$; $m_{z_{b,a}}$ - CR of node y_a, necessary for transaction processing z_b ; $s_{y_{b,a}}$ - penalty for distributing the transaction $z_b \in Z$ to the node $y_a \in Y$, defined by the formula $s_{y_{b,a}} = \sum_{i=1}^{h_y} \left(u_{z_{b,i}} \cdot h_{wa,i} \right) / \phi_{z_b}$; $h_{wa,i}$ - the length of the shortest route between the nodes y_a and y_i. The resulting distribution γ must satisfy the following conditions:

$$\forall y_a \in Y \left| \sum_{b=1}^{h_z} m_{z_{b,a}} \le \phi_{y_a} \right. ; \quad (11)$$

$$\forall z_b \in Z \left| \sum_{a=1}^{h_y} m_{z_{b,a}} \le \phi_{z_b} \right. ; \quad (12)$$

$$\sum_{a=1}^{h_y} \phi_{y_a} \ge \sum_{b=1}^{h_z} \phi_{z_b} ; \quad (13)$$

$$s_{y_{b,a}} \ge 0, \; m_{z_{b,a}} \ge 0$$

$$\text{for } \; 1 \le a \le h_y, \; 1 \le b \le h_z, \quad (14)$$

where ϕ_{y_a} - the available CR of the node ya $\in$ Y.

In view of the above conditions, the problem of finding a rational partition of the set of Z transactions processed in HCI into subsets and their distributions over nodes $y_a \in Y$ can be formulated as follows. Let the sets of transactions Z and Y nodes of the HCI, given by tuples

$$\langle Z, \phi_z, U_z \rangle \text{ and } \langle Y, \phi_y, H_w \rangle ,$$

where $\phi_z = \left(\phi_{z_1}, \ldots, \phi_{z_{h_z}} \right)$ is the vector of the required BP for processing the set of transactions Z; $U_z = \left\| u_{z_{b,i}} \right\|$ - transaction intensity matrix of the set Z with nodes of the set Y; $\phi_y = \left(\phi_{y_1}, \ldots, \phi_{y_{h_y}} \right)$ - the vector of the accessible BP of the set of nodes Y of the HCI; $H_w = \left\| h_{w_{a,i}} \right\|$ - the matrix of the lengths of the shortest routes between each pair of HCI nodes y_a and y_i, $1 \le a \le h_y$, $1 \le i \le h_y$.

It is required to find a distribution $\gamma \in \Im$, satisfying conditions (11) - (14), so that formula (10) assumes a minimum value.

V. Discussion of the results

To test the theoretical provisions of the third and fourth sections, a software package was developed that simulates the distribution of the computational resource in the hyper-converged environment of supporting EER. The planning time interval was chosen equal to the academic hour (45 minutes).

For fixed values h_y. h_z, ϕ_{y_a}, ϕ_{z_b}, ϕ_{t_i} the graduation of the scale T_z was changed. The results of the experiment are given in Table. 1 (τ - is the calculation time of the plan, the choice of the optimal distribution was carried out by the method of full search).

TABLE 1. EXPERIMENTAL RESULTS

h_t	card $\Im$	F(γ)	τ
$h_z = 20, h_y = 10$			
15	1	1	2
45	5	0,92	19
90	32	0,88	78
180	47	0,87	322
$h_z = 100, h_y = 30$			
15	3	0,95	124
45	124	0,88	
90	2842	–[a]	–[a]
180	–[a]	–[a]	–[a]

* Results were not received within an astronomical hour

The results of modeling allowed forming requirements to the developed algorithm for solving the problem of planning implementation of e-learning transactions in a hyper-converged environment supporting electronic educational resources:

to solve the problem of choosing the optimal plan from the set of permissible it is necessary to develop an approximate method that allows finding a solution close to optimal within acceptable time periods;

it is necessary to find the optimal graduation of the scale of the planning time interval, since too small scale significantly increases the calculation time, but does not lead to a significant improvement in the plan;

it is necessary to take into account the priorities of transactions, especially in the case of obtaining in the first stage an empty set of admissible solutions.

VI. CONCLUSIONS

The formal formulation of the task of planning the implementation of e-learning transactions in a hyper-converged environment supporting electronic educational resources has been proposed. The solution to this problem allows minimizing the costs of computing resource with the timely execution of transaction requests.

At the first stage of the research, the mathematical model for e-learning transaction planning was developed, which allows obtaining a set of feasible execution plans.

The second stage of the research allowed us to formulate the problem of choosing the optimal plan from the set of computational resources that are admissible by the criterion of minimization.

The direction of further research is related to the search for a method for solving the formulated problem, taking into account the requirements specified in the previous section.

References

1. "Babson Survey Research Group. Online Learning", available at: https://www.onlinelearningsurvey.com (accessed 23 April 2017).
2. Udaya, Sri, Vamsi, K. and Krishna T. V. (2014), "E-Learning: Technological Development in Teaching for school kids", International Journal of Computer Science and Information Technologies, pp. 6124—6126.
3. "Babson Study: Over 6.7 Million Students Learning Online", available at: http://sloanconsortium.org/news_press/january2013_new-study-over-67-million-students-learning-online (accessed 23 April 2017).
4. Welsh, E, Wanberg, C, Brown, K and Simmering, M (2003), "E-Learning: emerging issues, empirical results and future directions", International Journal of Training and Development, 8 (4), pp. 245-258.
5. Brodo, J. A. (2006) "Today's Ecosystem of E-learning", Trainer Talk. Vol. 3, No. 4, available at: : http://enewsbuilder.net/salesmarketing/ e_article000615779.cfm (accessed 23 April 2017).
6. White Paper: Riverbed Hyper-converged Edge, available at: https://www.riverbed.com/document-repository/white-paper--riverbed-hyper-converged-edge.html (accessed 23 April 2017).
7. Черняк, Л. (2012), "Время конвергентных инфраструктур", Открытые системы. СУБД, № 4, available at: https://www.osp.ru/os/2012/04/13015754/ (accessed 23 April 2017).
8. Ганьжа, Д. (2016), "Гиперконвергенция: ИТ-инфраструктура на раз, два, три", /Журнал сетевых решений,. № 5, available at: www.osp.ru/lan/2016/05/13049349 (accessed 23 April 2017).

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Математична модель процесу виконання транзакцій e-learning

Проблематика. При гіперконвергентному управлінні неможливо гранулярне оновлення системи. Крім того, час обробки транзакцій в порівнянні з розподіленими системами збільшується. Тому завдання забезпечення своєчасного виконання транзакцій є актуальним.

Мета досліджень. Розробити математичну модель процесу виконання транзакцій e-learning в гіперконвергентному середовищі підтримки електронних освітніх ресурсів. У даній моделі при своєчасному виконанні запитів транзакцій мінімізуються витрати обчислювального ресурсу.

Методика реалізації. Запропоновано двоетапна методика побудови математичної моделі. На першому етапі визначається безліч допустимих по ресурсам варіантів виконання транзакцій. На другому етапі знаходиться оптимальне за часом розподіл ресурсів між транзакціями.

Результати досліджень. Розроблено математичну модель процесу виконання транзакцій. Адекватність моделі перевірено за допомогою імітаційної моделі.

Висновки. Запропоновано математичну модель процесу виконання транзакцій e-learning у гіпорконвергентному середовищі підтримки електронних освітніх ресурсів. Дана модель дозволяє при своєчасному виконанні запитів транзакцій мінімізувати витрати обчислювального ресурсу.

Ключові слова: E-learning; електронні освітні ресурси; гіперконвергентна платформа.

Шматков С.И., Кучук Н.Г.

Математическая модель процесса выполнения транзакций e-learning

Проблематика. При гиперконвергентном управлении невозможно гранулярное обновление системы. Кроме того, время обработки транзакций по сравнению с распределенными системами увеличивается. Поэтому задача обеспечения своевременного выполнения транзакций является актуальной.

Цель исследований. Разработать математическую модель процесса выполнения транзакций e-learning в гиперконвергентной среде поддержки электронных образовательных ресурсов. В данной модели при своевременном выполнении запросов транзакций минимизируются затраты вычислительного ресурса.

Методика реализации. Предложена двухэтапная методика построения математической модели. На первом этапе определяется множество допустимых по ресурсам вариантов выполнения транзакций. На втором этапе находится оптимальное по времени распределение ресурсов между транзакциями.

Результаты исследований. Разработана математическая модель процесса выполнения транзакций. Адекватность модели проверена при помощи имитационной модели.

Выводы. Предложена математическая модель процесса выполнения транзакций e-learning в гипорконвергентной среде поддержки электронных образовательных ресурсов. Данная модель позволяет при своевременном выполнении запросов транзакций минимизировать затраты вычислительного ресурса.

Ключевые слова. E-learning; электронные образовательные ресурсы; гиперконвергентная платформа.